Linear Probing

If it takes a lot of time, then \( E \) very long full intervals

\[ L_I = \left| \{ x \in S \mid h(x) \in I \} \right| \]

\( I \): # items that hash here

same interval of length \( \geq k \) must be full

\[ E(L_I) = \frac{|I|}{2} \quad \text{and} \quad m = an \]

Chernoff Bound

\[ X = \sum_{i=1}^{n} X_i \quad X_i \text{ mutually indep indicator r.v.'s} \]

\[ p_i = \Pr(X_i = 1) \quad \Rightarrow \quad E(X) = \sum p_i = \mu \]

\[ \Pr(\frac{X}{\mu} > (1+\delta)) < \left[ \frac{e^\delta}{(1+\delta)^{1+\delta}} \right]^n \quad A \delta > 0 \]
\[ \Pr \left( L_I > 2E(L_I) \right) \leq \left( \frac{e}{4} \right)^{E(L_I)} \]  

take \( \delta = 1 \) in Chernoff

\[ E(\text{insertion time}) = \sum_{k=1}^{n} k \Pr(\text{insertion time } k) \]

\[ \leq \sum_{j=k}^{n} \Pr(\text{L}_I \text{ is full}) \]

\[ \leq \left( \frac{e}{4} \right)^{j/2} \]

\[ \leq 0.82^k \]

\[ = 0(1) \]

\[ \frac{80}{\sum_{k=0}^{\infty} x^k} = \frac{1}{1-x} \quad 0 \leq x < 1 \]

\[ \sum_{k=1}^{\infty} k x^{k-1} = \frac{1}{(1-x)^2} \]

\[ 0(1) \text{ expected insertion time for } 5\text{-wise minlp} \]

but not 4-wise