CSE 521: Design and Analysis of Algorithms **Fall 2023**

Problem Set 8

Deadline: Dec 6th in Canvas

1) You are given data containing grades in different courses for 5 students; say $G_{i,j}$ is the grade of student i in course j. (Of course, $G_{i,j}$ is not defined for all i, j since each student has only taken a few courses.) We are trying to "explain" the grades as a linear function of the student's innate aptitude, the easiness of the course and some error term.

$$G_{i,j} = \text{aptitude}_i + \text{easiness}_j + \epsilon_{i,j},$$

where $\epsilon_{i,j}$ is an error term of the linear model. We want to find the best model that minimizes the sum of the $|\epsilon_{i,j}|$'s.

a) Write a linear program to find aptitude_i and easiness_j for all i, j minimizing $\sum_{i,j} |\epsilon_{i,j}|$.

b) Use any standard package for linear programming (Matlab/CVX, Freemat, Sci-Python, Excel etc.; we recommend CVX on matlab) to fit the best model to this data. Include a printout of your code, the objective value of the optimum, $\sum_{i,j} |\epsilon_{i,j}|$, and the calculated easiness values of all the courses and the aptitudes of all the students.

	MAT	CHE	ANT	REL	POL	ECO	COS
Alex		C+	B	B+	A-	C	
Billy	B-	A-			A+	D+	B
Chris	B-		B+		A-	B	B+
David	A+		B-	A		A-	
Elise		B-	D+	B+		B	C+

Assume $A = 4, B = 3$ and so on. Also, let $B+ = 3.33$ and $A- = 3.66$.

2) Given an unweighted graph $G + (V, E)$ design a polynomial time algorithm to assign non-negative weights $w : E \to \mathbb{R}_{\geq 0}$ to the edges of G such that

i) The fractional degree of every vertex is 1, $\sum_{j \sim i} w_{i,j} = 1$.

ii) Let G_w be the corresponding weighted graph with corresponding adjacency matrix A. The second largest eigenvalue of A is as small as possible.

Prove that your algorithm is correct.

Note: In this problem we are finding the best possible expander out of a given graph G. This task has many applications in theory and practice.