Due:  Friday, January 18th at 5pm

This assignment contains a writing question and a programming question, which will also requires a written report of experimental results. Your final, combined writeup should be no more than four pages long. Good luck!

1 Problem 1 (5pt)

In class, we discussed how back off can improve the performance of language models. Consider the following back-off scheme. First, we define the sets

\[ A(w_{i-1}) = \{ w : c(w_{i-1}, w) > 0 \} \]
\[ B(w_{i-1}) = \{ w : c(w_{i-1}, w) = 0 \} \]
\[ A(w_{i-2}, w_{i-1}) = \{ w : c(w_{i-2}, w_{i-1}, w) > 0 \} \]
\[ B(w_{i-2}, w_{i-1}) = \{ w : c(w_{i-2}, w_{i-1}, w) = 0 \} \]

where \( c \) is a function that counts \( n \)-grams in the training set. For example, if the bigram "Honey Bunny" appears 22 times in the corpus, we will have \( c(\text{Honey, Bunny}) = 22 \).

Now, we can define a back-off trigram model:

\[
p(w_i | w_{i-2}, w_{i-1}) = \begin{cases} 
p_1(w_i | w_{i-2}, w_{i-1}) & \text{if } w_i \in A(w_{i-2}, w_{i-1}) \\
p_2(w_i | w_{i-2}, w_{i-1}) & \text{if } w_i \in A(w_{i-1}) \text{ and } w_i \in B(w_{i-2}, w_{i-1}) \\
p_3(w_i | w_{i-2}, w_{i-1}) & \text{if } w_i \in B(w_{i-1}) 
\end{cases}
\]

Where:

\[
p_1(w_i | w_{i-2}, w_{i-1}) = \frac{p_{ML}(w_i | w_{i-2}, w_{i-1})}{p_{ML}(w_{i-2}, w_{i-1})}
\]
\[
p_2(w_i | w_{i-2}, w_{i-1}) = \frac{\sum_{w \in B(w_{i-2}, w_{i-1})} p_{ML}(w | w_{i-1})}{p_{ML}(w_{i-1})}
\]
\[
p_3(w_i | w_{i-2}, w_{i-1}) = \frac{\sum_{w \in B(w_{i-1})} p_{ML}(w)}{p_{ML}(w_{i-1})}
\]

Question  Does the above model form a valid probability distribution? Prove your answer. If it does not form a valid probability distribution, suggest how to make it one by modifying \( p_1 \), \( p_2 \) and \( p_3 \), using \( p_{ML} \) (the maximum likelihood estimate) and/or the count function \( c \), and briefly explain why your modification works.

2 Problem 2 (10pt)

In this programming problem, you will build and evaluate two language models. The first language model is the one you discussed in Problem 1. If you found the given language model to be an invalid probability distribution, please use the correct one you suggested. You should also implement another smoothing approach based on linear interpolation between unigram, bi-gram, and tri-gram models. In the write-up, be sure to fully define each model and describe your approach for setting any hyper-parameters.
We provide three corpora to conduct the evaluation, each from a different domain. Compare the two language models using the different corpora. Additionally, explore issues of transferring your model between domains. How does transferring a model from a source domain to a different target domain influence performance? What does it tell you about the language used in the domains and their similarity? Provide graphs, tables, charts or other summary evidence to support any claims you make.

The data files are formatted with each line containing a tokenized sentence (white spaces mark token boundaries). In addition, each corpus is accompanied by a readme file describing its origin.

You may implement the language models in the programming language of your choice. However, please provide well commented code if you want any partial credit. Additionally, if you have multiple files, please provide a short description in the preamble of each file. Your submission will not be evaluated for efficiency, but we recommend keeping such issues in mind to better streamline the experiments.

**Bonus (2pt)** Design an approach for using a small fraction of the target domain data to adapt the model trained on the source domain. How does it influence performance? How close can you get to within-domain performance?