Course Summary (thus far)

- **Neural Encoding**
  - What makes a neuron fire? (STA, covariance analysis)
  - Poisson model

- **Neural Decoding**
  - Stimulus Discrimination based on firing rate
  - Spike-train based decoding of stimulus
  - Population decoding (Bayesian estimation)

- **Single Neuron Models**
  - RC circuit model of membrane
  - Integrate-and-fire model
  - Conductance-based and Compartmental Models
Today’s Agenda

✦ Computation in Networks of Neurons
  ➔ From spiking to firing-rate based networks
  ➔ Feedforward Networks
    ♦ E.g. Coordinate transformations in the brain
  ➔ Linear Recurrent Networks
    ♦ Can amplify inputs
    ♦ Can integrate inputs
    ♦ Can function as short-term memory

Flashback: Modeling Synaptic Inputs from other Neurons

\[ \tau_m \frac{dV}{dt} = -(V - E_L) - r_m g_s (V - E_s) + I_e R_m \]

\[ g_s = g_{s,max} P_{rel} P_s \]

Probability of transmitter release given an input spike

(= fraction of channels opened)
Flashback 2  Simplified Synapse Model

✦ “Alpha Function” model:

Synaptic kernel  \[ K(t) = \frac{t}{\tau_{peak}} e^{-\frac{t}{\tau_{peak}}} \]

Synaptic current:  \[ I_s(t) = w_s \int_{-\infty}^{t} K(t-\tau) \rho_s(\tau) d\tau \]

where \( \rho_s(t) \) is the input spike train:

\[ \rho_s(\tau) = \sum_i \delta(\tau-t_i) \]  (t_i are the spike times)

Modeling Networks of Neurons

✦ **Option 1**: Use spiking neurons (e.g. I & F neurons)
  ➤ **Advantages**: Allows computation and learning based on:
    ✦ Spike Timing
    ✦ Spike Correlations/Synchrony between neurons
  ➤ **Disadvantages**: Computationally expensive

✦ **Option 2**: Use neurons with firing-rate outputs
  ➤ **Advantages**: Greater efficiency, scales well to large networks
  ➤ **Disadvantages**: Ignores spike timing issues

✦ **Question**: How are these two approaches related?
Network Notation

- Output $\mathcal{U}$
- Weights $\mathbf{w}$
- Input $\mathbf{u}$

Current at synapse $b$

$$I_b(t) = w_b \int_{-\infty}^{t} K(t-\tau) \rho_b(\tau) d\tau$$

Spike train $\rho_b(t)$

$$\approx w_b \int_{-\infty}^{t} K(t-\tau) u_b(\tau) d\tau$$

Firing rate $u_b(t)$

Total synaptic current

$$I_s(t) = \sum_b I_b(t)$$

Synaptic Current Dynamics

- If synaptic kernel $K$ is an exponential function: $K(t) = e^{-t/\tau_s}$

Differentiating

$$I_s(t) = \sum_b w_b \int_{-\infty}^{t} K(t-\tau) u_b(\tau) d\tau$$

We get

$$\tau_s \frac{dI_s}{dt} = -I_s + \sum_b w_b u_b$$

$$= -I_s + \mathbf{w} \cdot \mathbf{u}$$
Output Firing-Rate Dynamics

✦ How is the output firing rate $v$ related to synaptic inputs?
✦ On-board derivations…

(see also pages 234-236 in the text)

How good are the Firing Rate Models?

Firing rate $v(t) = F(I(t))$ describes this well but not this case

Input $I(t) = I_0 + I_1 \cos(\omega t)$
Feedforward versus Recurrent Networks

\[ \tau \frac{dv}{dt} = -v + F(Wu + Mv) \]

Output Decay Input Feedback

(For feedforward networks, matrix M = 0)

The Problem of Coordinate Transformations

\[ g = \text{gaze angle relative to body} \]
\[ s = \text{stimulus or target angle relative to gaze (retinal coordinates)} \]
\[ s+g = \text{stimulus relative to body} \]

Same arm movement required in A and B but s and g are different

How does the brain solve this problem?
Body-Based Representation in the Monkey

Head fixed, gaze shifted to $g_1$, $g_2$, $g_3$

Same tuning curve regardless of gaze angle
Premotor cortex neuron responds to stimulus location relative to body, not retinal image location

When head is moved but gaze remains unchanged:

After head is moved 15°, objects approaching at 15° in retinal image now elicit the highest response → Tuning curve in retinal coordinates has shifted
Suggested Feedforward Network

Output: Premotor Cortex Neuron with Body-Based Tuning Curves

\[
\begin{align*}
\text{output: } & \mathcal{V} \\
\text{weights: } & \mathbf{w} \\
\text{input: } & \mathbf{u}
\end{align*}
\]

Input: Area 7a Neurons with Gaze-Dependent Tuning Curves

Input neurons exhibit \textit{gaze-dependent gain modulation}

Gaze-Dependent Gain Modulation

Responses of Area 7a neuron

Example of a gain-modulated tuning curve

\[\xi = -20\]
What should the weights be?

Output: Premotor Cortex Neuron with Body-Based Tuning Curves

Output of a Simulated Feedforward Network

Retinal tuning curves shift to compensate for $g$ (i.e. stable for $s + g$)
Next Class: More on Networks

✦ Things to do:
  ➔ Finish reading Chapter 7
  ➔ Homework #3 due next Tuesday
  ➔ Start working on mini-project