Ray Tracing

Reading

Foley *et al.*, 16.12

Optional:

- Glassner, An introduction to Ray Tracing, Academic Press, Chapter 1.
- T. Whitted. “An improved illumination model for shaded display”. *Communications of the ACM* 23(6), 343-349, 1980.

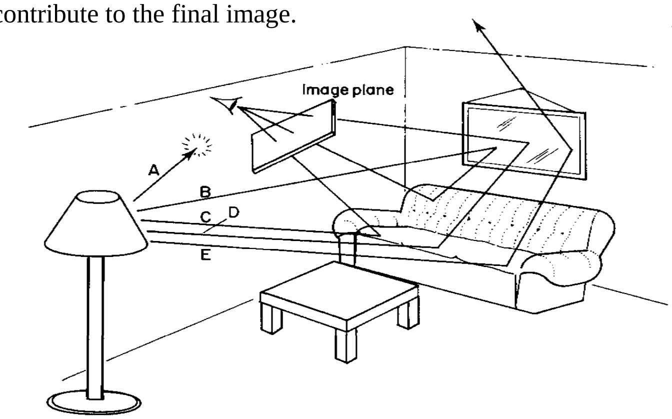
Geometric optics

We will take the view of **geometric optics**

- Light is a flow of photons with wavelengths. We'll call these flows ``light rays."
- Light rays travel in straight lines in free space.
- Light rays do not interfere with each other as they cross.
- Light rays obey the laws of reflection and refraction.
- Light rays travel form the light sources to the eye, but the physics is invariant under path reversal (reciprocity).

Forward Ray Tracing

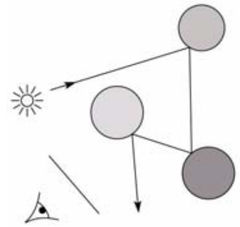
- Rays emanate from light sources and bounce around in the scene.
- Rays that pass through the projection plane and enter the eye contribute to the final image.



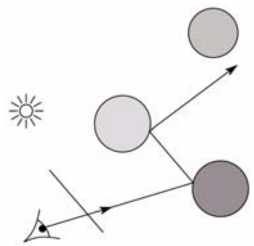
- What’s wrong with this method?

Eye vs. Light

- Starting at the light (a.k.a. forward ray tracing, photon tracing)

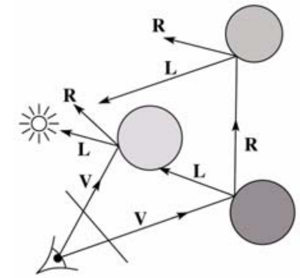


- Starting at the eye (a.k.a. backward ray tracing)



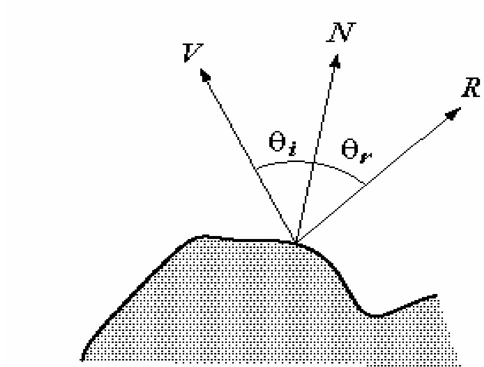
Whitted ray-tracing algorithm

1. For each pixel, trace a **primary ray** to the first visible surface
2. For each intersection trace **secondary rays**:
   - **Shadow rays** in directions Li to light sources
   - **Reflected ray** in direction R
   - **Refracted ray** (**transmitted ray**) in direction T

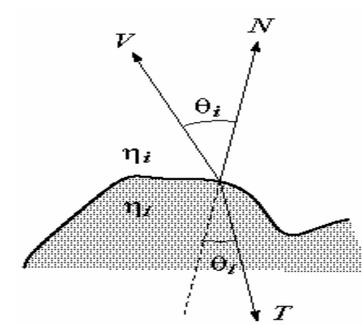


Reflection

- Reflected light from objects behaves like specular reflection from light sources
  - Reflectivity is just specular color
  - Reflected light comes from direction of perfect specular reflection



Refraction

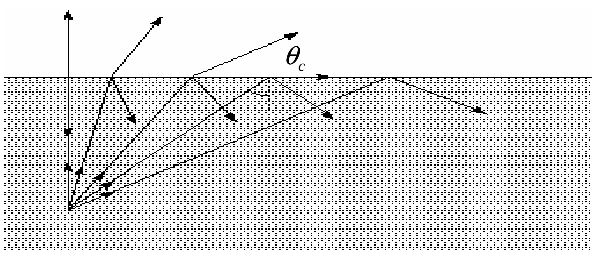


- Amount to transmit determined by transparency coefficient, which we store explicitly
- *T* comes from Snell’s law

$$\eta_i \sin(\theta_i) = \eta_t \sin(\theta_t)$$

Total Internal Reflection

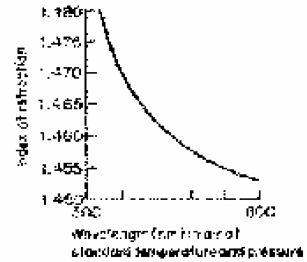
- When passing from a dense medium to a less dense medium, light is bent further away from the surface normal
- Eventually, it can bend right past the surface!
- The θ_i that causes θ_t to exceed 90 degrees is called the **critical angle (θ_c)**. For θ_i greater than the critical angle, no light is transmitted.
- A check for TIR falls out of the construction of T



Index of Refraction

- Real-world index of refraction is a complicated physical property of the material

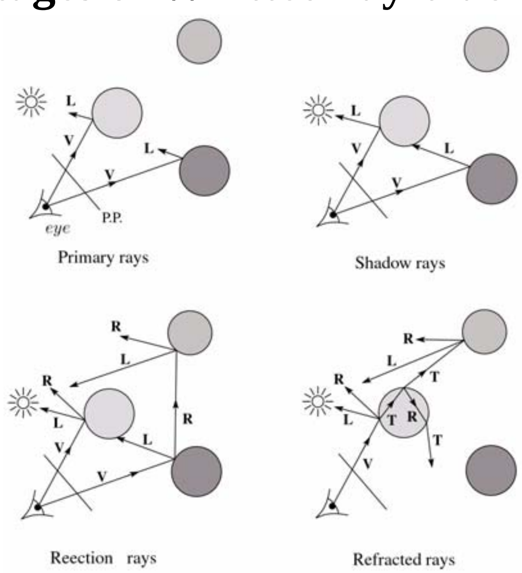
Medium	Index of refraction
Vaccum	1
Air	1.0003
Water	1.33
Fused quartz	1.46
Glass, crown	1.52
Glass, dense flint	1.66
Diamond	2.42



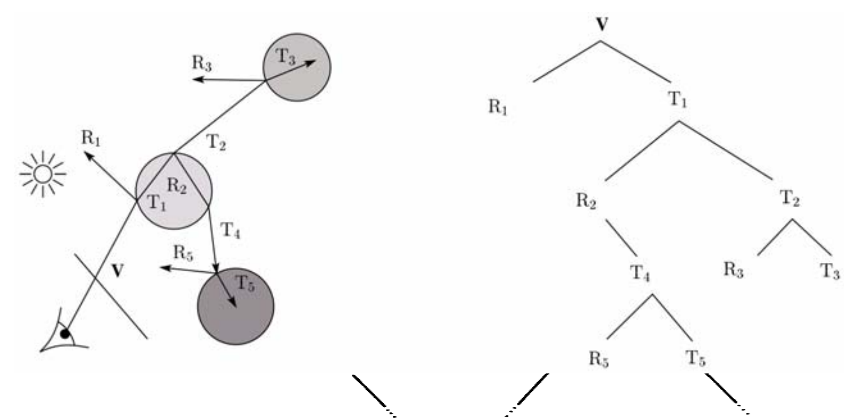
Index of refraction variation for fused quartz

- IOR also varies with wavelength, and even temperature!
- How can we account for wavelength dependence when ray tracing?

Stages of Whitted ray-tracing



The Ray Tree



Shading

If $I(P_0, \mathbf{u})$ is the intensity seen from point P_0 along direction $\mathbf{u}$

$$I(P_0, \mathbf{u}) = I_{direct} + I_{reflected} + I_{transmitted}$$

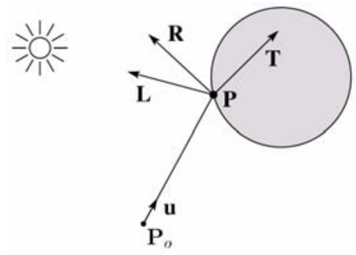
where

I_{direct} = Shade(**N**, **L**, **u**, **R**) (e.g. Phong shading model)

$$I_{reflected} = k_r I(P, \mathbf{R})$$

$$I_{transmitted} = k_t I(P, \mathbf{T})$$

Typically, we set $k_r = k_s$ and k_t



Parts of a Ray Tracer

- What major components make up the core of a ray tracer?
  - Outer loop sends primary rays into the scene
  - Trace arbitrary ray and compute its color contribution as it travels through the scene
  - Shading model

$$I = k_e + k_a I_a + \sum_i f(d_i) I_{li} \left[k_d (\mathbf{N} \cdot \mathbf{L}_i)_+ + k_s (\mathbf{V} \cdot \mathbf{R})_+^{n_s} \right]$$

Outer Loop

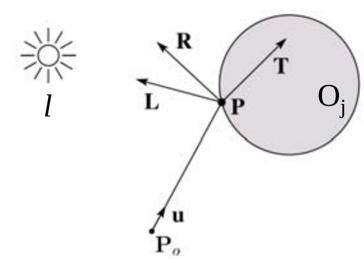
```
void traceImage (scene)
{
        for each pixel (i,j) in the image {

                p = pixelToWorld(i,j)
                c = COP
                u = (p - c)/||p - c||
                I(i,j) = traceRay (scene, c, u)

        }
}
```

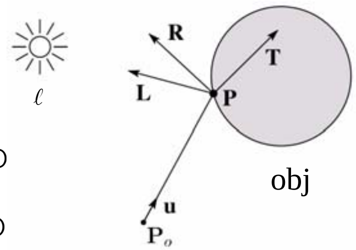

Trace Pseudocode

```
color traceRay(point P0, direction u )
{
        (P,Oi) = intersect( P0, u);
        I = 0
        for each light source l {
                (P', LightObj) = intersect(P, dir(P,l))
                if LightObj = l {
                        I = I + I(l)
                }
        }
        I = I + Obj.Kr * traceRay(P, R)
        I = I + Obj.Kt * traceRay(P, T)
        return I
}
```



TraceRay Pseudocode

```
function traceRay(scene, P_0, u) {
   (t, P, N, obj)  ← scene.intersect (P_0, u)
   I = shade( u, N, scene )
   R = reflectDirection( u, N )
   I ← I + obj.k_r * traceRay(scene, P, R)
   if ray is entering object {
      (n_i, n_t) ← (index_of_air, obj.index)
   } else {
      (n_i, n_t) ← (obj.index, index_of_air)
   }
   if (notTIR ( u, N, n_i, n_t ) {
      T = refractDirection ( u, N, n_i, n_t )
      I ← I + obj.k_t * traceRay(scene, P, T)
   }
   return I
}
```

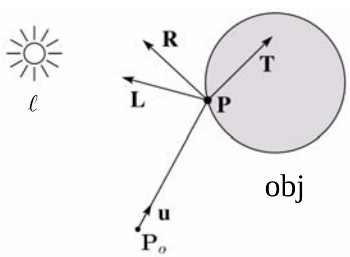


Controlling Tree Depth

- Ideally, we'd spawn child rays at every object intersection forever, getting a "perfect" color for the primary ray.
- In practice, we need heuristics for bounding the depth of the tree (i.e., recursion depth)
- ?

Shading Pseudocode

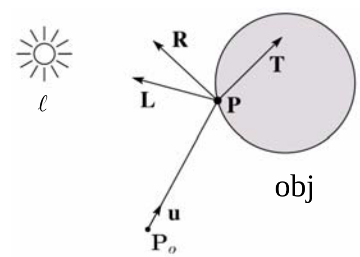
```
function shade(obj, scene, P, N, u) {
   I ← obj.k_e + obj. k_a * scene->I_a
   for each light source ℓ {
      atten = distanceAttenuation(ℓ, P) *
         shadowAttenuation(ℓ, Scene, P)
      I ← I + atten*(diffuse term + spec term)
   }
   return I
}
```



Shadow attenuation pseudocode

Check to see if a ray makes it to the light source.

```
function shadowAttenuation(ℓ, scene, P) {
   d = (ℓ.position - P).normalize()
   (t, P_1, N, obj) ← scene.intersect(P, d)
   if P_1 is before the light source {
      atten = 0
   } else {
      atten = 1
   }
   return atten
}
```

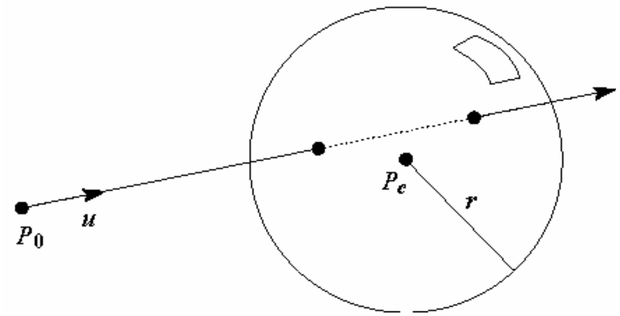


Q: What if there are transparent objects along a path to the light source?

Ray-Object Intersection

- Must define different intersection routine for each primitive
- The bottleneck of the ray tracer, so make it fast!
- Most general formulation: find all roots of a function of one variable
- In practice, many optimized intersection tests exist (see Glassner)

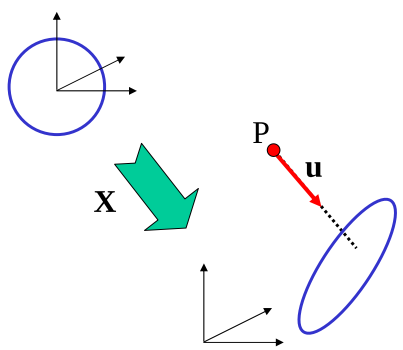
Ray-Sphere Intersection



- Given a sphere centered at P_c =[0,0,0] with radius r and a ray $P(t) = P_0 + t\boldsymbol{u}$, find the intersection(s) of $P(t)$ with the sphere.

Object hierarchies and ray intersection

How do we intersect with primitives transformed with affine transformations?



$$\mathbf{u}' = \begin{bmatrix} u_x \\ u_y \\ u_z \\ 0 \end{bmatrix} \mathbf{X}^{-1}$$

$$P' = \begin{bmatrix} P_x \\ P_y \\ P_z \\ 1 \end{bmatrix} \mathbf{X}^{-1}$$

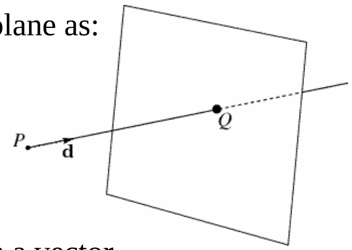
Numerical Error

- Floating-point roundoff can add up in a ray tracer, and create unwanted artifacts
  - Example: intersection point calculated to be ever-so-slightly *inside* the intersecting object. How does this affect child rays?
- Solutions:
  - Perturb child rays
  - Use global ray epsilon

Plane Intersection

- We can write the equation of a plane as:

$$ax + by + cz + d = 0$$

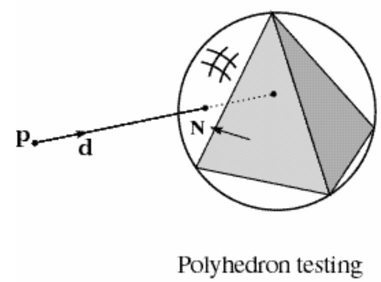


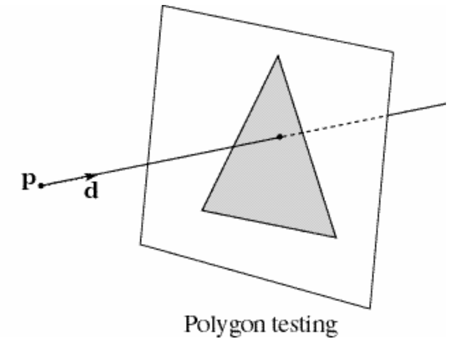
- The coefficients a, b, and c form a vector that is normal to the plane, $\mathbf{n} = [a\ b\ c]^T$. Thus, we can re-write the plane equation as:

$$\mathbf{n} \cdot (\mathbf{P} + t\mathbf{u}) + d = 0$$

- We can solve for the intersection parameter (and thus the point):

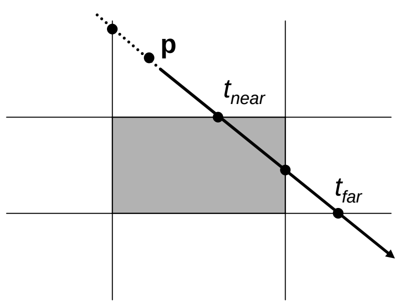
Ray-Polymesh Intersection

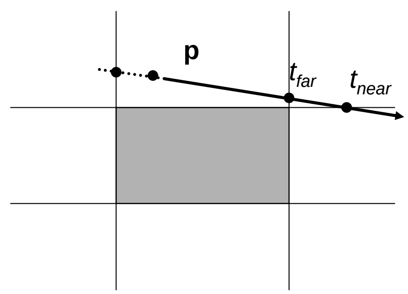




1. Use bounding sphere for fast failure
2. Test only front-facing polygons
3. Intersect ray with each polygon's supporting plane
4. Use a point-in-polygon test
5. Intersection point is smallest t

Axis-Aligned Cube Intersection





- for each pair of parallel planes, compute t intersection values for both
- Let t_{near} be the smaller, t_{far} be the larger
- let t_1 = largest t_{near}, t_2 = smallest t_{far}
- ray intersections cube if $t_1 <= t_2$
- intersection point given by t_1

Goodies

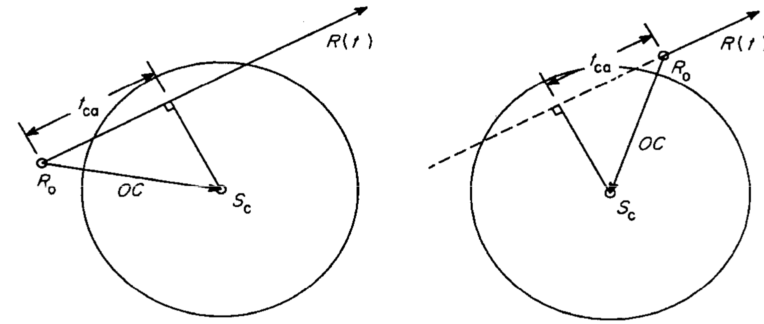
- There are some advanced ray tracing feature that self-respecting ray tracers shouldn't be caught without:
  - Acceleration techniques
  - Antialiasing
  - CSG
  - Distribution ray tracing

Acceleration Techniques

- Problem: ray-object intersection is very expensive
  - make intersection tests faster
  - do fewer tests

Fast Failure

- We can greatly speed up ray-object intersection by identifying cheap tests that guarantee failure
- Example: if origin of ray is outside sphere and ray points away from sphere, fail immediately.

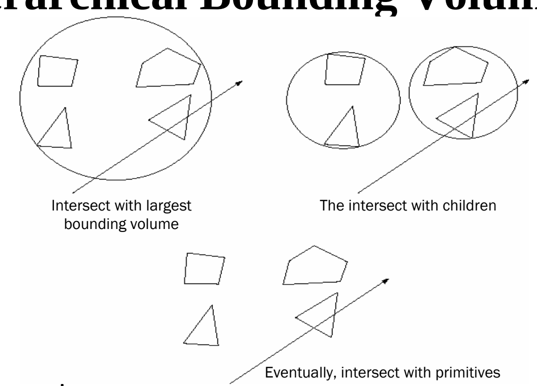


$t_{ca} > 0$, so the ray points toward the sphere

$t_{ca} < 0$, so the ray points away from the sphere

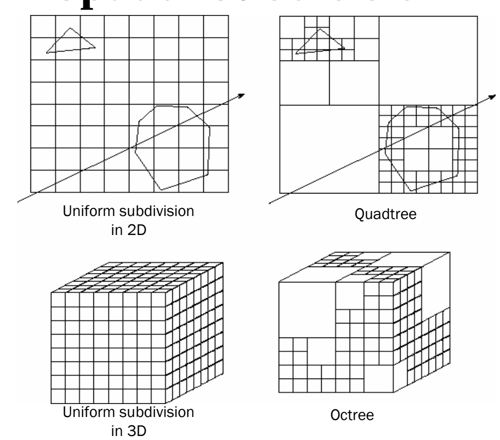
- Many other fast failure conditions are possible!

Hierarchical Bounding Volumes



- Arrange scene into a tree
  - Interior nodes contain primitives with very simple intersection tests (e.g., spheres). Each node's volume contains all objects in subtree
  - Leaf nodes contain original geometry
- Like BSP trees, the potential benefits are big but the hierarchy is hard to build

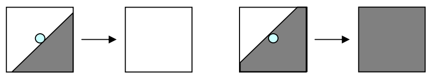
Spatial Subdivision



- Divide up space and record what objects are in each cell
- Trace ray through **voxel** array

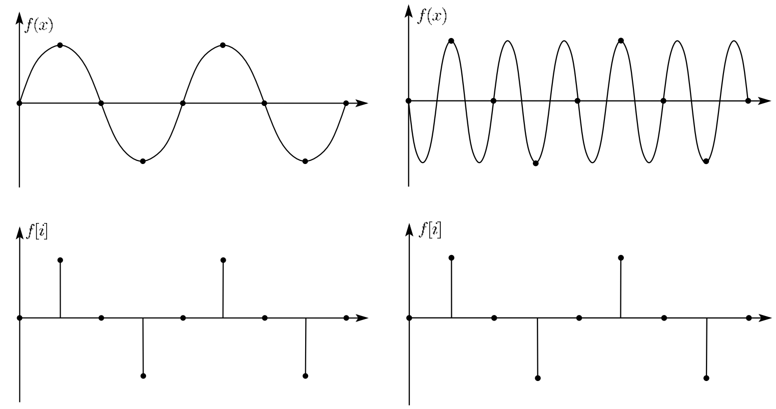
Antialiasing

- So far, we have traced one ray through each pixel in the final image. Is this an adequate description of the contents of the pixel?



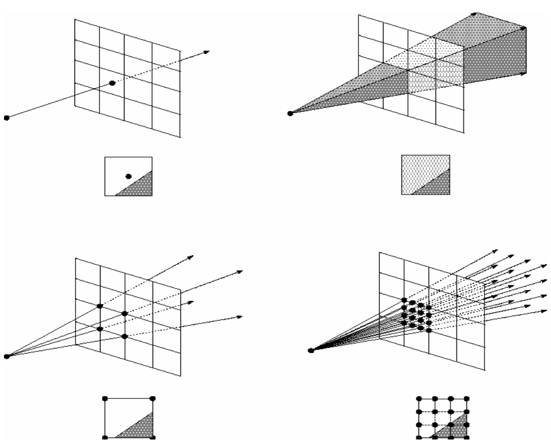
- This quantization through inadequate sampling is a form of **aliasing**. Aliasing is visible as "jaggies" in the ray-traced image.
- We really need to colour the pixel based on the *average*

Aliasing



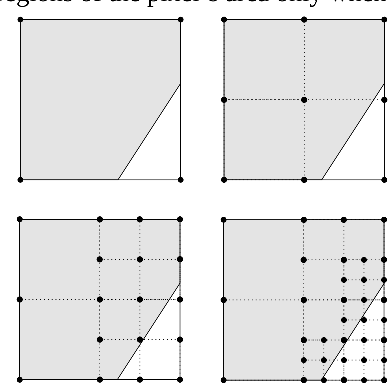
Supersampling

- We can approximate the average colour of a pixel's area by firing multiple rays and averaging the result.



Adaptive Sampling

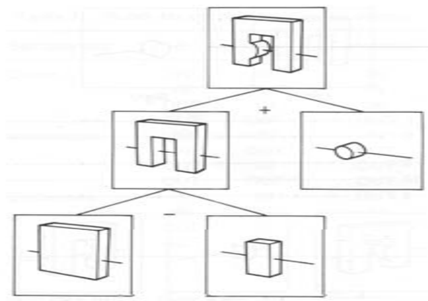
- Uniform supersampling can be wasteful if large parts of the pixel don't change much.
- So we can subdivide regions of the pixel's area only when the image changes in that area:



- How do we decide when to subdivide?

CSG

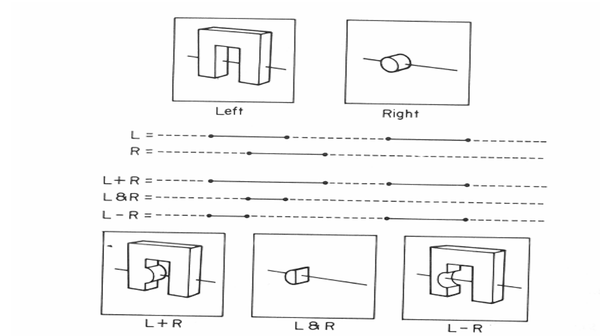
- CSG (constructive solid geometry) is an incredibly powerful way to create complex scenes from simple primitives.



- CSG is a modeling technique; basically, we only need to modify ray-object intersection.

CSG Implementation

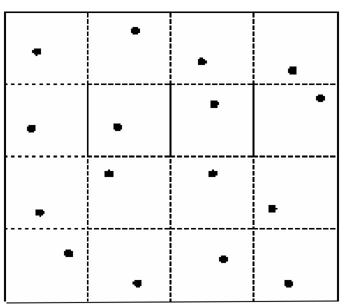
- CSG intersections can be analyzed using “Roth diagrams”.
  – Maintain description of *all intersections* of ray with primitive
  – Functions to combine Roth diagrams under CSG operations



- An elegant and extremely slow system

Distribution Ray Tracing

- Usually known as “distributed ray tracing”, but it has nothing to do with distributed computing
- General idea: instead of firing one ray, fire multiple rays in a jittered grid



- Distributing over different dimensions gives different effects
- Example: what if we distribute rays over pixel area?

Noise

•**Noise** can be thought of as randomness added to the signal.

•The eye is relatively insensitive to noise.

DRT pseudocode

traceImage() looks basically the same, except now each pixel records the average color of jittered sub-pixel rays.

```
function traceImage (scene):
    for each pixel (i, j) in image do
        I(i, j) ← 0
        for each sub-pixel id in (i,j) do
            s ← pixelToWorld(jitter(i, j, id))
            p ← COP
            u ←(s - p).normalize()
            I(i, j) ← I(i, j) + traceRay(scene, p, u, id)
        end for
        I(i, j) ← I(i, j)/numSubPixels
    end for
end function
```

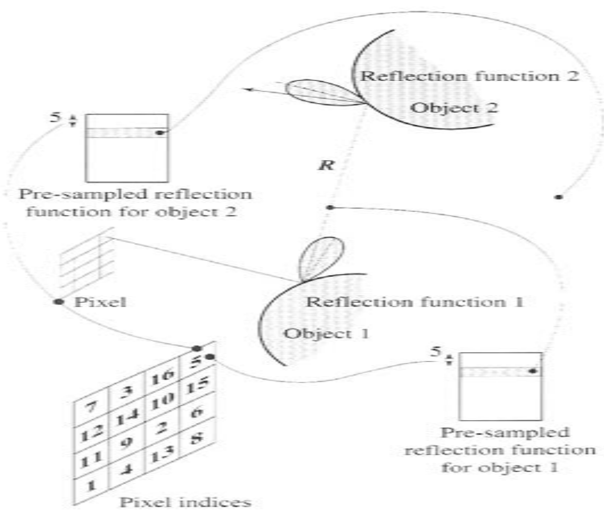
•A typical choice is numSubPixels = 4*4.

DRT pseudocode (cont'd)

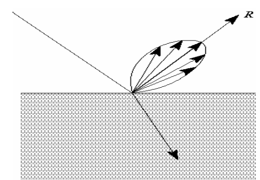
•Now consider *traceRay*(), modified to handle (only) opaque glossy surfaces:

```
function traceRay(scene, p, u, id):
    (q, N, obj) ← intersect (scene, p, u)
    I ← shade(…)
    R ← jitteredReflectDirection(N, -u, id)
    I ← I + obj.k_r * traceRay(scene, q, R, id)
    return I
end function
```

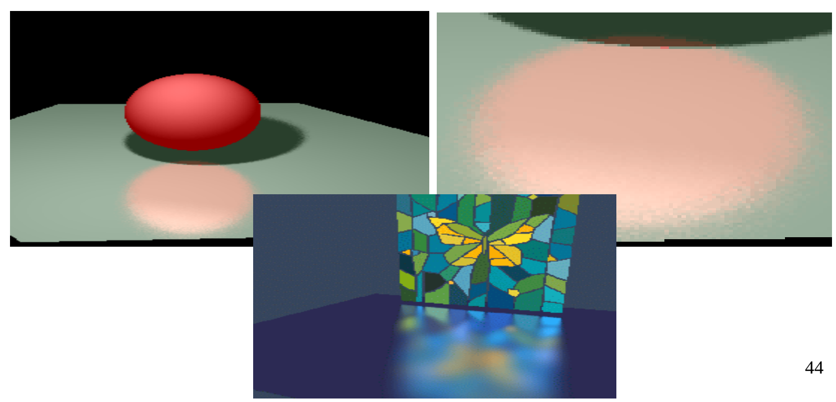

Pre-sampling glossy reflections



Distributing Reflections

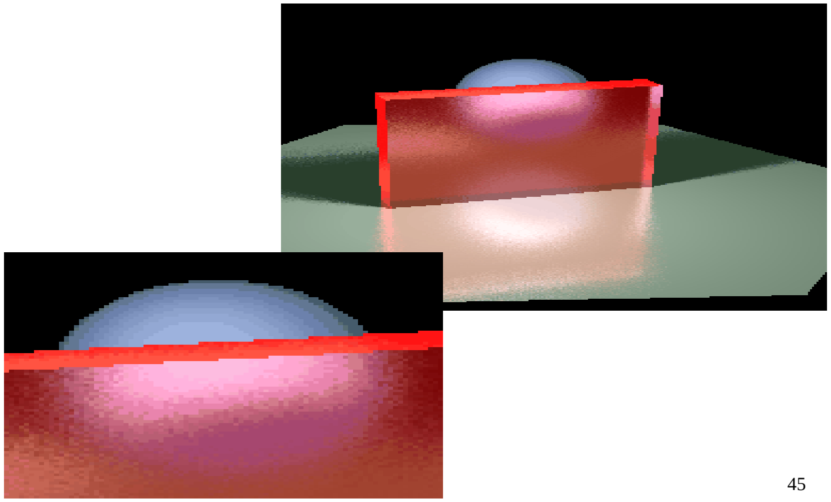


- Distributing rays over reflection direction gives:



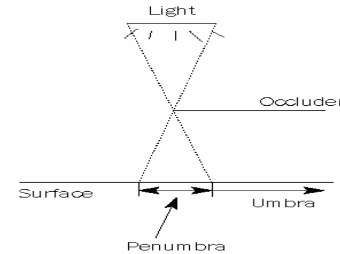
Distributing Refractions

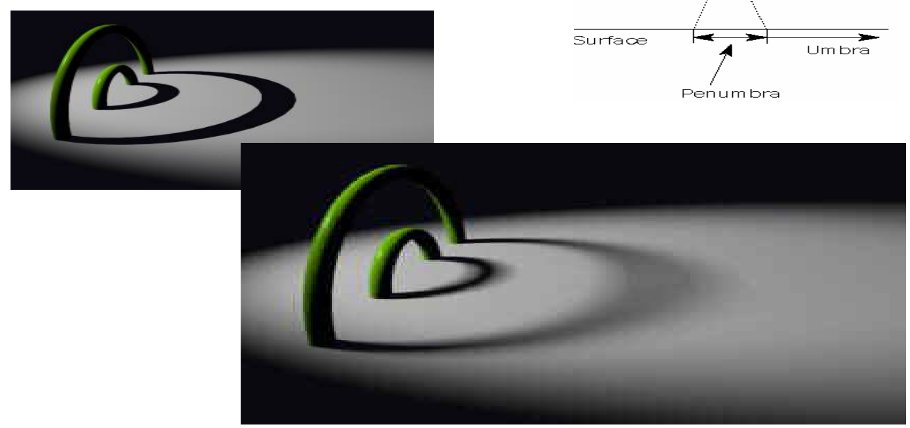
- Distributing rays over transmission direction gives:



Distributing Over Light Area

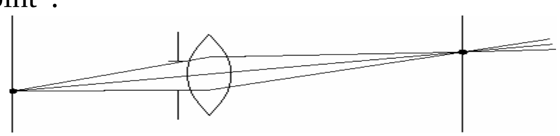
- Distributing over light area gives:

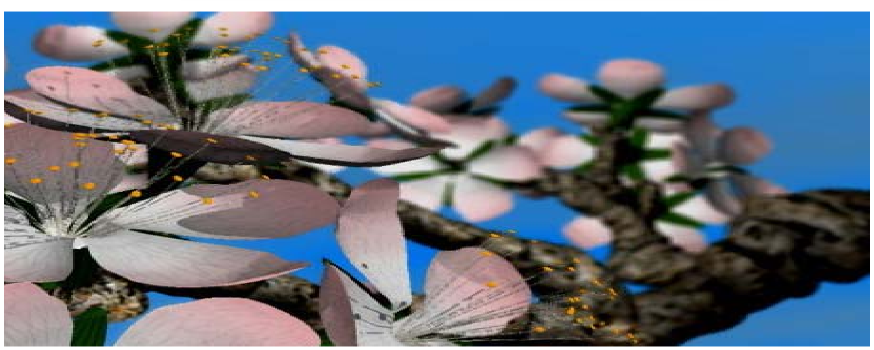




Distributing Over Aperature

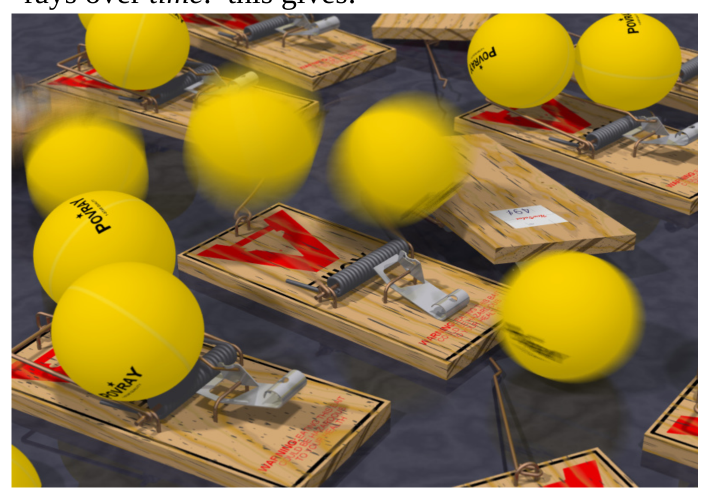
- We can fake distribution through a lens by choosing a point on a finite aperature and tracing through the “in-focus point”.

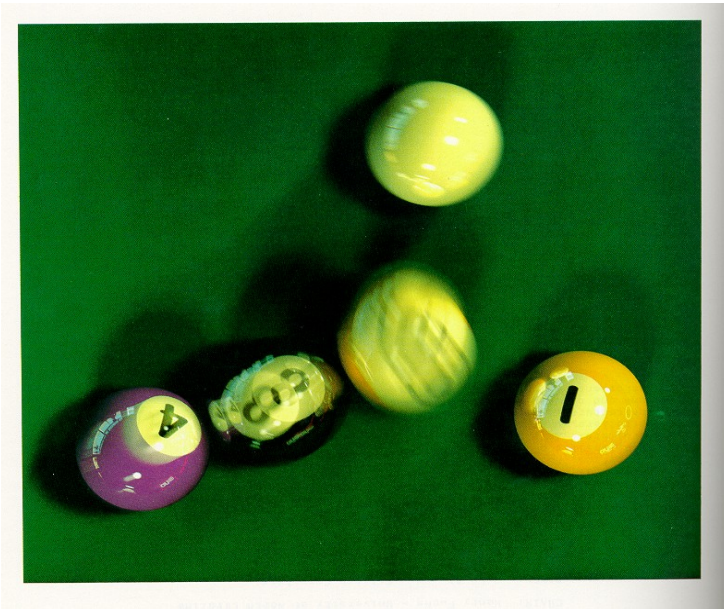




Distributing Over Time

- We can endow models with velocity vectors and distribute rays over *time*. this gives:





Chaining the ray id's

- In general, you can trace rays through a scene and keep track of their id's to handle *all* of these effects: