1. Define the class $\text{polyL} = \bigcup_{k \geq 1} \text{SPACE}(\log^k n)$ consisting of languages that can decided in polylogarithmic space.
   
   (a) Prove that $\text{NL} \neq \text{polyL}$.

   (b) Prove that $\text{polyL} \neq \text{P}$.

   (Hint: You are permitted to to use the result of Theorem 10.40 from Sipser’s book, or 
   that of Problem 7 from Problem Set 4.)

2. (a) Prove that if $\text{NEXPTIME} \neq \text{EXPTIME}$, then $\text{P} \neq \text{NP}$. (Problem 9.19, Sipser’s book)

   (b) Prove that if $\text{BPP} = \text{EXPTIME}$, then $\text{P} \neq \text{NP}$.

3. Prove that there exists an oracle $C$ for which $\text{NP}^C \neq \text{coNP}^C$. (Problem 9.12, Sipser’s book)

4. Prove the following version of the Schwarz-Zippel lemma. Let $\mathbb{F}$ be any field (finite or infinite) 
   and let $Q(x_1, x_2, \ldots, x_m) \in \mathbb{F}[x_1, x_2, \ldots, x_m]$ be a non-zero $m$-variate polynomial over $\mathbb{F}$ 
   of total degree $d$. Fix any finite set $S \subseteq \mathbb{F}$. Prove that

   $$\text{Prob}[Q(r_1, r_2, \ldots, r_m) = 0] \leq \frac{d}{|S|}$$

   where the probability is taken over $r_1, r_2, \ldots, r_m$ that are chosen independently and uniformly 
   at random from $S$.

5. State and prove a hierarchy theorem for circuit size. Your result should at least prove that 
   circuits of size $O(n^a)$ are strictly more powerful than circuits of size $O(n^{a-1})$ for every integer 
   $a \geq 2$ (and this will receive a good portion of the credit). The question as posed is deliberately 
   vague, and solutions which are creative and/or establish the finest hierarchies will receive 
   bonus points.

6. (a) Prove that if $L \in \text{BPP}$, then there is a polynomially bounded function $p : \mathbb{N} \to \mathbb{N}$ and a 
   polynomial time deterministic Turing machine $V$ such that

   $$x \in L \quad \Rightarrow \quad \text{Prob}_{r \in \{0,1\}^p(|x|)}[V \text{ accepts } (x, r)] \geq (1 - 2^{-2|x|})$$

   $$x \notin L \quad \Rightarrow \quad \text{Prob}_{r \in \{0,1\}^p(|x|)}[V \text{ accepts } (x, r)] \leq 2^{-2|x|}.$$ 

   (b) Prove that every language in $\text{BPP}$ has a circuit family of polynomial size that decides it. 

   (Hint: Use (a) above)