# Natural Language Processing (CSE 517): Sequence Models

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- ► Idea: for each position i, calculate the score of the best label prefix y<sub>1:i</sub> ending in each possible value for Y<sub>i</sub>.
- With a little bookkeeping, we can then trace backwards and recover the best label sequence.

First, think about the *score* of the best sequence.

Let  $s_i(y)$  be the score of the best label sequence for  $x_{1:i}$  that ends in y. It is defined recursively:

$$s_{\ell}(y) = \gamma_{\bigcup | y} \cdot \theta_{x_{\ell} | y} \cdot \max_{y' \in \mathcal{L}} \gamma_{y | y'} \cdot \boxed{s_{\ell-1}(y')}$$

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	$x_1$	$x_2$	 $x_\ell$
y			
y'			
:			
$y^{last}$			

	$x_1$	$x_2$	 $x_\ell$
y	$s_1(y)$		
y'	$s_1(y')$		
:			
$y^{last}$	$s_1(y^{last})$		

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	$x_1$	$x_2$	 $x_\ell$
y	$s_1(y)$	$s_2(y)$	
y'	$s_1(y')$	$s_2(y')$	
:			
$y^{last}$	$s_1(y^{last})$	$s_2(y^{last})$	

$$s_i(y) = \theta_{x_i|y} \cdot \max_{y' \in \mathcal{L}} \gamma_{y|y'} \cdot \boxed{s_{i-1}(y')}$$

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y	$s_1(y)$	$s_2(y)$	$s_\ell(y)$
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$$\max_{y \in \mathcal{L}} s_{\ell}(y) = \max_{y \in \mathcal{L}} \gamma_{\bigcup | y} \cdot \theta_{x_{\ell} | y} \cdot \max_{y' \in \mathcal{L}} \gamma_{y | y'} \cdot \boxed{s_{\ell-1}(y')}$$

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$$\begin{aligned} \max_{y \in \mathcal{L}} s_{\ell}(y) &= \max_{y \in \mathcal{L}} \gamma_{\bigcirc |y} \cdot \theta_{x_{\ell}|y} \cdot \max_{y' \in \mathcal{L}} \gamma_{y|y'} \cdot \boxed{s_{\ell-1}(y')} \\ &= \max_{y \in \mathcal{L}} \gamma_{\bigcirc |y} \cdot \theta_{x_{\ell}|y} \cdot \max_{y' \in \mathcal{L}} \gamma_{y|y'} \cdot \boxed{\theta_{x_{\ell-1}|y'} \cdot \max_{y'' \in \mathcal{L}} \gamma_{y'|y''} \cdot \boxed{s_{\ell-2}(y'')}} \\ &= \max_{y \in \mathcal{L}} \gamma_{\bigcirc |y} \cdot \theta_{x_{\ell}|y} \cdot \max_{y' \in \mathcal{L}} \gamma_{y|y'} \cdot \underbrace{\theta_{x_{\ell-2}|y''} \cdot \max_{y'' \in \mathcal{L}} \gamma_{y''|y'''} \cdot \boxed{s_{\ell-3}(y''')}}_{y'' \in \mathcal{L}} \end{aligned}$$

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Claim:  $\max_{y \in \mathcal{L}} s_{\ell}(y) = \max_{y \in \mathcal{L}^{\ell+1}} p(\boldsymbol{x}, \boldsymbol{y})$  $\max_{y \in \mathcal{L}} s_{\ell}(y) = \max_{y \in \mathcal{L}} \gamma_{\bigcirc |y} \cdot \theta_{x_{\ell}|y} \cdot \max_{y' \in \mathcal{L}} \gamma_{y|y'} \cdot \boxed{s_{\ell-1}(y')}$  $= \max_{y \in \mathcal{L}} \gamma_{\bigcirc |y} \cdot \theta_{x_{\ell}|y} \cdot \max_{y' \in \mathcal{L}} \gamma_{y|y'} \cdot \boxed{\theta_{x_{\ell-1}|y'} \cdot \max_{y'' \in \mathcal{L}} \gamma_{y'|y''} \cdot \boxed{s_{\ell-2}(y'')}}$ 

$$= \max_{y \in \mathcal{L}} \gamma_{\bigcirc |y} \cdot \theta_{x_{\ell}|y} \cdot \max_{y' \in \mathcal{L}} \gamma_{y|y'} \cdot \left[ \theta_{x_{\ell-2}|y''} \cdot \max_{y'' \in \mathcal{L}} \gamma_{y''|y'''} \cdot \boxed{\theta_{x_{\ell-2}|y''} \cdot \max_{y''' \in \mathcal{L}} \gamma_{y''|y'''} \cdot \boxed{s_{\ell-3}(y''')}} \right]$$
  
= max  $\gamma_{\bigcirc i} \cdot \theta_{i+1} \cdot \gamma_{i+1} \cdot \theta_{i+1} \cdot \gamma_{i+1} \cdot \theta_{i+1} \cdot \gamma_{i+1} \cdot \theta_{i+1}$ 

$$= \max_{\boldsymbol{y} \in \mathcal{L}^{\ell+1}} \gamma_{\boldsymbol{y}_{\ell}} \cdot \boldsymbol{\theta}_{x_{\ell}|y_{\ell}} \cdot \gamma_{y_{\ell}|y_{\ell-1}} \cdot \boldsymbol{\theta}_{x_{\ell-1}|y_{\ell-1}} \cdot \gamma_{y_{\ell-1}|y_{\ell-2}}$$
$$\boldsymbol{\theta}_{x_{\ell-2}|y_{\ell-2}} \cdots \boldsymbol{\theta}_{x_{1}|y_{1}} \cdot \gamma_{y_{1}|y_{0}} \cdot \pi_{y_{0}}$$

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Claim: 
$$\max_{y \in \mathcal{L}} s_{\ell}(y) = \max_{y \in \mathcal{L}^{\ell+1}} p(\boldsymbol{x}, \boldsymbol{y})$$
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$$= \max_{y \in \mathcal{L}} \gamma_{\bigcirc |y} \cdot \theta_{x_{\ell}|y} \cdot \max_{y' \in \mathcal{L}} \gamma_{y|y'} \cdot \boxed{\theta_{x_{\ell-1}|y'} \cdot \max_{y'' \in \mathcal{L}} \gamma_{y'|y''} \cdot \underbrace{s_{\ell-2}(y'')}_{y'' \in \mathcal{L}} \gamma_{y'|y''} \cdot \underbrace{\theta_{x_{\ell-2}|y''} \cdot \max_{y'' \in \mathcal{L}} \gamma_{y'|y''} \cdot \underbrace{s_{\ell-3}(y'')}_{y'' \in \mathcal{L}} \gamma_{y'|y''} \cdot \underbrace{\theta_{x_{\ell-2}|y''} \cdot \max_{y'' \in \mathcal{L}} \gamma_{y'|y''} \cdot \underbrace{s_{\ell-3}(y'')}_{y \in \mathcal{L}^{\ell+1}} \gamma_{y_{\ell-2}} \cdot \underbrace{\theta_{x_{\ell-2}|y_{\ell-2}} \cdot \cdot \cdot \theta_{x_{\ell}|y_{\ell}} \cdot \gamma_{y_{\ell}|y_{\ell-1}} \cdot e_{x_{\ell-1}|y_{\ell-1}} \cdot \gamma_{y_{\ell-1}|y_{\ell-2}} \cdot e_{x_{\ell-2}|y_{\ell-2}} \cdot \cdot \cdot \cdot e_{x_{1}|y_{1}} \cdot \gamma_{y_{1}|y_{0}} \cdot \pi_{y_{0}}}$$

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	$x_1$	$x_2$	 $x_\ell$
y			
y'			
÷			
$y^{last}$			

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	$x_1$	$x_2$	 $x_\ell$
y	$s_1(y)$		
	$b_1(y)$		
y'	$s_1(y')$		
	$b_1(y')$		
:			
$y^{last}$	$s_1(y^{last})$		
	$b_1(y^{last})$		

$$s_1(y) = \theta_{x_1|y} \cdot \max_{y' \in \mathcal{L}} \gamma_{y|y'} \cdot \pi_{y'}$$
$$b_1(y) = \operatorname*{argmax}_{y' \in \mathcal{L}} \gamma_{y|y'} \cdot \pi_{y'}$$

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	$x_1$	$x_2$	 $x_\ell$
y	$s_1(y)$	$s_2(y)$	
	$b_1(y)$	$b_2(y)$	
y'	$s_1(y')$	$s_2(y')$	
	$b_1(y')$	$b_2(y')$	
:			
$y^{last}$	$s_1(y^{last})$	$s_2(y^{last})$	
	$b_1(y^{last})$	$b_2(y^{last})$	

$$s_{i}(y) = \theta_{x_{i}|y} \cdot \max_{y' \in \mathcal{L}} \gamma_{y|y'} \cdot \boxed{s_{i-1}(y')}$$
$$b_{i}(y) = \operatorname*{argmax}_{y' \in \mathcal{L}} \gamma_{y|y'} \cdot s_{i-1}(y')$$

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	$x_1$	$x_2$	 $x_\ell$
y	$s_1(y)$	$s_2(y)$	$s_\ell(y)$
	$b_1(y)$	$b_2(y)$	$b_\ell(y)$
y'	$s_1(y')$	$s_2(y')$	$s_\ell(y')$
	$b_1(y')$	$b_2(y')$	$b_\ell(y')$
÷			
$y^{last}$	$s_1(y^{last})$	$s_2(y^{last})$	$s_{\ell}(y^{last})$
	$b_1(y^{last})$	$b_2(y^{last})$	$b_\ell(y^{last})$

$$s_{\ell}(y) = \gamma_{\bigcup|y} \cdot \theta_{x_{\ell}|y} \cdot \max_{y' \in \mathcal{L}} \gamma_{y|y'} \cdot \left[ s_{\ell-1}(y') \right]$$
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# Full Viterbi Procedure

Input: x, heta,  $\gamma$ ,  $\pi$ 

Output:  $\hat{y}$ 

- 1. For  $i \in \langle 1, \ldots, \ell \rangle$ :
  - Solve for  $s_i(*)$  and  $b_i(*)$ .
    - Special base case for i=1 to handle  $\pi$
    - General recurrence for  $i \in \langle 2, \dots, \ell 1 \rangle$
    - Special case for  $i = \ell$  to handle stopping probability
- 2.  $\hat{y}_{\ell} \leftarrow \operatorname*{argmax}_{y \in \mathcal{L}} s_{\ell}(y)$
- 3. For  $i \in \langle \ell, \dots, 1 \rangle$ :
  - $\blacktriangleright \hat{y}_{i-1} \leftarrow b(y_i)$

# Full Viterbi Procedure

Input:  $x, heta, \gamma, \pi$ Output:  $\hat{y}$ 

- 1. For  $i \in \langle 1, \ldots, \ell \rangle$ :
  - Solve for  $s_i(*)$  and  $b_i(*)$ .
    - Special base case for i = 1 to handle  $\pi$  (base case)
    - General recurrence for  $i \in \langle 2, \dots, \ell 1 \rangle$

$$s_i(y) = \theta_{x_i|y} \cdot \max_{y' \in \mathcal{L}} \gamma_{y|y'} \cdot s_{i-1}(y')$$
$$b_i(y) = \operatorname*{argmax}_{y' \in \mathcal{L}} \gamma_{y|y'} \cdot s_{i-1}(y')$$

• Special case for  $i = \ell$  to handle stopping probability

2. 
$$\hat{y}_{\ell} \leftarrow \operatorname*{argmax}_{y \in \mathcal{L}} s_{\ell}(y)$$
  
3. For  $i \in \langle \ell, \dots, 1 \rangle$ :  
 $\blacktriangleright \hat{y}_{i-1} \leftarrow b(y_i)$ 

# Viterbi Asymptotics

Space:  $O(|\mathcal{L}|\ell)$ 

Runtime:  $O(|\mathcal{L}|^2 \ell)$ 

	$x_1$	$x_2$	 $x_\ell$
y			
y'			
:			
$y^{last}$			

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▶ Instead of HMM parameters, we can use the featurized variant.

$$s_i(y) = \max_{y' \in \mathcal{L}} \exp\left(\mathbf{w} \cdot \boldsymbol{\phi}(\boldsymbol{x}, i, y, y')\right) \cdot s_{i-1}(y')$$

More features may increase runtime, but asymptotic dependence on  $\ell$  and  $|\mathcal{L}|$  is the same.

- ► For this case and for the HMM case, taking logarithms is a good idea.
- ▶ Note that dependence on entirety of *x* doesn't affect asymptotics.

- ► Instead of HMM parameters, we can use the featurized variant.
- Viterbi instantiates an general algorithm called max-product variable elimination for inference along a chain of variables with pairwise links.
  - Applicable to Bayesian networks and Markov networks.

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- ► Higher-order dependencies among *Y* are also possible.

$$s_i(y, y') = \max_{y'' \in \mathcal{L}} \exp\left(\mathbf{w} \cdot \boldsymbol{\phi}(\boldsymbol{x}, i, y, y', y'')\right) \cdot s_{i-1}(y', y'')$$

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- Dynamic programming algorithms.
- ► Weighted finite-state analysis.

# Applications of Sequence Models

- part-of-speech tagging (Church, 1988)
- supersense tagging (Ciaramita and Altun, 2006)
- named-entity recognition (Bikel et al., 1999)
- multiword expressions (Schneider and Smith, 2015)
- base noun phrase chunking (Sha and Pereira, 2003)

Along the way, we'll briefly mention two ways to learn sequence models.

# Parts of Speech

#### http://mentalfloss.com/article/65608/master-particulars-grammar-pop-culture-primer



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# Parts of Speech

- "Open classes": Nouns, verbs, adjectives, adverbs, numbers
- "Closed classes":
  - Modal verbs
  - Prepositions (on, to)
  - ▶ Particles (*off*, *up*)
  - Determiners (*the*, *some*)
  - Pronouns (she, they)
  - Conjunctions (and, or)

# Parts of Speech in English: Decisions

Granularity decisions regarding:

- verb tenses, participles
- plural/singular for verbs, nouns
- proper nouns
- comparative, superlative adjectives and adverbs

Some linguistic reasoning required:

- Existential there
- ► Infinitive marker to
- ▶ wh words (pronouns, adverbs, determiners, possessive whose)

Interactions with tokenization:

- Punctuation
- Compounds (Mark'll, someone's, gonna)

Penn Treebank: 45 tags,  $\sim$ 40 pages of guidelines (Marcus et al., 1993)

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- ► *wh* words (pronouns, adverbs, determiners, possessive *whose*) Interactions with tokenization:
  - Punctuation
  - Compounds (Mark'll, someone's, gonna)
  - Social media: hashtag, at-mention, discourse marker (*RT*), URL, emoticon, abbreviations, interjections, acronyms

Penn Treebank: 45 tags, ~40 pages of guidelines (Marcus et al., 1993) TweetNLP: 20 tags, 7 pages of guidelines (Gimpel et al., 2011) Example: Part-of-Speech Tagging

ikr smh he asked fir yo last name

so he can add u on fb lololol

Example: Part-of-Speech Tagging

I know, right shake my head for your ikr smh he asked fir yo last name

you Facebook laugh out loud so he can add u on fb lololol

# Example: Part-of-Speech Tagging



# Why POS?

- ► Text-to-speech: *record*, *lead*, *protest*
- Lemmatization:  $saw/V \rightarrow see$ ;  $saw/N \rightarrow saw$
- Quick-and-dirty multiword expressions: (Adjective | Noun)\* Noun (Justeson and Katz, 1995)
- Preprocessing for harder disambiguation problems:
  - ► The Georgia branch had taken on loan commitments ....
  - ► The average of interbank offered rates plummeted ....

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All datasets have some errors; estimated upper bound for Penn Treebank is 98%.

## Supervised Training of Hidden Markov Models

Given: annotated sequences  $\langle\langle m{x}_1, m{y}_1, 
angle, \dots, \langlem{x}_n, m{y}_n 
angle 
angle$ 

$$p(\boldsymbol{x}, \boldsymbol{y}) = \pi_{y_0} \prod_{i=1}^{\ell+1} \theta_{x_i|y_i} \cdot \gamma_{y_i|y_{i-1}}$$

Parameters: for each state/label  $y \in \mathcal{L}$ :

- $\pi$  is the "start" distribution
- $\theta_{*|y}$  is the "emission" distribution
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Maximum likelihood estimate: count and normalize!



TnT, a trigram HMM tagger with smoothing: 96.7% (Brants, 2000)

<ロ > < 部 > < 言 > < 言 > こ ジ < で 50 / 60

## Back to POS

TnT, a trigram HMM tagger with smoothing: 96.7% (Brants, 2000)

State of the art:  $\sim$ 97.5% (Toutanova et al., 2003); uses a feature-based model with:

- capitalization features
- spelling features
- name lists ("gazetteers")
- context words
- hand-crafted patterns

Parts of speech are a minimal *syntactic* representation.

Sequence labeling can get you a lightweight *semantic* representation, too.



A problem with a long history: word-sense disambiguation.

#### Supersenses

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Ciaramita and Johnson (2003) and Ciaramita and Altun (2006) used a lexicon called WordNet to define 41 semantic classes for words.

WordNet (Fellbaum, 1998) is a fascinating resource in its own right! See http://wordnetweb.princeton.edu/perl/webwn to get an idea.

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This represents a coarsening of the annotations in the Semcor corpus (Miller et al., 1993).

# Example: box's Thirteen Synonym Sets, Eight Supersenses

- 1. box: a (usually rectangular) container; may have a lid. "he rummaged through a box of spare parts"
- 2. box/loge: private area in a theater or grandstand where a small group can watch the performance. "the royal box was empty"
- 3. box/boxful: the quantity contained in a box. "he gave her a box of chocolates"
- 4. corner/box: a predicament from which a skillful or graceful escape is impossible. "his lying got him into a tight corner"
- 5. box: a rectangular drawing. "the flowchart contained many boxes"
- 6. box/boxwood: evergreen shrubs or small trees
- 7. box: any one of several designated areas on a ball field where the batter or catcher or coaches are positioned. "the umpire warned the batter to stay in the batter's box"
- 8. box/box seat: the driver's seat on a coach. "an armed guard sat in the box with the driver"
- 9. box: separate partitioned area in a public place for a few people. "the sentry stayed in his box to avoid the cold"
- 10. box: a blow with the hand (usually on the ear). "I gave him a good box on the ear"
- 11. box/package: put into a box. "box the gift, please"
- 12. box: hit with the fist. "I'll box your ears!"
- 13. box: engage in a boxing match.

# Example: box's Thirteen Synonym Sets, Eight Supersenses

- 1. box: a (usually rectangular) container; may have a lid. "he rummaged through a box of spare parts"  $\rightsquigarrow$  N.ARTIFACT
- 2. box/loge: private area in a theater or grandstand where a small group can watch the performance. "the royal box was empty"  $\rightsquigarrow$  N.ARTIFACT
- 3. box/boxful: the quantity contained in a box. "he gave her a box of chocolates"  $\rightsquigarrow$  N.QUANTITY
- 4. corner/box: a predicament from which a skillful or graceful escape is impossible. "his lying got him into a tight corner" → N.STATE
- 5. box: a rectangular drawing. "the flowchart contained many boxes"  $\rightsquigarrow$  N.SHAPE
- 6. box/boxwood: evergreen shrubs or small trees  $\rightarrow$  N.PLANT
- 7. box: any one of several designated areas on a ball field where the batter or catcher or coaches are positioned. "the umpire warned the batter to stay in the batter's box" → N.ARTIFACT
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- 11. box/package: put into a box. "box the gift, please"  $\rightsquigarrow$  V.CONTACT
- 12. box: hit with the fist. "I'll box your ears!"  $\rightsquigarrow$  V.CONTACT
- 13. box: engage in a boxing match.  $\rightsquigarrow$  V.COMPETITION

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