

Online Algorithms

- Algorithms that make decisions without full knowledge of input.
- Full knowledge of past
- No knowledge of future.
- How to analyze? Two common approaches:
 - Competitive analysis (multiplicative)
 - Regret (additive)

Ski-rental

- You are about to go skiing for the first time. You need to decide between renting skis, say for \$50 or buying skis for \$500.
- What do you do?
- How about on your second time?
- Third time?
- What if you knew in advance exactly how many times you'd go skiing?

Rent until you've paid the cost of buying

- Has **competitive ratio 2**.
- An online algorithm A is **c-competitive** if there is a constant b such that for all possible input sequences σ

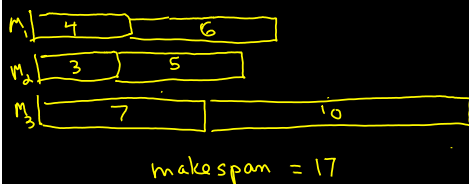
$$A(\sigma) \leq c \text{OPT}(\sigma) + b$$

Example 2: scheduling

- m identical machines
- Sequence of jobs arrives one at a time.
- Upon arrival, learn processing time of job j.
- But don't know the future jobs.
- Goal: minimize the makespan (maximum load = sum of processing times on any machine)

Example

- m=3, sequence of processing times: 7, 3, 4, 5, 6, 10



Most obvious algorithm

- Greedy: schedule the job on the least loaded machine.
- Theorem: This algorithm is $2 - 1/m$ competitive.

List update

- Using a linked list to store a set of items.
- Implementation of Dictionary data structure.

Storing a set S of items, each with key.

Operations supported

Insert(k) – add item with key k to the set S

Find (k) – is the key k in S ?

Delete (k) – remove the key k from S .

Find(k)

- Search through the list until find the key (or reach the end)
- Cost model:
 - If item is at position j in list, cost is j
 - Can move requested element anywhere in first j positions for free.
 - Can exchange adjacent items for free at cost $1/\text{swap}$.

Online algorithm for list update?

- Move To Front:
 - On each request, move the requested element to the front of the list.
- Theorem: MTF is 2-competitive

In class exercise

- Doubly linked list: Find the element, given a pointer somewhere in the list.