Homework #5

February 27, 1998

1. Let $RT_{\text{start}}$ and $DL$ be real-time bounds, and $S$ a program statement.

(a) Argue informally the truth of

$$\{ (RT = RT_{\text{start}}) \land (T(S) \in DL) \} \implies RT \in (RT_{\text{start}} + DL)$$

After the execution of $S$, $RT$ becomes $RT_{\text{start}} + T(S)$. But $T(S) \in DL$ implies $RT_{\text{start}} + T(S) \in RT_{\text{start}} + DL$. That is, $RT \in RT_{\text{start}} + DL$.

(b) Give bounds on $rt_{\text{min}}$ and $rt_{\text{max}}$ after the execution of $S$, assuming the postcondition of (a)

Since $RT$ is in $RT_{\text{start}} + DL$, we have $rt_{\text{min}} \geq rt_{\text{start-min}} + dl_{\text{min}}$ and $rt_{\text{max}} \leq rt_{\text{start-max}} + dl_{\text{max}}$.

2.

(a) $[x, y] = [20, 30] - [10, 15] = [5, 20]$

(b) $[x, y] + [10, 20] = [35, 55]$

   $x+10 = 35$ so $x = 25$

   $y+20 = 55$ so $y = 35$

   Therefore $[x, y] = [25, 35]$

(c) $[x, y] - [3, 12] = [5, 20]$

   $x-12 = 5$ so $x = 17$

   $y-3 = 20$ so $y = 23$

   Therefore $[x, y] = [17, 23]$

3. Let $CT$ be an interval that bounds the value returned by a call to $Clock()$. Let $RT_{\text{start}} = [rt_{\text{start-min}}, rt_{\text{start-max}}]$ be bounds on the value of real-time at the start event of a call to $Clock()$, and let $RT_{\text{end}} = [rt_{\text{end-min}}, rt_{\text{end-max}}]$ be bounds on the value of real-time at the end event of the same call. Prove that $CT = RT_{\text{end}} + [-t_{\text{min}}(Clock()), 0] + E$.

Since $RT_{\text{end}} = RT_{\text{start}} + T(Clock())$, it follows that

$$rt_{\text{end-min}} = rt_{\text{start-min}} + t_{\text{min}}(Clock())$$

(1)
Inside `Clock()`, the value of real-time can only be sampled some time between the start and end events of the call to `Clock()` and hence the value of computer time returned from a call to `Clock()` will be in the range

\[
CT = [rt_{\text{start-min}} - \epsilon, rt_{\text{end-max}} + \epsilon] = [rt_{\text{start-min}}, rt_{\text{end-max}}] + E \tag{2}
\]

Substituting \(rt_{\text{end-min}} - t_{\text{min}}(Clock())\) from (1) for \(rt_{\text{start-min}}\) into (2) yields

\[
CT = [rt_{\text{start-min}}, rt_{\text{end-max}}] + E
\]
\[
= [rt_{\text{end-min}} - t_{\text{min}}(Clock()), rt_{\text{end-max}}] + E
\]
\[
= [rt_{\text{end-min}}, rt_{\text{end-max}}] + [-t_{\text{min}}(Clock()), 0] + E
\]
\[
= RT_{\text{end}} + [-t_{\text{min}}(Clock()), 0] + E
\]

4.

(a) \(RT = RT_{\text{start}} + n(T(TS) + T(FH) + T(SR) + T(\text{delay(50)}) + T(\text{loop}))\)

(b) \(T(TS) + T(FH) + T(SR) + T(\text{delay(50)}) + T(\text{loop}) \leq 100\)

\[
[x, y] + 50 + [1, 2] + 1 \leq 100
\]
\[
[x, y] + [1, 2] \leq 49
\]
\[
[x + 1, y + 2] \leq 49
\]

therefore \(y \leq 47\), but \(0 \leq x \leq y\), so \(0 \leq x \leq y \leq 47\)