

We wish to show the following:

Lemma 0.1 *Let $f : \{-1, 1\}^m \rightarrow [-1, 1]$. Then*

$$\#\left\{i : \text{Inf}_i^{\leq C}(f) \geq \tau\right\} \leq C/\tau.$$

We instead show the following, which is clearly stronger:

Lemma 0.2 *Let $f : \{-1, 1\}^m \rightarrow [-1, 1]$. Then*

$$\sum_{i=1}^m \text{Inf}_i^{\leq C}(f) \leq C.$$

Proof:

$$\sum_{i=1}^m \text{Inf}_i^{\leq C}(f) = \sum_{i=1}^m \sum_{\substack{|S| \leq C \\ S \ni i}} \hat{f}(S)^2 = \sum_{|S| \leq C} \sum_{i \in S} \hat{f}(S)^2 = \sum_{|S| \leq C} |S| \hat{f}(S)^2 \leq C \sum_S \hat{f}(S)^2 \leq C.$$

□