Case Study 3: fMRI Prediction

Coping with Large Covariances: Latent Factor Models

Machine Learning for Big Data
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Multivariate Normal Models

- So far, we looked at univariate multiple regression

- If one has a multivariate response $y^f \in \mathbb{R}^d$
  - Assuming independence between dimensions
Multivariate Normal Models

- If one has a multivariate response \( y^i \in \mathbb{R}^d \)
  - Assuming correlation between the output dimensions

- Assume linear (or other mean regression) is removed and focus on the correlation structure

- Matrix valued parameter!

High-Dimensional Covariance

- What if \( d \) is large?

  - A few common approaches:
    - Low-rank approximations
    - Sparsity assumptions
Low-Rank Approximations

- In general, assume some matrix parameter

- Here, $\Sigma_0$ must be a symmetric, positive definite matrix

$\Sigma = \Lambda \Lambda' + \Sigma_0$

$\Sigma_0 = \text{diag}(\sigma_1^2, \ldots, \sigma_d^2)$

- Number of parameters:
Latent Factor Models

- Original multivariate regression
  \[ y^i = B^T x^i + e^i, \quad e^i \sim N(0, \Sigma) \]

- Latent factor model assumption: \[ \Sigma = \Lambda \Lambda' + \Sigma_0 \]
- Low-rank approximation arises from a latent factor model

Proof:

Lower-dim Embeddings

Sharing information in 
*low-dim subspace*
Sparsity Assumptions

- What if we assume $\Sigma$ is sparse?

- More often, we can reasonably make statements about conditional independence.