CSE 531: Introduction to Complexity Theory — Winter 2007
Problem Set #**0** — Instructor: Venkatesan Guruswami
January 3, 2007

Does NOT have to be turned in.

Important: Subscribe to CSE 531 email group ASAP by visiting the course webpage at `http://www.cs.washington.edu/531`

Note: The goal of this problem set is to aid you in reviewing material on finite automata, context-free grammars, and (un)decidability. Though we will not cover this material in lecture, background on these topics is necessary for the some of the stuff we will cover in the class. This problem set will also serve as a good warm-up for the skills you will need in this class. So, while you do not have to turn it in, I strongly urge you to work through this problem set *at your own pace.*

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1. Problem 1.35, Sipser's book (A language is regular iff it has finite index.)
2. Problem 1.44, Sipser's book (Exponential blow-up is necessary when converting NFAs to DFAs.)
   - Here is a suggestion for such a language:
     $E_n = \{x \in \{0,1\}^* : x$ has a 1 in the n'th position from the right $\}$.
3. Give algorithms for answering the following questions about finite automata. (For a DFA M, $L(M)$ is the language of strings accepted by M.)
   (a) Given a DFA M, is $L(M) = \Sigma^*$ (i.e. does M accept *every* string)?
   (b) Given two DFAs M_1 and M_2, is $L(M_1) \subseteq L(M_2)$?
   (c) Given two DFAs M_1 and M_2, is $L(M_1) = L(M_2)$?
4. (a) Prove that the intersection of a context-free language with a regular language is context-free.
   (b) Give an example to show that context-free languages are **not** closed under intersection. Deduce that CFLs are not closed under complementation.
5. Given a context-free grammar G, give an algorithm to decide if $L(G) = \emptyset$.
6. Prove that a language C is Turing-recognizable iff there exists a decidable language D such that $C = \{x : \exists y(\langle x, y\rangle \in D)\}$)
7. Which of the following problems about Turing machines are decidable and which are not? Briefly justify your answers.

(a) To determine, given a Turing machine M, whether M has the property that it accepts a string $w \in \{0,1\}^*$ if and only if it accepts the sring $\overline{w}$ (here $\overline{w}$ denotes the bitwise complement of w; eg. $\overline{100110} = 011001$).

(b) To determine, given a Turing machine M and a string w, whether M ever moves it head to the left when it is run on input w.

(c) To determine, given a Turing machine M and a string w, whether M on input w ever tries to move its head left when its head is on the left-most tape cell.

(d) To determine, given a Turing machine M, whether the tape ever contains four consecutive 1's during the course of M's computation when it is run on input 01.

8. Problem 5.20, Sipser's book (Acceptance and emptiness problems for two headed finite automata)

   - Suggestion: Reading Theorems 5.8 and 5.9 on linear bounded automata will help.