CSE/NEUBEH 528
Lecture 9: Computation by Networks
(Chapter 7)

Lecture figures are from Dayan & Abbott’s book
Course Summary (thus far)

- **Neural Encoding**
  - What makes a neuron fire? (STA, covariance analysis)
  - Poisson model

- **Neural Decoding**
  - Stimulus Discrimination based on firing rate
  - Spike-train based decoding of stimulus
  - Population decoding (Bayesian estimation)

- **Single Neuron Models**
  - RC circuit model of membrane
  - Integrate-and-fire model
  - Conductance-based and Compartmental Models
Today’s Agenda

- Computation in Networks of Neurons
  - From spiking to firing-rate based networks
  - Feedforward Networks
    - E.g. Coordinate transformations in the brain
  - Linear Recurrent Networks
    - Can amplify inputs
    - Can integrate inputs
    - Can function as short-term memory
Modeling Networks of Neurons

- **Option 1**: Use *spiking* neurons (e.g. I & F neurons)
  - *Advantages*: Allows computation and learning based on:
    - Spike Timing
    - Spike Correlations/Synchrony between neurons
  - *Disadvantages*: Computationally expensive

- **Option 2**: Use neurons with *firing-rate outputs*
  - *Advantages*: Greater efficiency, scales well to large networks
  - *Disadvantages*: Ignores spike timing issues

- **Question**: How are these two approaches related?
Flashback

1-Compartment Membrane Model

\[ I_e = \frac{I_e}{A} \]

\[ c_m \frac{dV}{dt} = -\frac{(V - E_L)}{r_m} + \frac{I_e}{A}, \text{ or equivalently} \]

\[ \tau_m \frac{dV}{dt} = -(V - E_L) + I_e R_m \]

\( \tau_m = r_m c_m = \) membrane time constant
Flashback: Modeling Synaptic Inputs from other Neurons

Synaptic conductance

\[ \tau_m \frac{dV}{dt} = -(V - E_L) - r_m g_s (V - E_s) + I_e R_m \]

\[ g_s = g_{s,\text{max}} P_{rel} P_s \]

- Probability of postsynaptic channel opening
- Probability of transmitter release given an input spike

R. Rao, CSE528: Lecture 9
Basic Synapse Model

- Assume $P_{rel} = 1$
- Model the effect of a single spike input on $P_s$
- **Kinetic Model:**
  
  Closed $\xrightarrow{\alpha_s} \text{open}$
  
  Open $\xrightarrow{\beta_s} \text{closed}$

  \[
  \frac{dP_s}{dt} = \alpha_s (1 - P_s) - \beta_s P_s
  \]

  - Fraction of channels open
  - Opening rate
  - Closing rate
  - Fraction of channels closed
Postsynaptic Data

Exponential function $K(t)$ gives reasonable fit to biological data (other options: difference of exponentials, “alpha” function)
Modeling Synaptic Input Current

Synaptic kernel: \[ K(t) = e^{-\frac{t}{\tau_s}} / \tau_s \]

Synaptic current: \[ I_s(t) = w_s \int_{-\infty}^{t} K(t - \tau) \rho_s(\tau) d\tau \]

where \( \rho_s(t) \) is the input spike train:

\[ \rho_s(\tau) = \Sigma_i \delta(\tau - t_i) \quad (t_i \text{ are the spike times}) \]
From Spiking to Firing Rate Models

output $\mathcal{V}$

weights $\mathbf{w}$

input $\mathbf{u}$

Current at synapse $b$

$$I_b(t) = w_b \int_{-\infty}^{t} K(t - \tau) \rho_b(\tau) d\tau$$

Spike train $\rho_b(t)$

$$\approx w_b \int_{-\infty}^{t} K(t - \tau) u_b(\tau) d\tau$$

Firing rate $u_b(t)$

Total synaptic current

$$I_s(t) = \sum_b I_b(t)$$
Synaptic Current Dynamics

If synaptic kernel \( K \) is an exponential function: \( K(t) = e^{-\frac{t}{\tau_s}} / \tau_s \)

Differentiating \( I_s(t) = \sum_b w_b \int_{-\infty}^{t} K(t-\tau)u_b(\tau)d\tau \)

We get \( \tau_s \frac{dI_s}{dt} = -I_s + \sum_b w_b u_b \)

\( = -I_s + \mathbf{w} \cdot \mathbf{u} \)
Output Firing-Rate Dynamics

- How is the output firing rate $v$ related to synaptic inputs?

$$\tau_r \frac{dv}{dt} = -v + F(I_s(t))$$

- On-board derivations of special cases obtained from comparing $\tau_r$ and $\tau_s$ ...

(see also pages 234-236 in the text)
How good are the Firing Rate Models?

Firing rate $v(t) = F(I(t))$ describes this well but not this case

Input $I(t) = I_0 + I_1 \cos(\omega t)$
Feedforward versus Recurrent Networks

\[ \tau \frac{dv}{dt} = -v + F(Wu + Mv) \]

Output Decay Input Feedback

(For feedforward networks, matrix M = 0)
The Problem of Coordinate Transformations

\( g = \text{gaze angle relative to body} \)
\( s = \text{stimulus or target angle relative to gaze (retinal coordinates)} \)
\( s+g = \text{stimulus relative to body} \)

Same arm movement required in A and B but \( s \) and \( g \) are different

How does the brain solve this problem?
Body-Based Representation in the Monkey

Head fixed

gaze shifted to $g_1$, $g_2$, $g_3$

Premotor cortex neuron responds to stimulus location relative to body, not retinal image location

Same tuning curve regardless of gaze angle

Objects approaching at different angles

Objects

$s + g$ (deg)

firing rate (% max)

-30 -15 0 15 30 45 60
Body-Based Representation in the Monkey

When head is moved but gaze remains unchanged:

After head is moved 15°, objects approaching at 15° in retinal image now elicit the highest response
Tuning curve in retinal coordinates has shifted

\[ s \text{ (deg)} \]

---

R. Rao, CSE528: Lecture 9
Suggested Feedforward Network

Output: Premotor Cortex Neuron with Body-Based Tuning Curves

Input: Area 7a Neurons with Gaze-Dependent Tuning Curves

Input neurons exhibit \textit{gaze-dependent gain modulation}
Gaze-Dependent Gain Modulation

Responses of Area 7a neuron

Example of a gain-modulated tuning curve

Gaze 1

Gaze 2

\[ \xi = -20 \]

\[ \gamma = 20 \]
What should the weights be?

Output: Premotor Cortex Neuron with Body-Based Tuning Curves

Input: Area 7a Neurons with Gaze-Dependent Tuning Curves

Weights $w(\xi, \gamma)$ need to be a function of $\xi + \gamma$
Output of a Simulated Feedforward Network

Retinal tuning curves shift to compensate for $g$ (i.e. stable for $s + g$)

Head fixed
gaze shifted to $g_1$  $g_2$  $g_3$
Next Class: Recurrent Networks

Things to do:
- Finish reading Chapter 7
- Homework #3 due next Thursday May 14
- Start working on mini-project