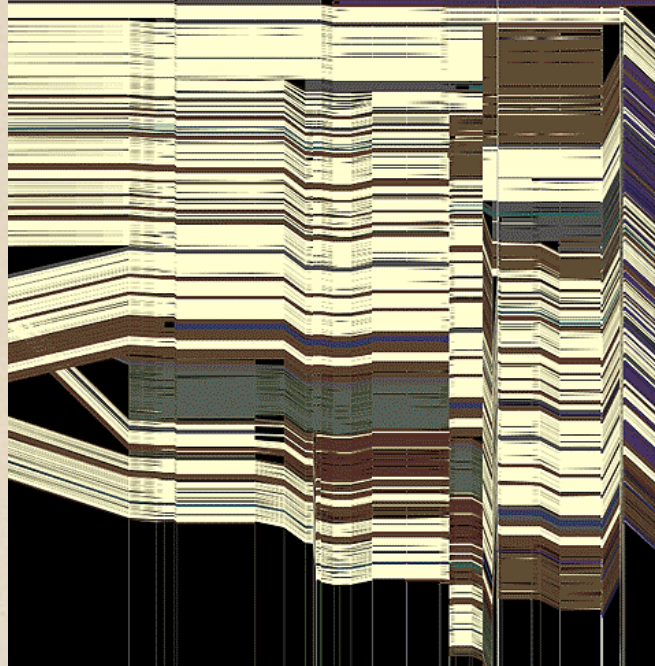
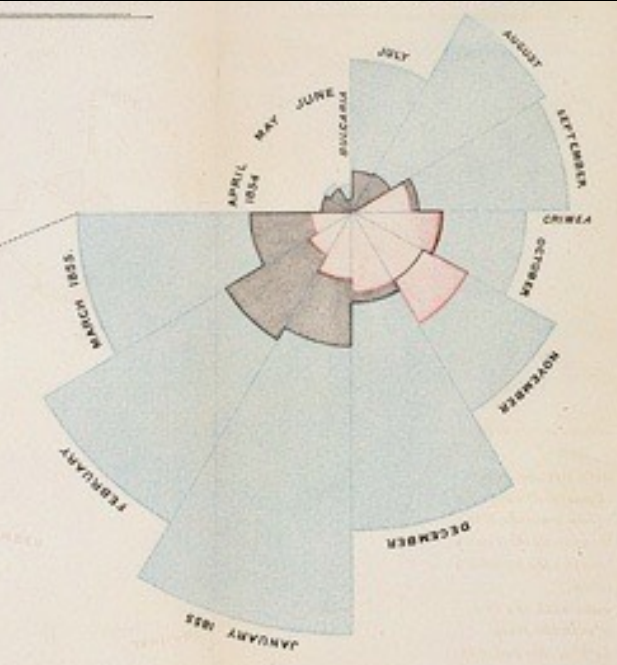


CSE 512 - Data Visualization

Exploratory Data Analysis



Jeffrey Heer University of Washington

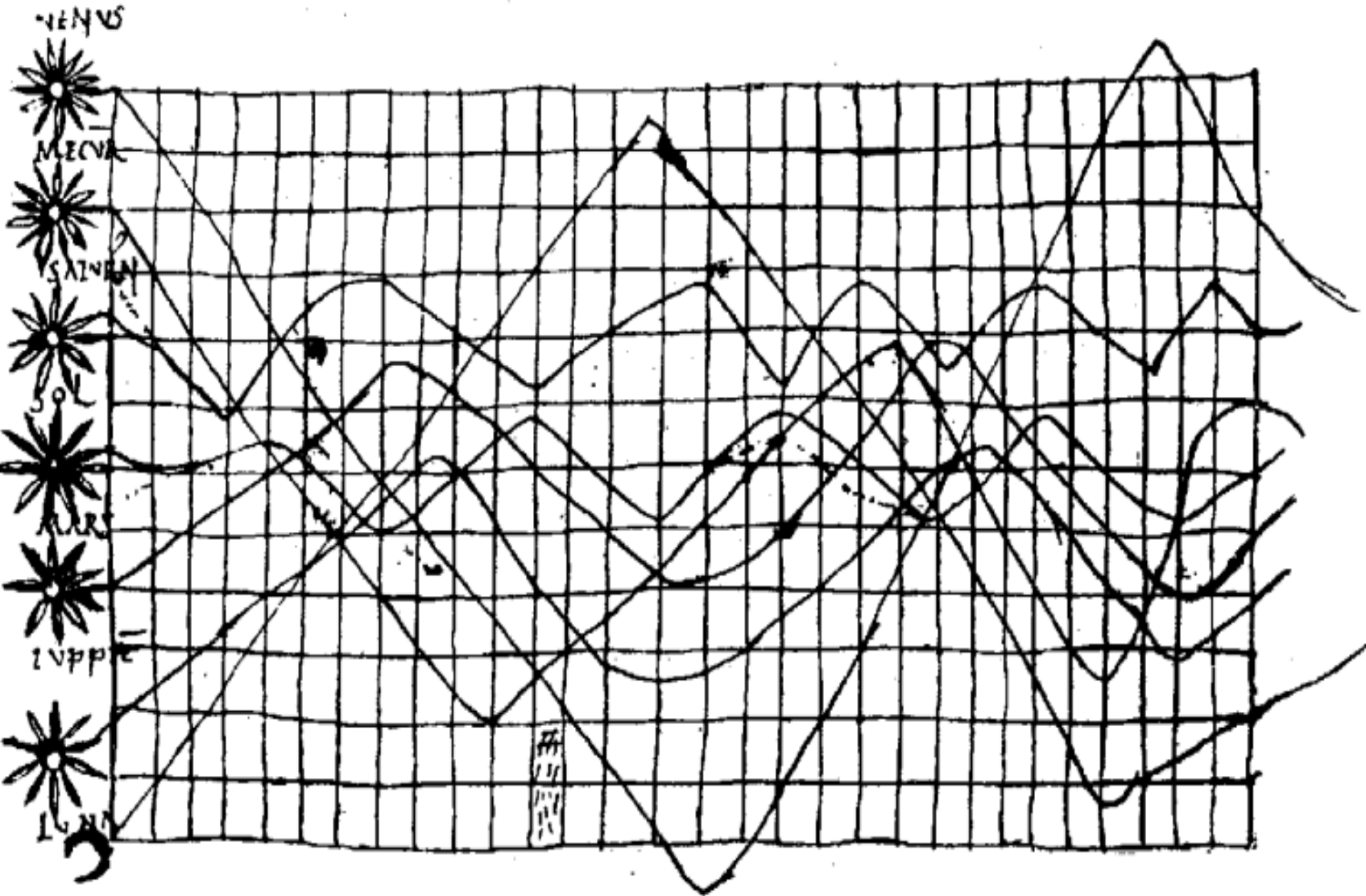
What was the **first**
data visualization?

0 BC



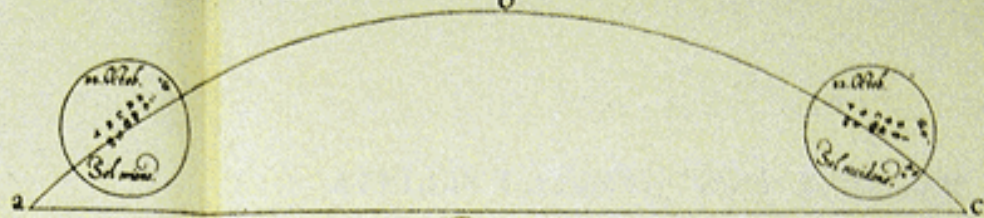
~6200 BC Town Map of Catal Hyük, Konya Plain, Turkey

0 BC



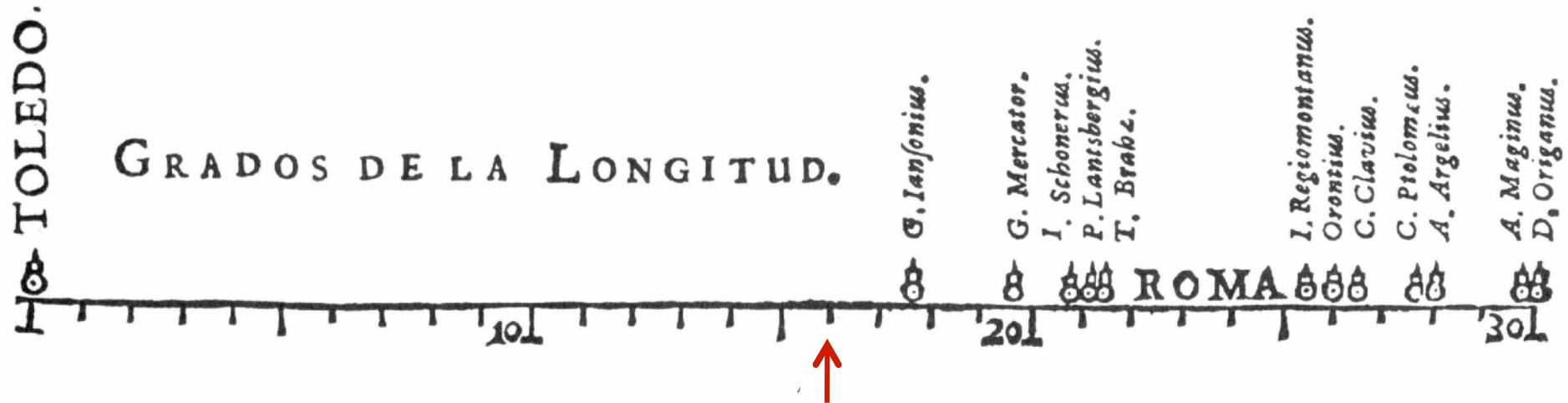
~950 AD Position of Sun, Moon and Planets

MACVLAE IN SOLE APPARENTES, OBSERVATAE
anno 1611. ad latitudinem grad. 48. min. 40.

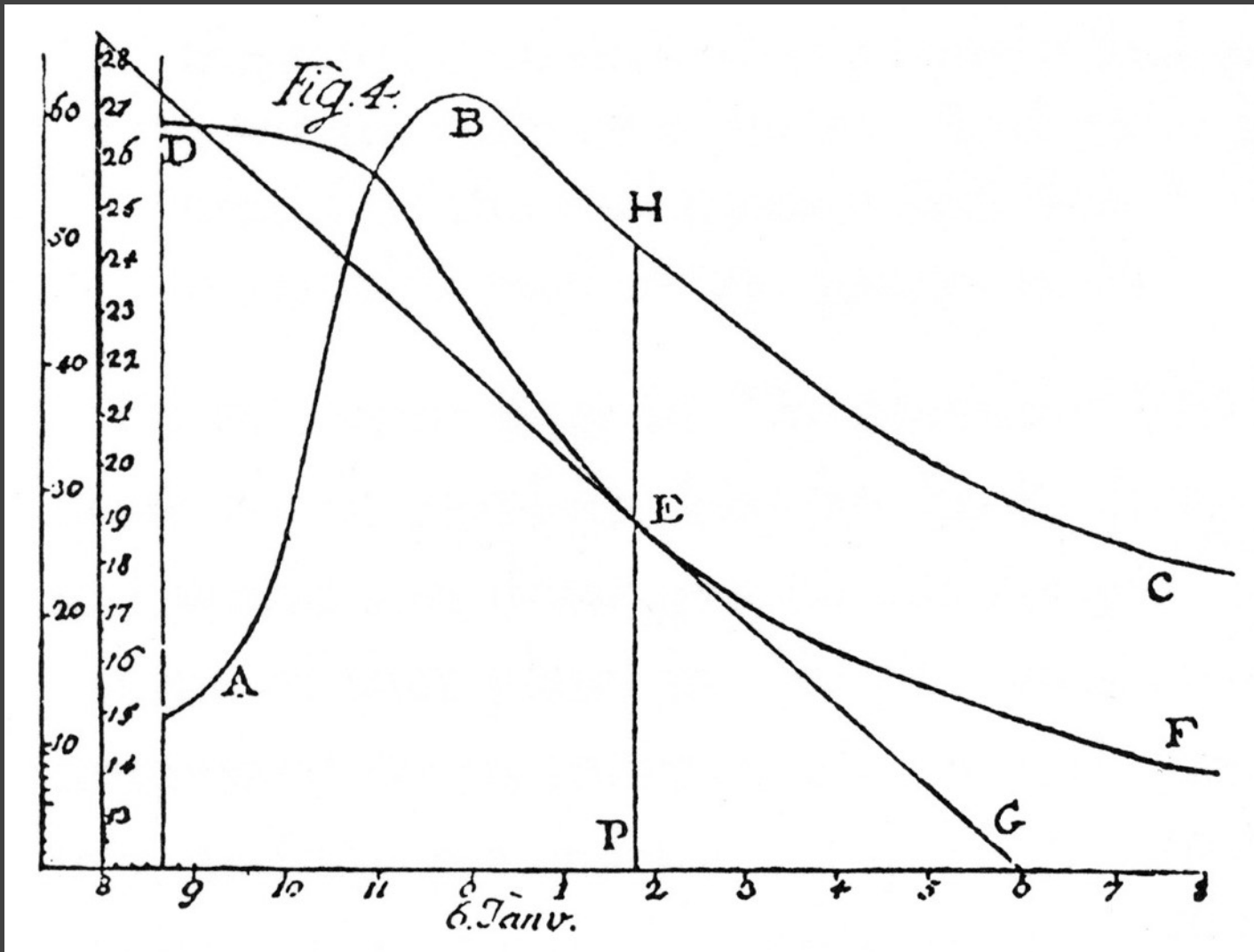


Macula M, es
hac tenus in sarum
mat. oima, nullig
prima magnitudinis
sideri fisco cedit.

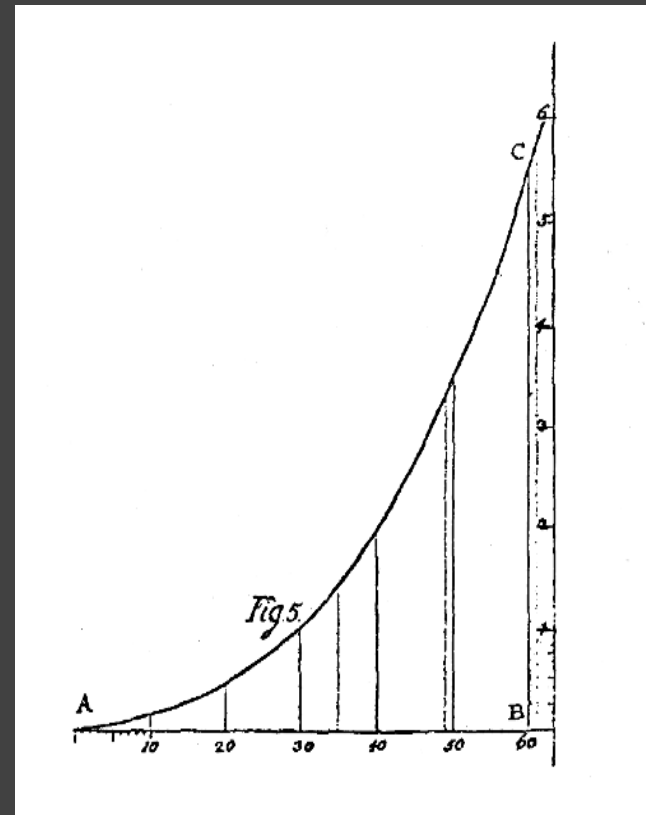
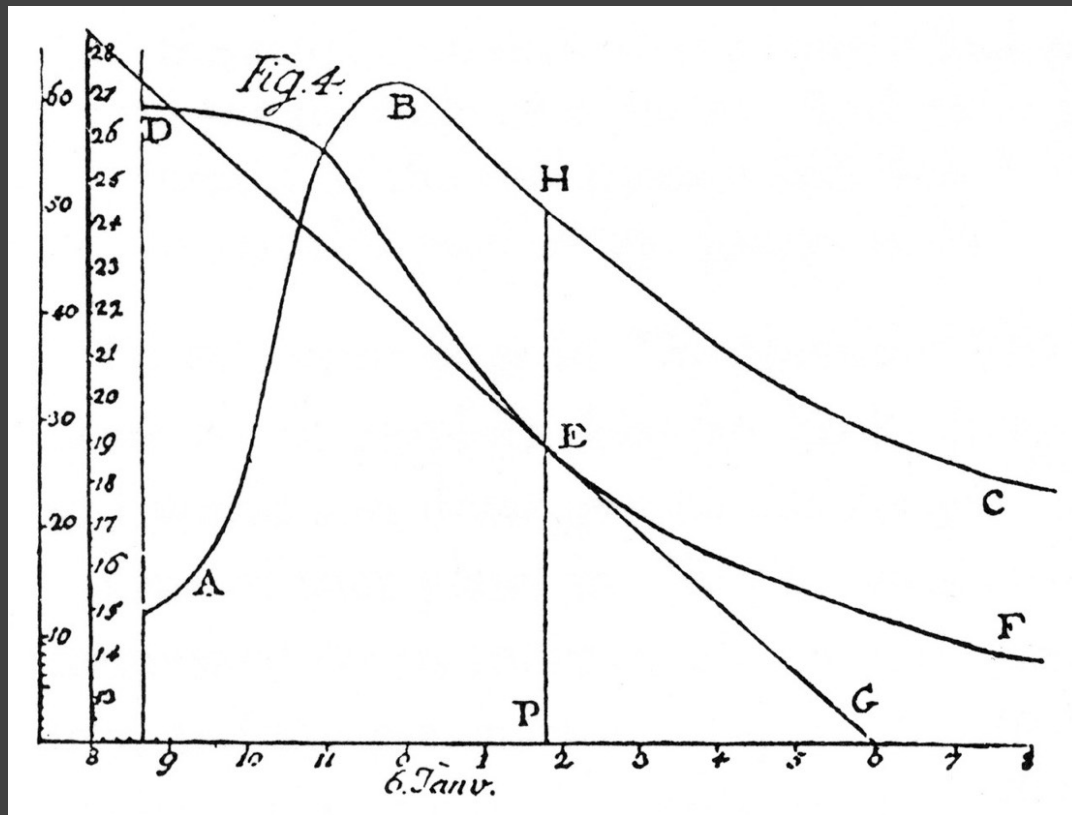
Figura qua
habet annexum
signum X, et
O littera.



Longitudinal distance between Toledo and Rome, van Langren 1644



The Rate of Water Evaporation, Lambert 1765



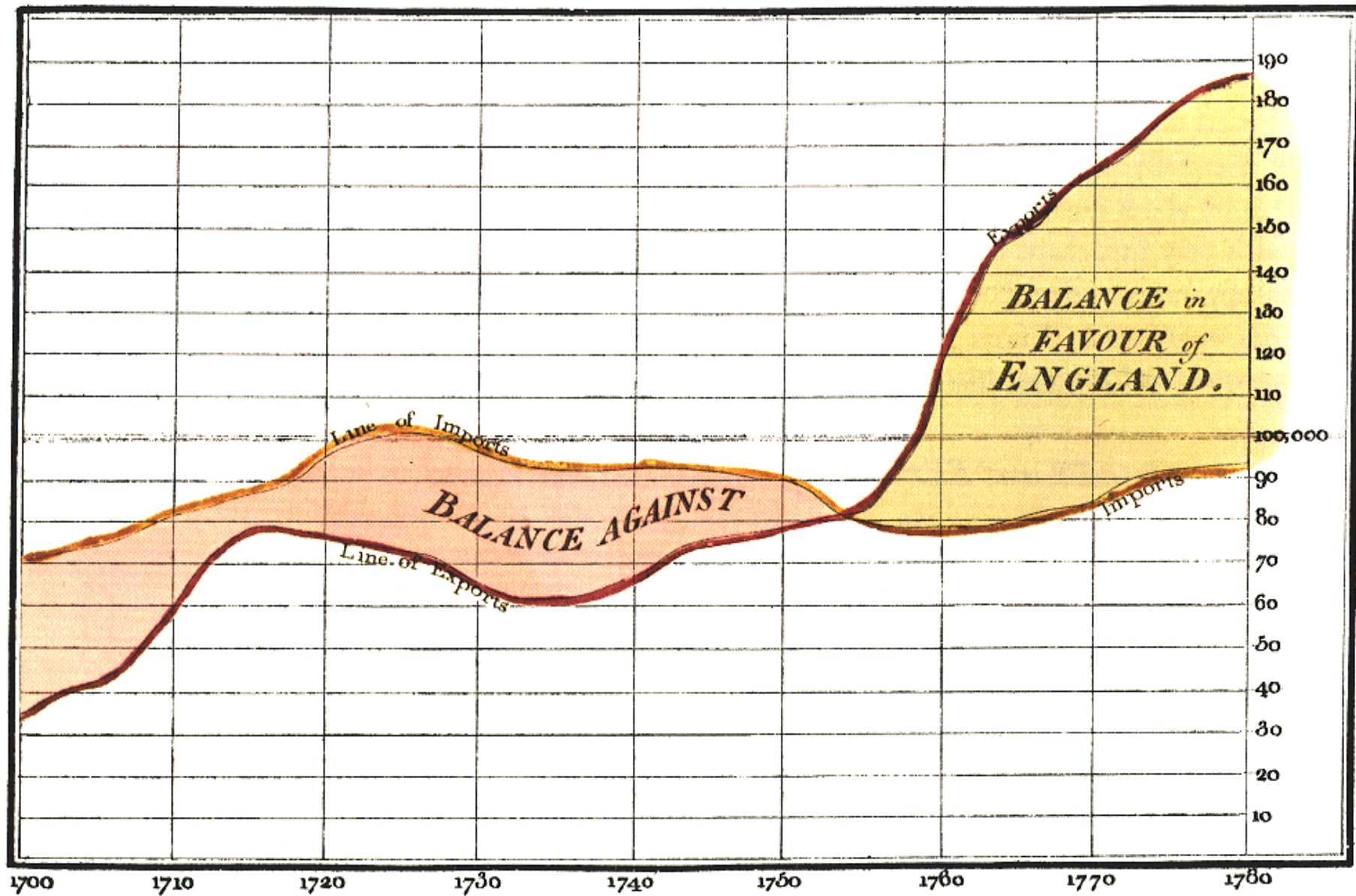
The Rate of Water Evaporation, Lambert 1765

The **Golden Age** of Data Visualization

1786 1900

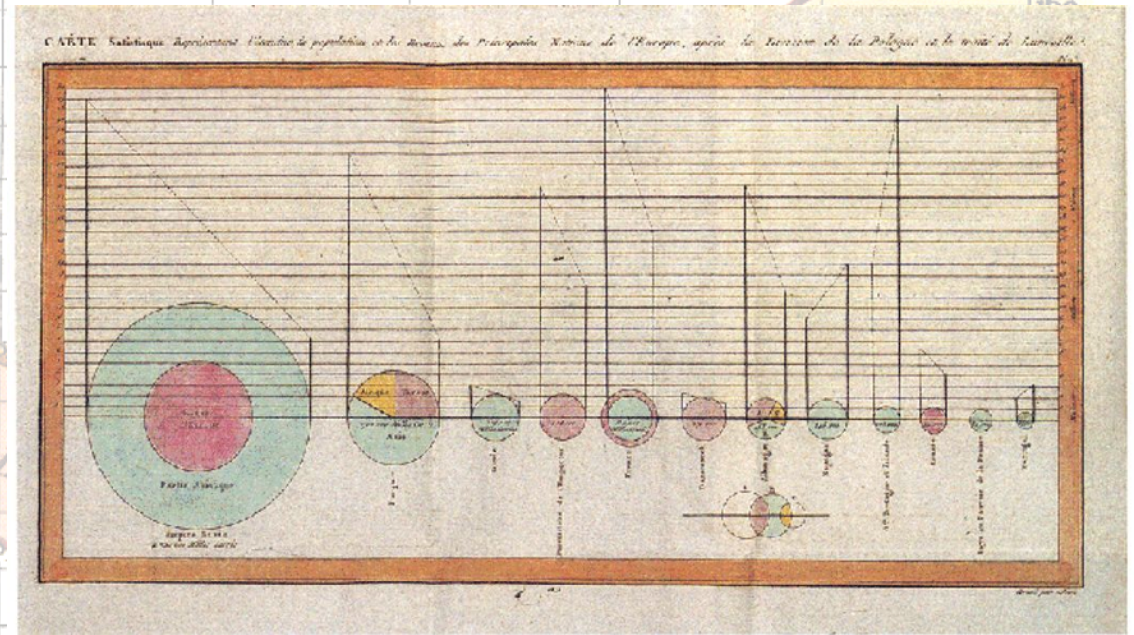
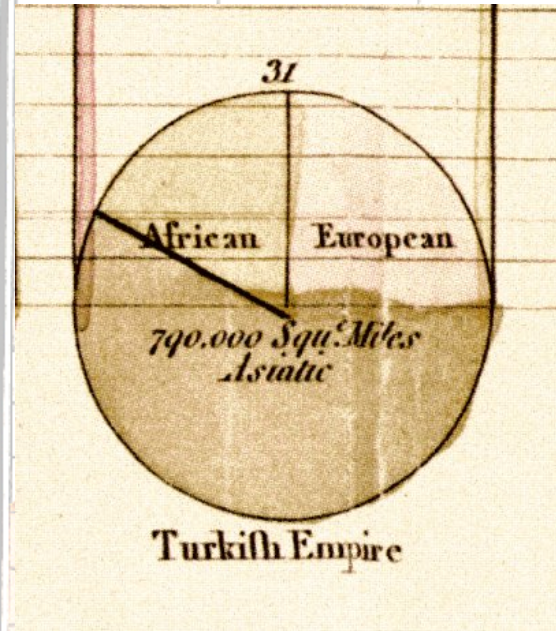
A horizontal timeline bar at the bottom of the slide. It is a thin white line with a small vertical tick mark at the left end. A red rectangular highlight is positioned on the right side of the bar, corresponding to the year 1900.

Exports and Imports to and from DENMARK & NORWAY from 1700 to 1780.

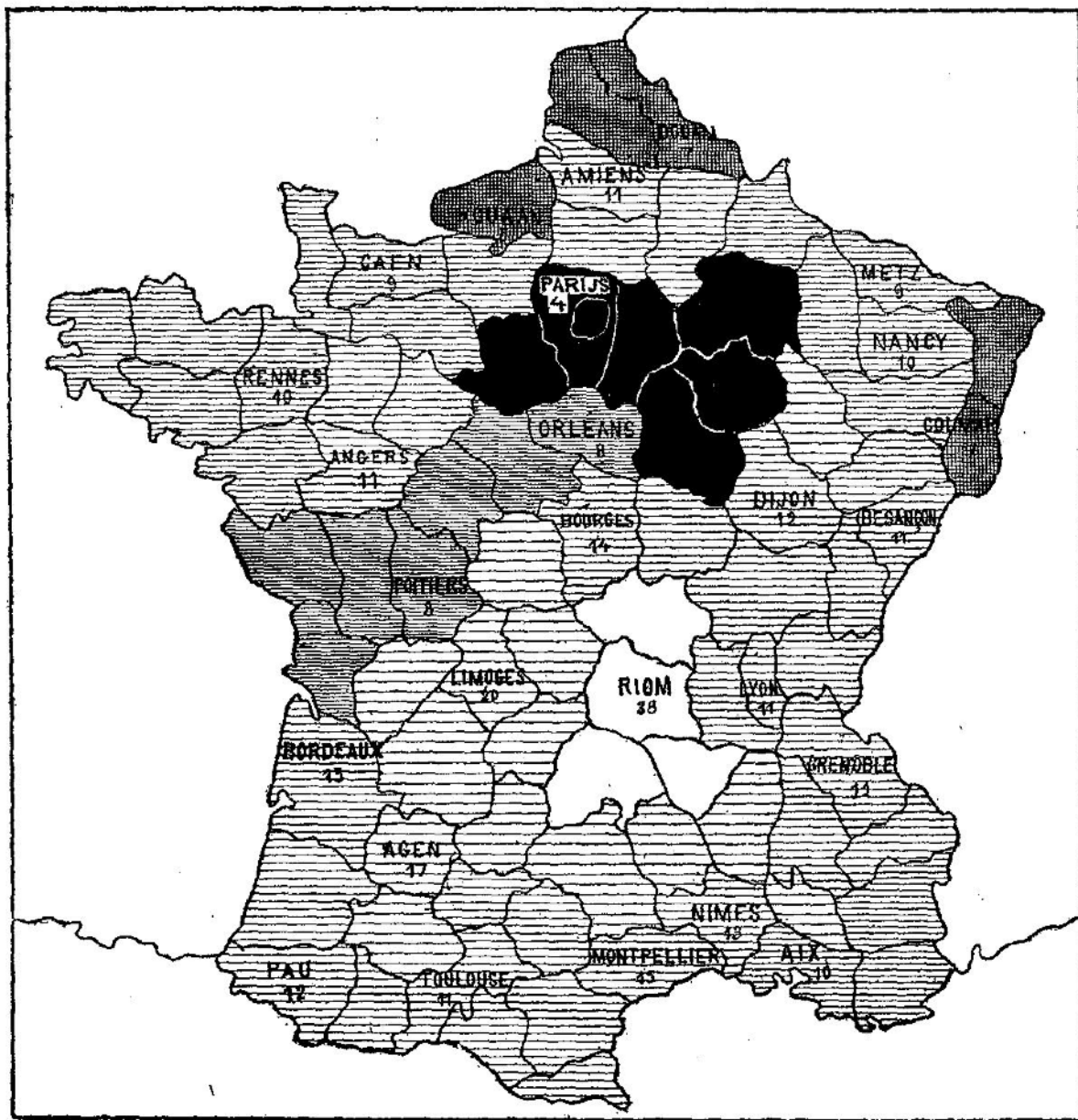


The Commercial and Political Atlas, William Playfair 1786

Exports and Imports to and from DENMARK & NORWAY from 1700 to 1780.



Statistical Breviary, William Playfair 1801



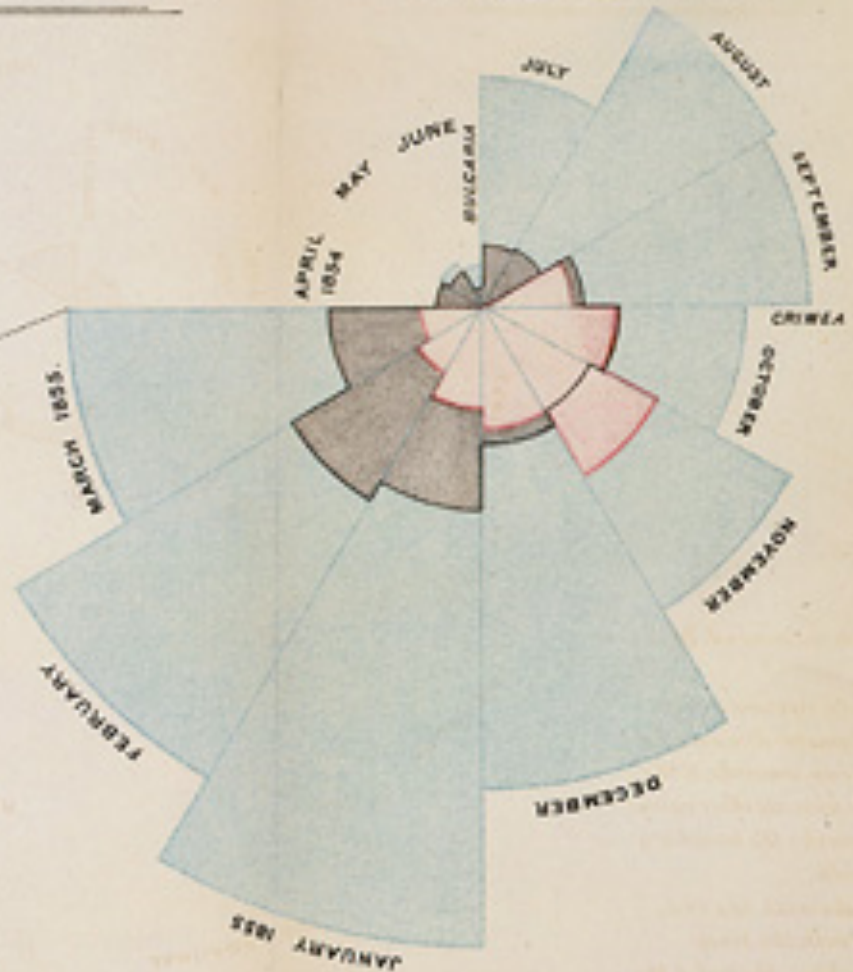
1786

1826(?) Illiteracy in France, Pierre Charles Dupin

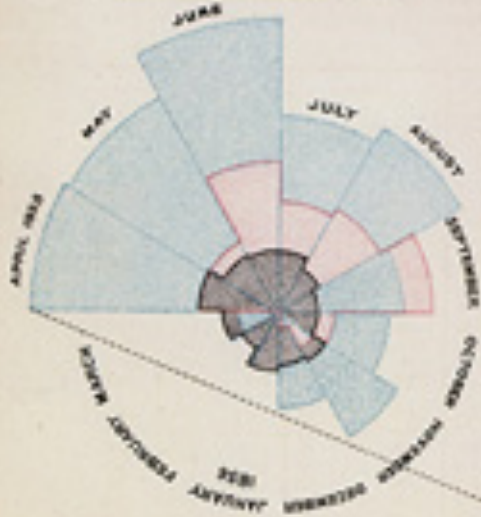


DIAGRAM OF THE CAUSES OF MORTALITY IN THE ARMY IN THE EAST.

1.
APRIL 1854 TO MARCH 1855.



2.
APRIL 1855 TO MARCH 1856.



"to affect thro' the Eyes
what we fail to convey to
the public through their
word-proof ears"

1786

1856 "Coxcomb" of Crimean War Deaths, Florence Nightingale



CARTE descriptive et approximative de la **Houille Anglaise** exportée en 1884 dessinée par M^r MINARD, Ingénieur Civil des Ponts et Chaussées et membre de l'Académie des Sciences.

Les courbes représentent dans cette carte les quantités de houille exportées de la Grande-Bretagne par les différents ports, en 1884, par rapport à la production totale de la Grande-Bretagne en 1884.

Observation. — Les courbes des quantités de houille exportées de la Grande-Bretagne par les différents ports, en 1884, par rapport à la production totale de la Grande-Bretagne en 1884, sont représentées par des courbes de même couleur que les courbes de la production totale de la Grande-Bretagne en 1884.

Les courbes de la production totale de la Grande-Bretagne en 1884, sont représentées par des courbes de même couleur que les courbes de la production totale de la Grande-Bretagne en 1884.

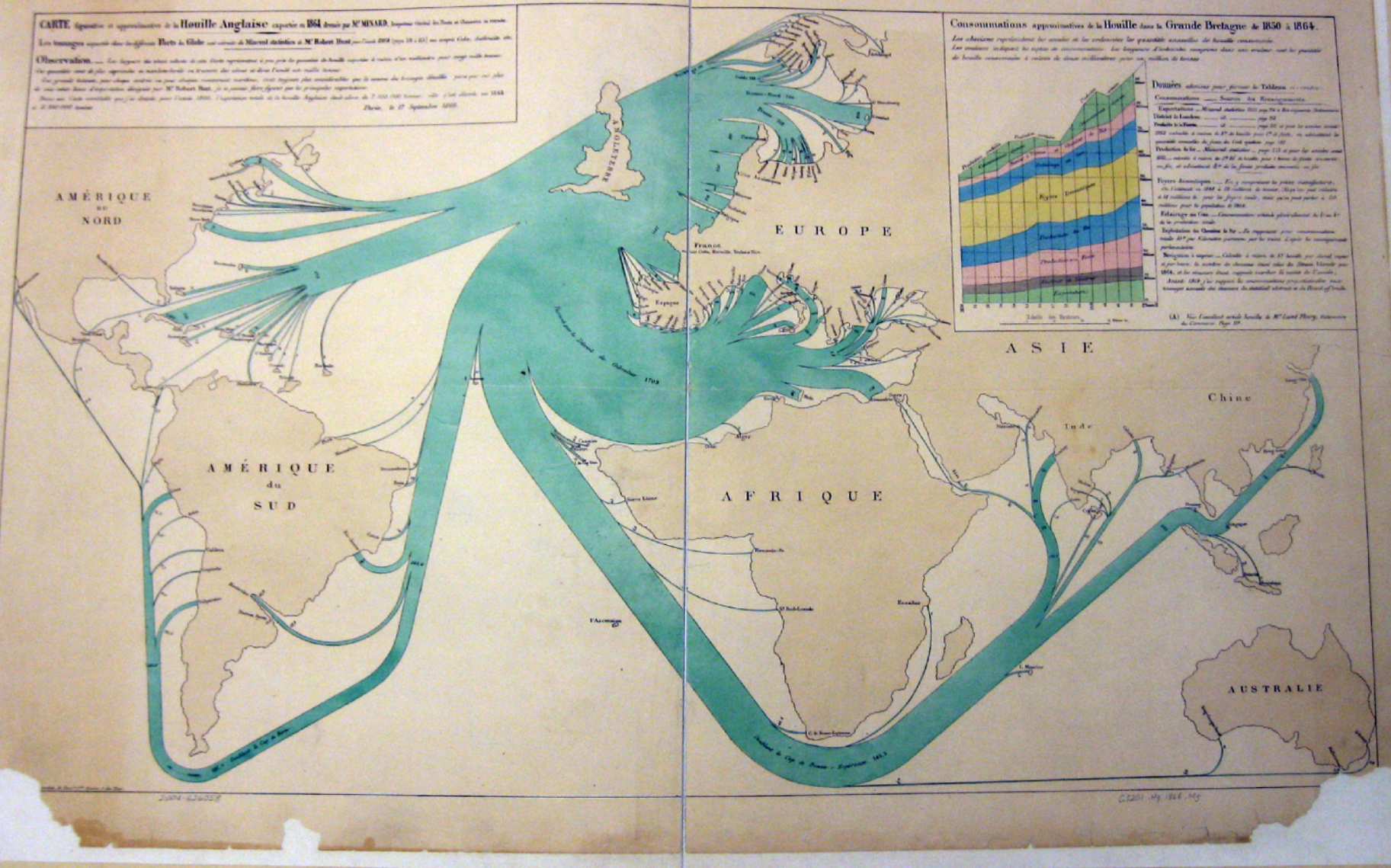
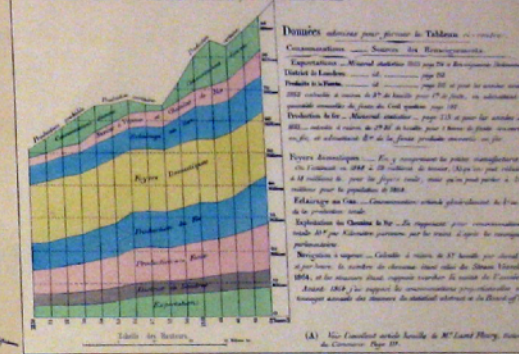
Données pour l'année 1884. — La production totale de la houille en Grande-Bretagne en 1884, a été de 100 millions de tonnes.

Données pour l'année 1884. — La production totale de la houille en Grande-Bretagne en 1884, a été de 100 millions de tonnes.

Consommations approximatives de la Houille dans la Grande-Bretagne & 1850 à 1864.

Les courbes représentent les années et les ordonnées les quantités annuelles de houille consommées.

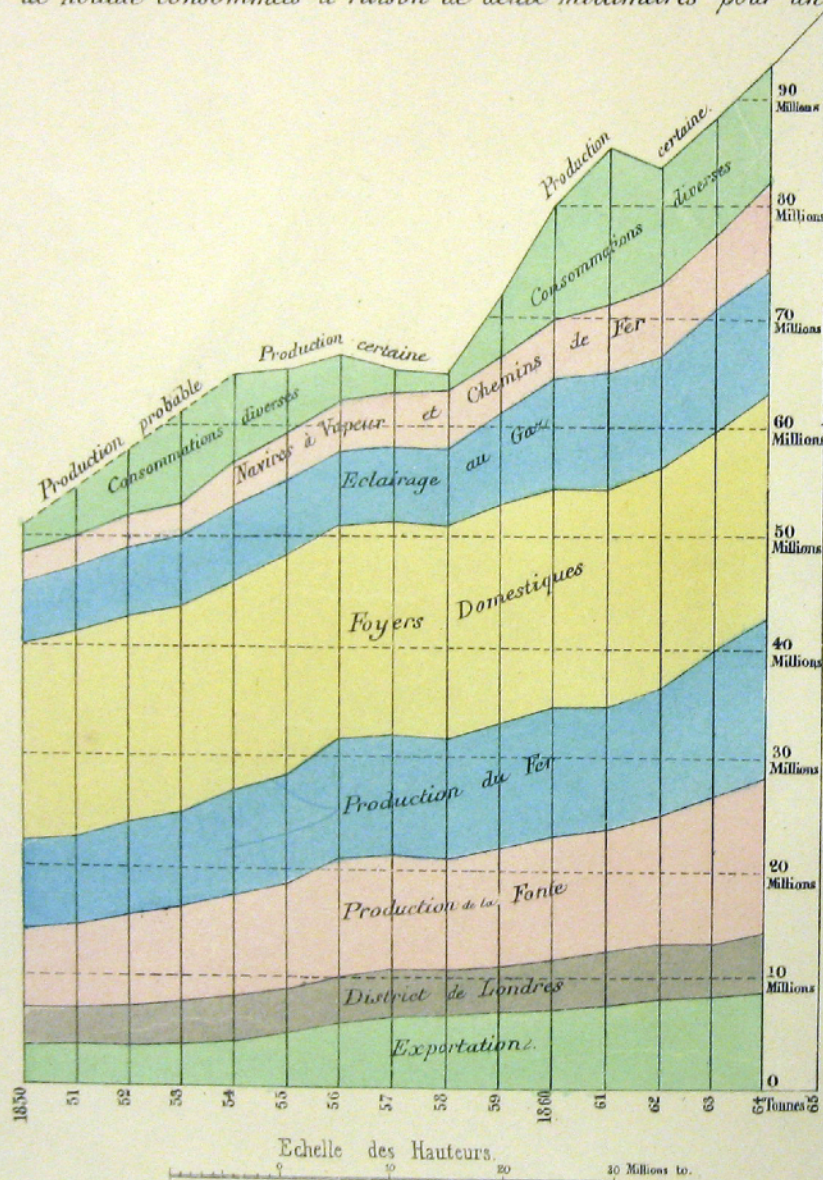
Les courbes indiquent les années de consommation. Les courbes d'ordonnées indiquent les quantités annuelles de houille consommées.



Consommations approximatives de la Houille dans la Grande Bretagne de 1850 à 1864.

Les abscisses représentent les années et les ordonnées les quantités annuelles de houille consommée.

Les couleurs indiquent les espèces de consommations. Les longueurs d'ordonnées comprises dans une couleur sont les quantités de houille consommées à raison de deux millimètres pour un million de tonnes.



Données admises pour former le Tableau ci-contre.

Consommations. — Sources des Renseignements.

Exportations. — *Mineral statistics* 1865 page 214 et Renseignements Parlementaires.

District de Londres. — *id.* — page 213

Produits de la Fonte. — *id.* — page 215 et pour les années avant 1855 calculée à raison de 3^{es} de houille pour 1^{re} de fonte, en admettant les quantités annuelles de fonte du Coal question page 192.

Production du fer — *Mineral statistics* — page 215 et pour les années avant 1855 — calculée à raison de 3^{es} 35 de houille pour 1 tonne de fonte convertie en fer, et admettant $\frac{2}{10}$ de la fonte produite convertie en fer.

Foyers domestiques. — En y comprenant les petites manufactures.

On l'estimait en 1848 à 19 millions de tonnes, (A) qu'on peut réduire à 18 millions to. pour les foyers seuls, mais qu'on peut porter à 20 millions pour la population de 1864.

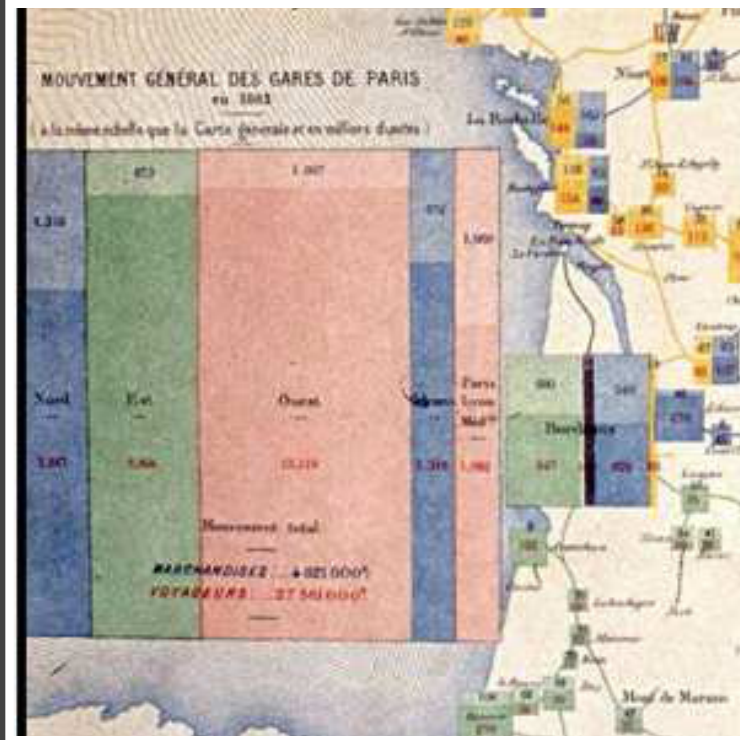
Eclairage au Gaz. — Consommation estimée généralement du $\frac{3}{5}$ au $\frac{1}{3}$ de la production totale.

Exploitation des Chemins de Fer. — En supposant pour consommation totale 10^{es} par Kilomètre parcouru par les trains d'après les renseignements parlementaires.

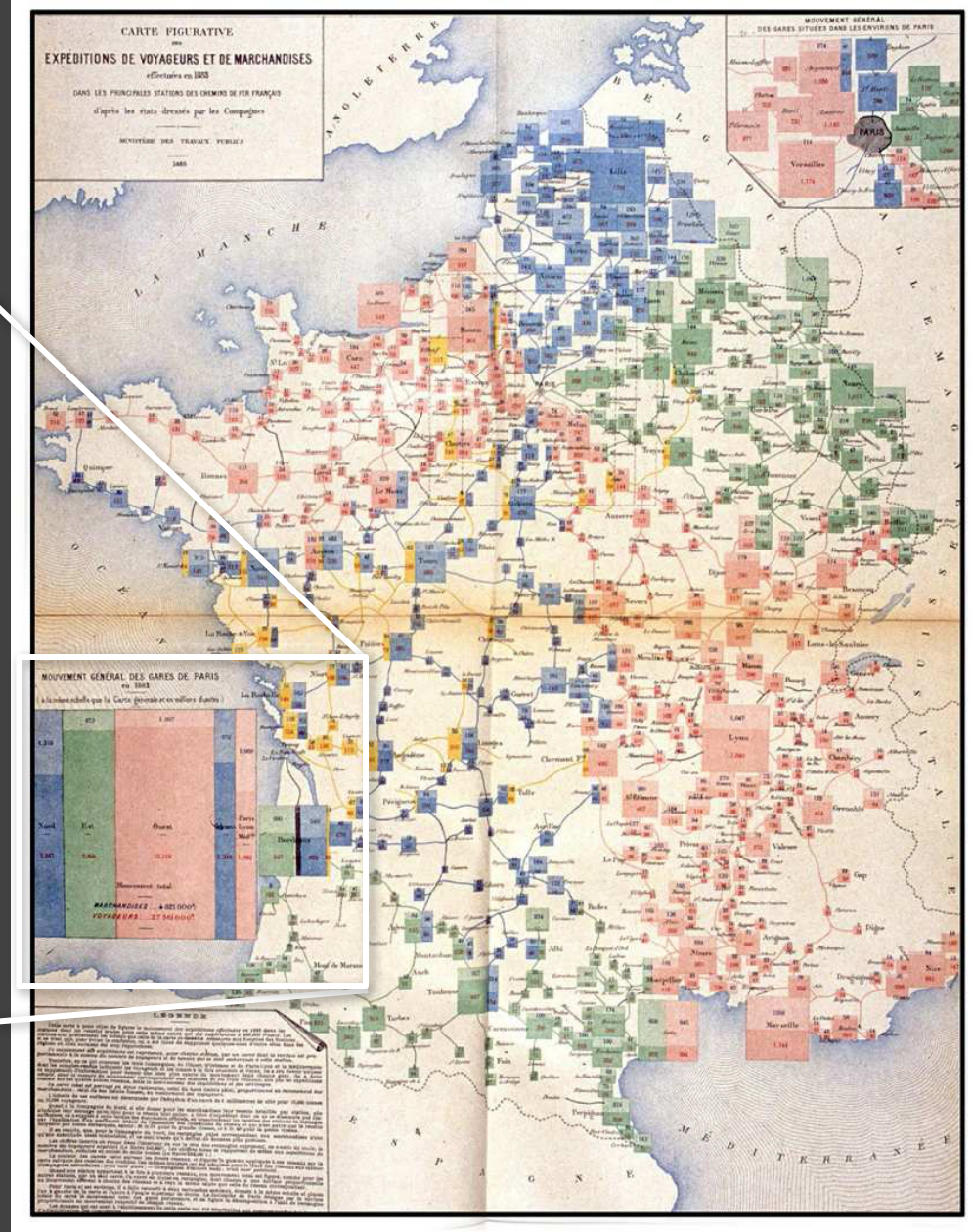
Navigation à vapeur. — Calculée à raison de 5^{es} houille par cheval vapeur et par heure, le nombre de chevaux étant celui du Steam Vessels pour 1864, et les steamers étant supposés marcher la moitié de l'année;

Avant 1864 j'ai supposé les consommations proportionnelles aux tonnages annuels des steamers du statistical abstract et du Board of trade.

(A) Voir l'excellent article houille de M^r Lamé Fleury, Dictionnaire du Commerce Page III.



1786



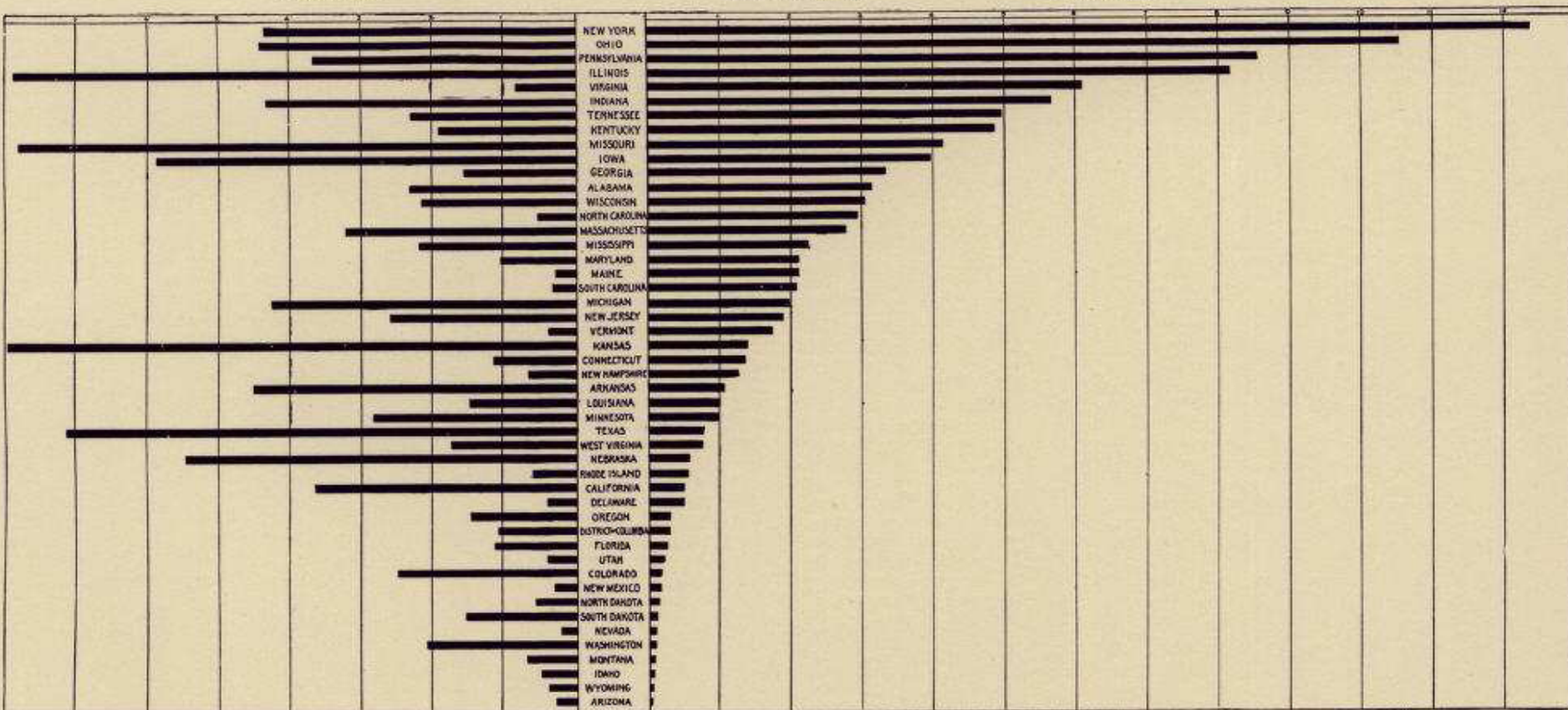
1884 Rail Passengers and Freight from Paris

66. INTERSTATE MIGRATION—NUMBER OF NATIVE IMMIGRANTS AND NATIVE EMIGRANTS, BY STATES AND TERRITORIES: 1890.

Native immigrants.

[Hundreds of thousands.]

Native emigrants.



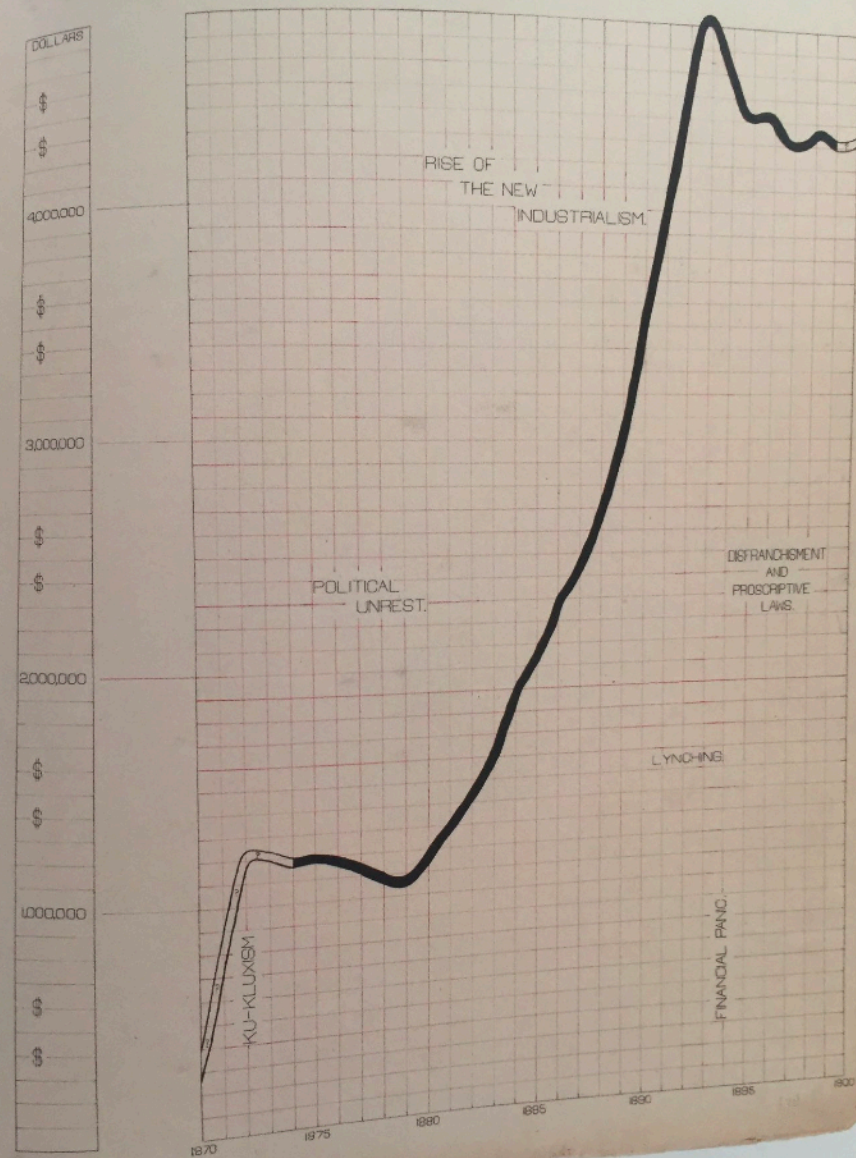
Negro business men in the United States.

Nègres Américains dans les affaires.

Done by Atlanta University.



VALUATION OF TOWN AND CITY PROPERTY OWNED BY GEORGIA NEGROES.



1786

1900 Visualizing Black America, W. E. B. DuBois et al.

The Rise of Statistics

1786



1900



1950

Rise of **formal statistical methods** in the physical and social sciences

Little innovation in graphical methods

A period of **application and popularization**

Graphical methods enter textbooks, curricula, and **mainstream use**

1786

1900

1950





LIFE

1786

Data Analysis & Statistics, Tukey 1962





Four major influences act on data analysis today:

1. The formal theories of statistics.
2. Accelerating developments in computers and display devices.
3. The challenge, in many fields, of more and larger bodies of data.
4. The emphasis on quantification in a wider variety of disciplines.

LIFE



The last few decades have seen the rise of formal theories of statistics, "legitimizing" variation by confining it by assumption to random sampling, often assumed to involve tightly specified distributions, and restoring the appearance of security by emphasizing narrowly optimized techniques and claiming to make statements with "known" probabilities of error.

LIFE



While some of the influences of statistical theory on data analysis have been helpful, others have not.

LIFE



Exposure, the effective laying open of the data to display the unanticipated, is to us a major portion of data analysis. Formal statistics has given almost no guidance to exposure; indeed, it is not clear how the **informality** and **flexibility** appropriate to the **exploratory character of exposure** can be fitted into any of the structures of formal statistics so far proposed.

LIFE



Nothing - not the careful logic of mathematics, not statistical models and theories, not the awesome arithmetic power of modern computers - nothing can substitute here for the **flexibility of the informed human mind.**

Accordingly, both approaches and techniques need to be structured so as to **facilitate human involvement and intervention.**

LIFE

Set A

X	Y
10	8.04
8	6.95
13	7.58
9	8.81
11	8.33
14	9.96
6	7.24
4	4.26
12	10.84
7	4.82
5	5.68

Set B

X	Y
10	9.14
8	8.14
13	8.74
9	8.77
11	9.26
14	8.1
6	6.13
4	3.1
12	9.11
7	7.26
5	4.74

Set C

X	Y
10	7.46
8	6.77
13	12.74
9	7.11
11	7.81
14	8.84
6	6.08
4	5.39
12	8.15
7	6.42
5	5.73

Set D

X	Y
8	6.58
8	5.76
8	7.71
8	8.84
8	8.47
8	7.04
8	5.25
19	12.5
8	5.56
8	7.91
8	6.89

Summary Statistics

$$u_X = 9.0 \quad \sigma_X = 3.317$$

$$u_Y = 7.5 \quad \sigma_Y = 2.03$$

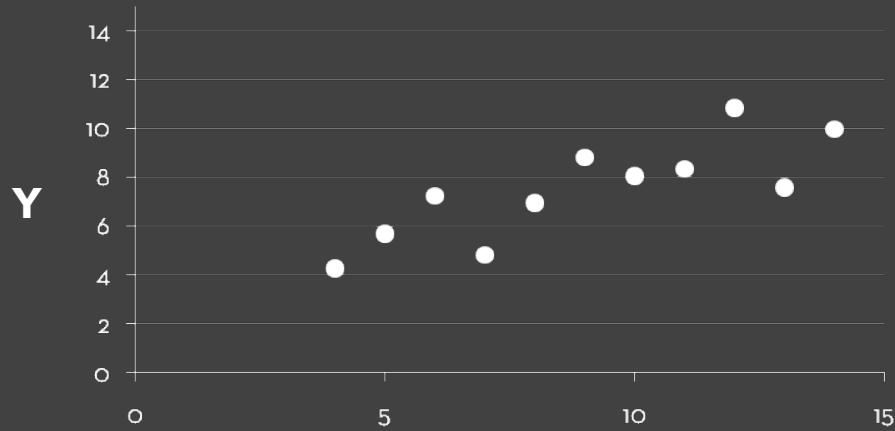
Linear Regression

$$Y = 3 + 0.5 X$$

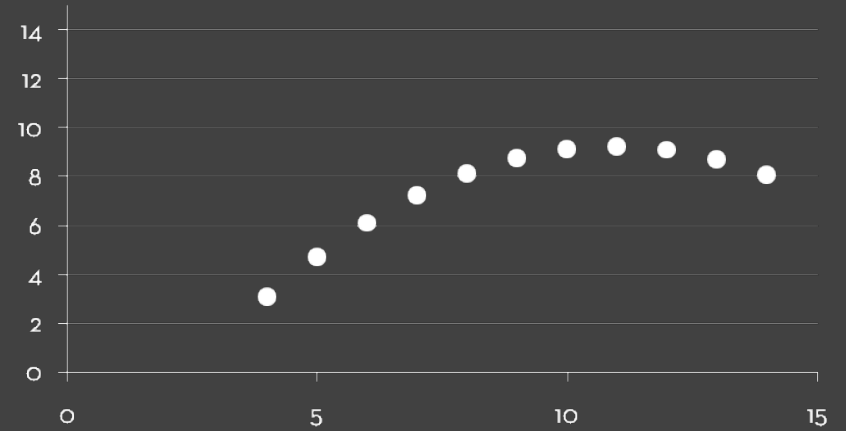
$$R^2 = 0.67$$

[Anscombe 1973]

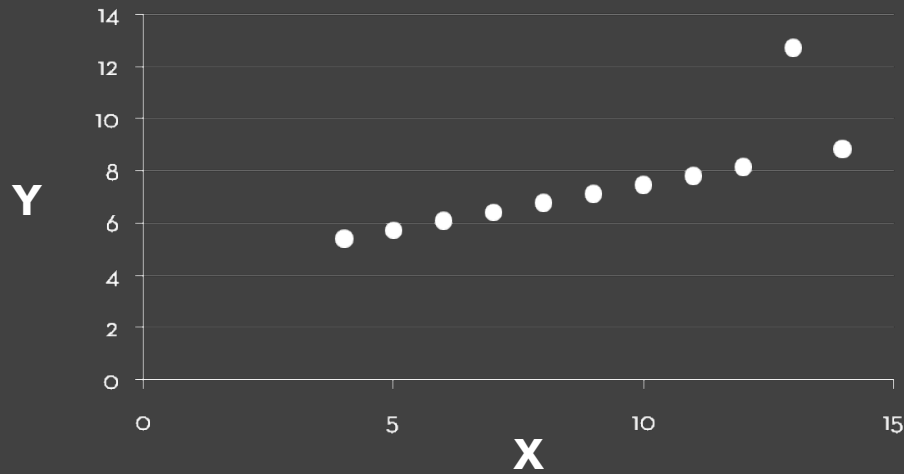
Set A



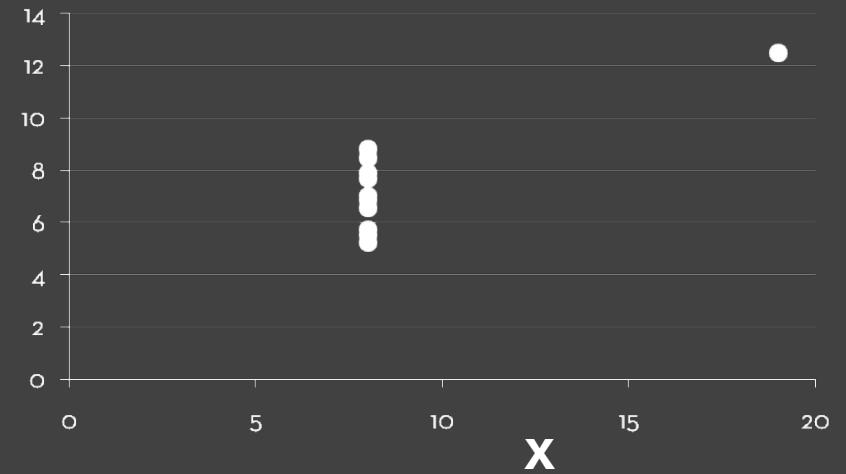
Set B



Set C



Set D



Topics

Exploratory Data Analysis

Data Wrangling

Exploratory Analysis Examples

Tableau / Polaris

Data Wrangling

I spend more than half of my time integrating, cleansing and transforming data without doing any actual analysis. Most of the time I'm lucky if I get to do any "analysis" at all.

Anonymous Data Scientist

[Kandel et al. '12]





**Big Data
Borat**

@BigDataBorat



Following

In Data Science, 80% of time spent prepare data, 20% of time spent complain about need for prepare data.



Reported crime in Alabama

Year	Population	Property crime rate	Burglary rate	Larceny-theft rate	Motor vehicle theft rate
2004	4525375 4029.3	987 2732.4 309.9			
2005	4548327 3900	955.8 2656 289			
2006	4599030 3937	968.9 2645.1 322.9			
2007	4627851 3974.9	980.2 2687 307.7			
2008	4661900 4081.9	1080.7 2712.6 288.6			

Reported crime in Alaska

Year	Population	Property crime rate	Burglary rate	Larceny-theft rate	Motor vehicle theft rate
2004	657755 3370.9	573.6 2456.7 340.6			
2005	663253 3615	622.8 2601 391			
2006	670053 3582	615.2 2588.5 378.3			
2007	683478 3373.9	538.9 2480 355.1			
2008	686293 2928.3	470.9 2219.9 237.5			

Reported crime in Arizona

Year	Population	Property crime rate	Burglary rate	Larceny-theft rate	Motor vehicle theft rate
2004	5739879 5073.3	991 3118.7 963.5			
2005	5953007 4827	946.2 2958 922			
2006	6166318 4741.6	953 2874.1 914.4			
2007	6338755 4502.6	935.4 2780.5 786.7			
2008	6500180 4087.3	894.2 2605.3 587.8			

Reported crime in Arkansas

Year	Population	Property crime rate	Burglary rate	Larceny-theft rate	Motor vehicle theft rate
2004	2750000 4033.1	1096.4 2699.7 237			
2005	2775708 4068	1085.1 2720 262			
2006	2810872 4021.6	1154.4 2596.7 270.4			
2007	2834797 3945.5	1124.4 2574.6 246.5			
2008	2855390 3843.7	1182.7 2433.4 227.6			

Reported crime in California

Year	Population	Property crime rate	Burglary rate	Larceny-theft rate	Motor vehicle theft rate
2004	35842038	3423.9 686.1 2033.1 704.8			
2005	36154147	3321 692.9 1915 712			
2006	36457549	3175.2 676.9 1831.5 666.8			
2007	36553215	3032.6 648.4 1784.1 600.2			
2008	36756666	2940.3 646.8 1769.8 523.8			

Reported crime in Colorado

Year	Population	Property crime rate	Burglary rate	Larceny-theft rate	Motor vehicle theft rate
2004	4601821 3918.5	717.3 2679.5 521.6			

DataWrangler

Suggestions		rows: 408 prev next	
Delete rows 8,10			
Delete empty rows			
Delete rows where Property_crime_rate is null			
Delete rows where Year is null			
Script		Export	
▶ Split data repeatedly on newline into rows			
▶ Split data repeatedly on ','			
		#	Property_crime_rate
		Year	
		Reported crime in Alabama	
		2004	4029.3
		2005	3900
		2006	3937
		2007	3974.9
		2008	4081.9
		Reported crime in Alaska	
		2004	3370.9
		2005	3615
		2006	3582
		2007	3373.9

**Wrangler: Interactive Visual Specification
of Data Transformation Scripts**

Sean Kandel et al. *CHI'11*

Data Wrangling

One often needs to manipulate data prior to analysis. Tasks include reformatting, cleaning, quality assessment, and integration.

Approaches include:

Manual manipulation in spreadsheets

Code: arquero (JS), dplyr (R), pandas (Python)

Trifacta Wrangler <http://www.trifacta.com/products/wrangler/>

Open Refine <http://openrefine.org/>

Tidy Data [Wickham 2014]

How do rows, columns, and tables match up with observations, variables, and types? In “tidy” data:

1. Each variable forms a column.
2. Each observation forms a row.
3. Each type of observational unit forms a table.

The advantage is that this provides a flexible starting point for analysis, transformation, and visualization.

Our pivoted table variant was not “tidy”!

(This is a variant of normalized forms in DB theory)

Data Quality

"The first sign that a visualization is good is that it shows you a problem in your data...

...every successful visualization that I've been involved with has had this stage where you realize, "Oh my God, this data is not what I thought it would be!" So already, you've discovered something."

Martin Wattenberg

**Violent
Infants!**

**Marauding
Centenarians!**

???

County (Res):	Prince Georges
Zip Code (Res):	20770
Received:	940706
Complaint Sequence:	1
Source:	Citizen
Reason:	Delinquent
Alleged Offense:	HABAS
Offense Level:	2 - Misdemeanor
County (Off):	Prince Georges
Zip Code (Off):	20770
Area:	V
Office:	71610
Intake Decision Date:	940729
Intake Decision:	Closed
Days to ID:	23
Court Finding:	NONE
Disposition Date:	0
Disposition:	

Query Result: 4792 out of 4792 (100%)

Graph Viewer

Roll-up by:

All

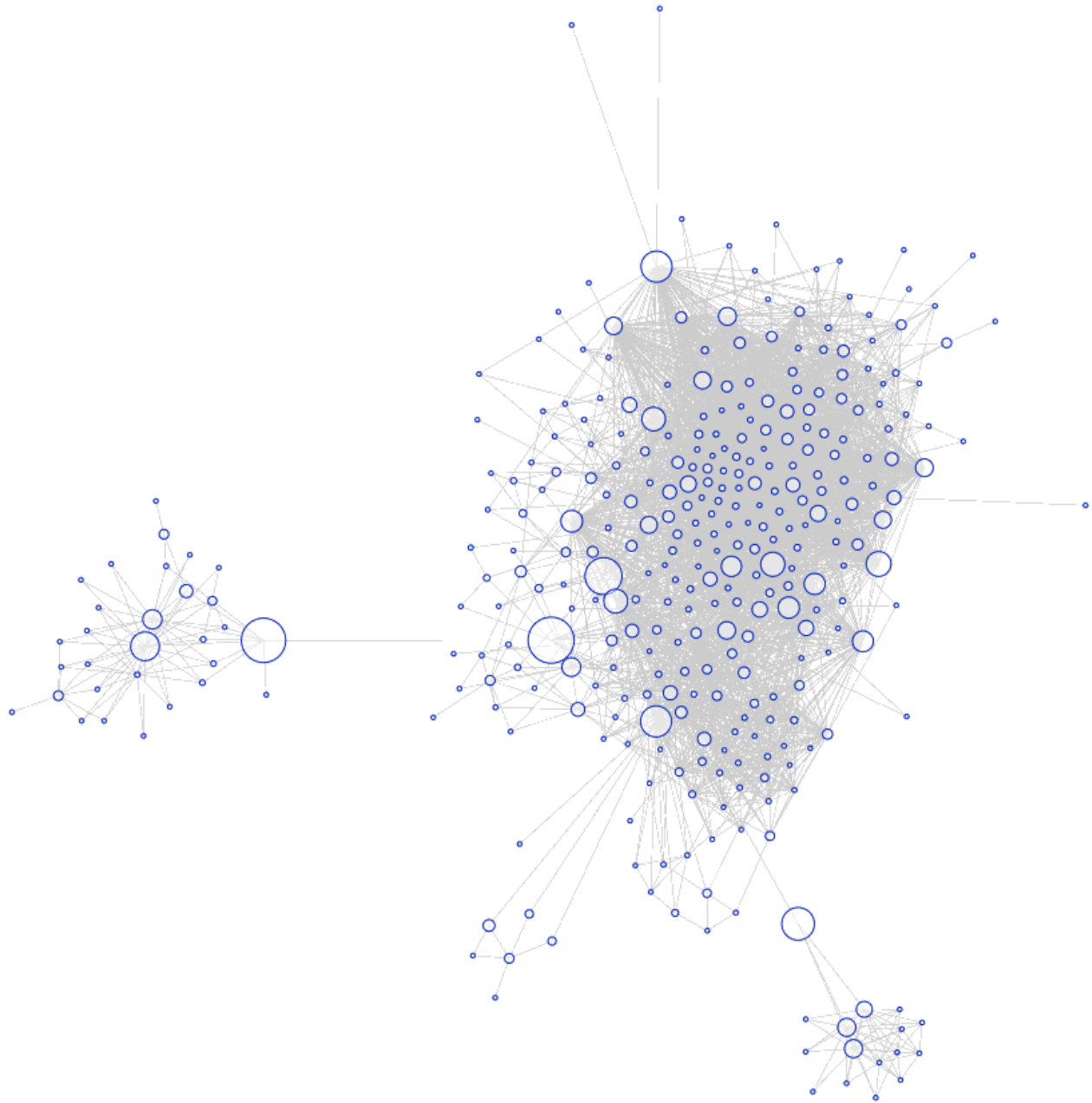
Visualization:

Node-Link

Sort by:

None

Edge centrality filters:

☐ Images☒ Animate

Graph Viewer

Roll-up by:

All

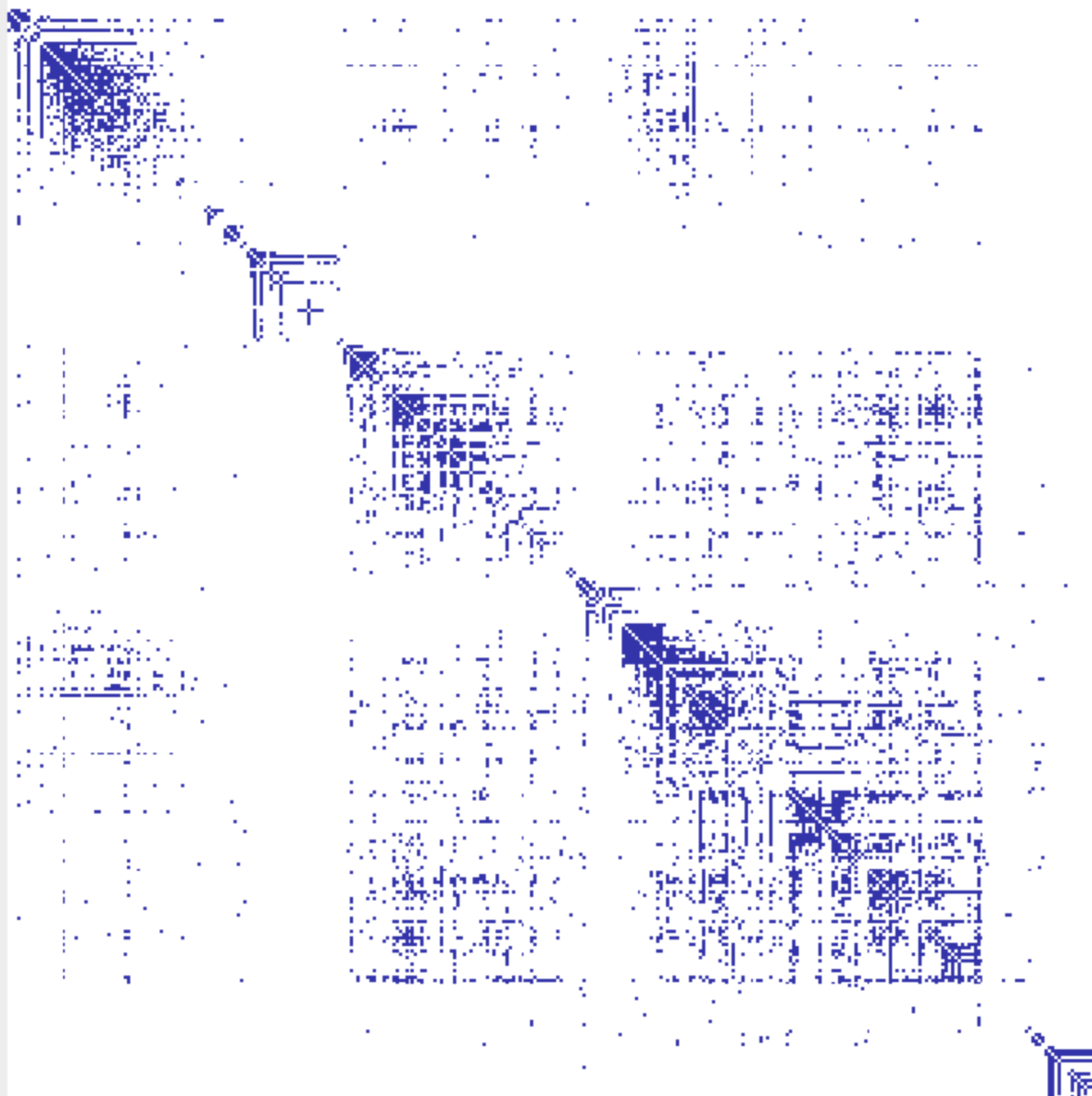
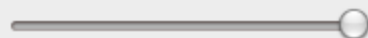
Visualization:

Matrix

Sort by:

Linkage

Edge centrality filters:



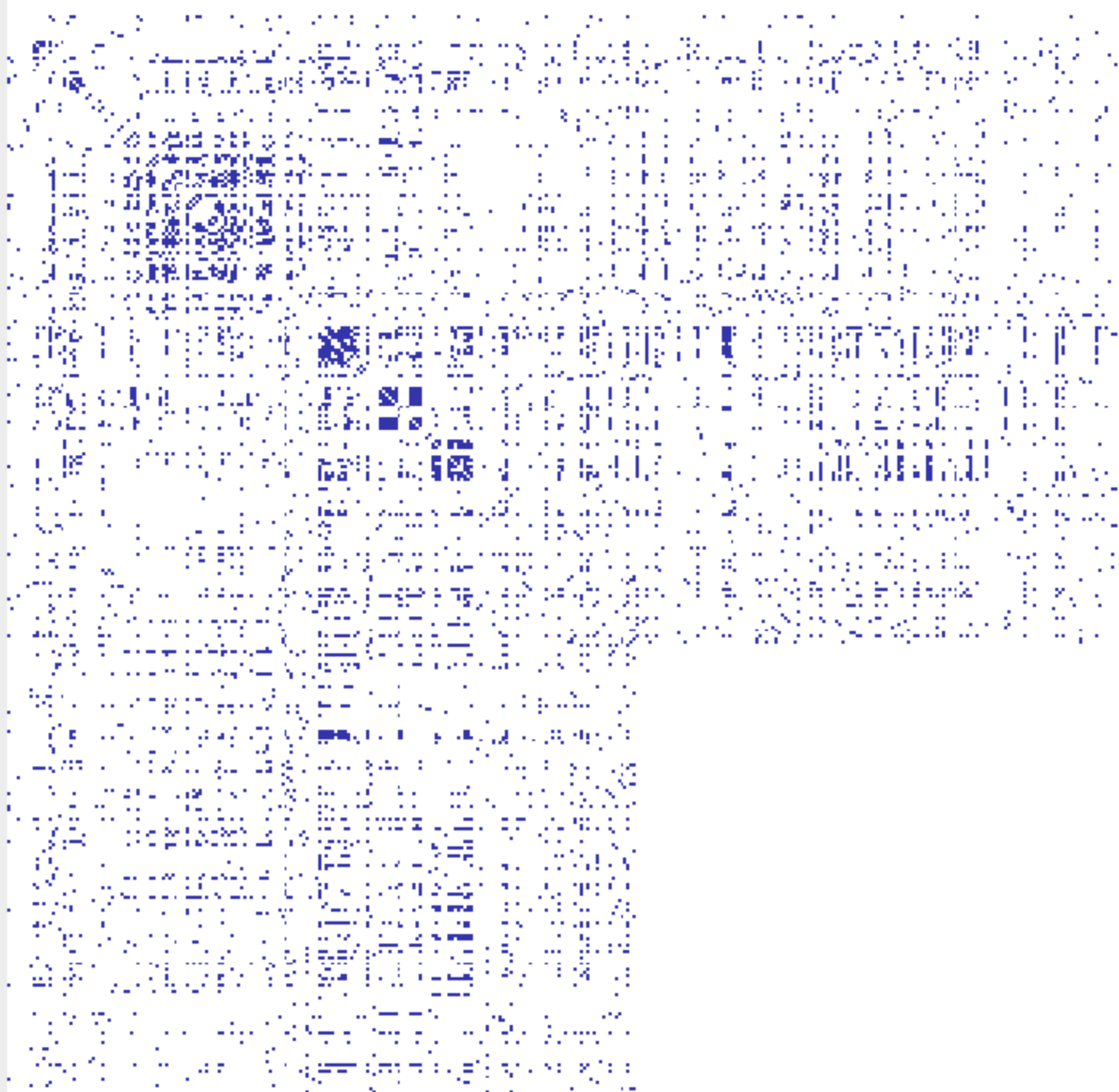
Graph Viewer

Roll-up by:

Visualization:

Sort by:

Edge centrality filters:



Visualize Friends by School?



Data Quality Hurdles

Missing Data	no measurements, redacted, ...?
Erroneous Values	misspelling, outliers, ...?
Type Conversion	e.g., zip code to lat-lon
Entity Resolution	diff. values for the same thing?
Data Integration	effort/errors when combining data

LESSON: Anticipate problems with your data.
Many research problems around these issues!

Administrivia

A1: Visualization Design

Pick a **guiding question**, use it to title your vis.
Design a **static visualization** for that question.
You are free to **use any tools** (inc. pen & paper).

Deliverables (upload via Canvas; see A1 page)

Image of your visualization (PNG or JPG format)

Short description + design rationale (≤ 4 paragraphs)

Due by **11:59 pm, Wednesday April 7.**

A2: Exploratory Data Analysis

Use visualization software to form & answer questions

First steps:

Step 1: Pick domain & data

Step 2: Pose questions

Step 3: Profile the data

Iterate as needed

Create visualizations

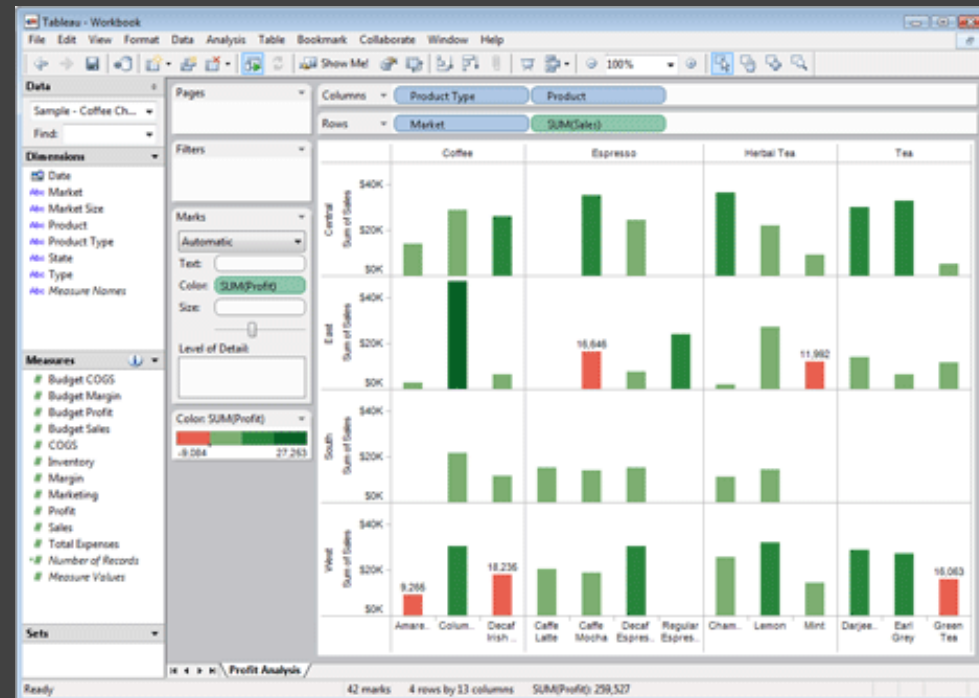
Interact with data

Refine your questions

Author a report

Screenshots of most insightful views (8+)

Include titles and captions for each view



Due by 11:59pm

Friday, Apr 23

Tableau Tutorial (Optional)

Monday April 12, 4:30-5:30pm

Zoom link available on Canvas

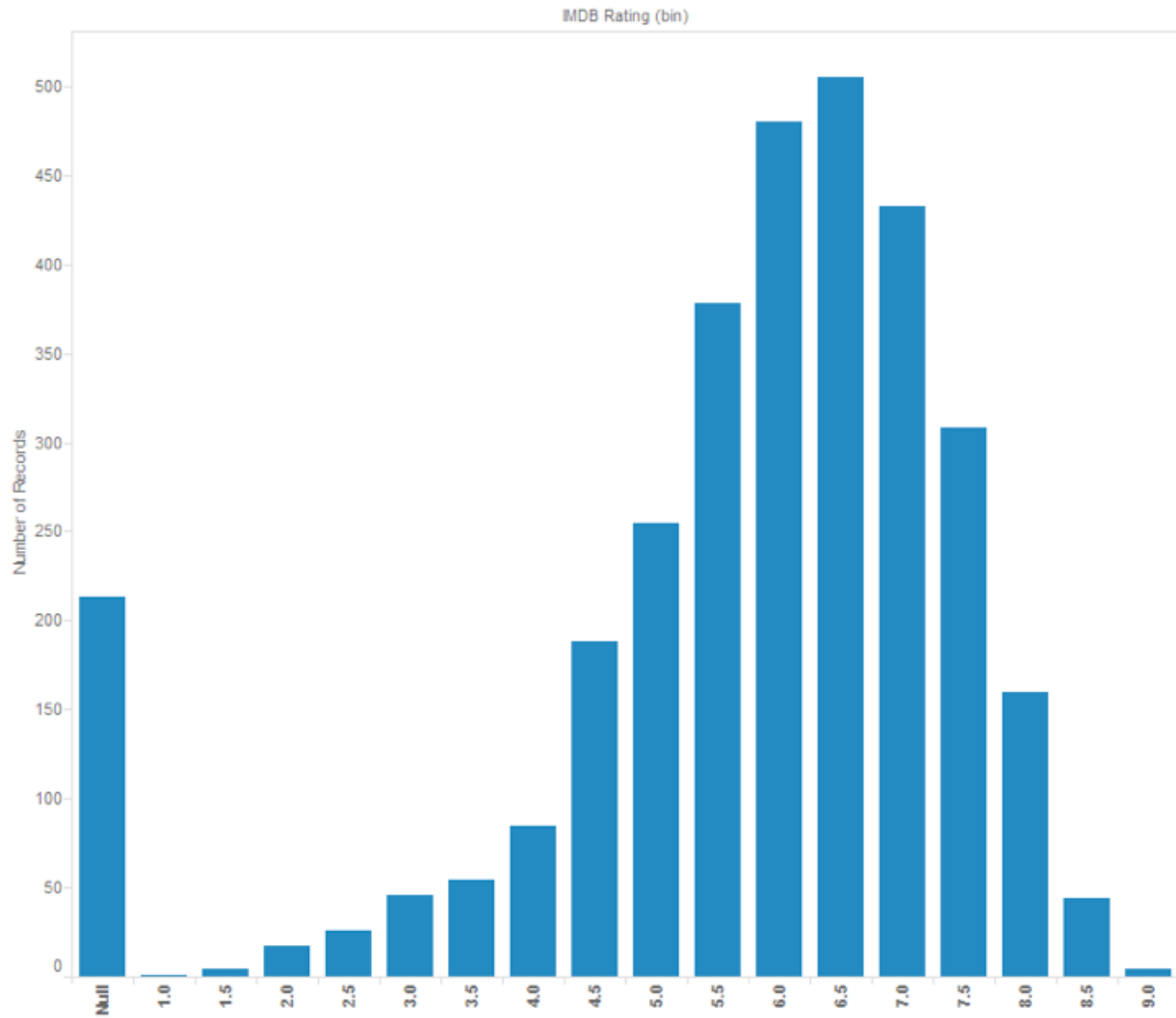
Session will be recorded.

Break Time!

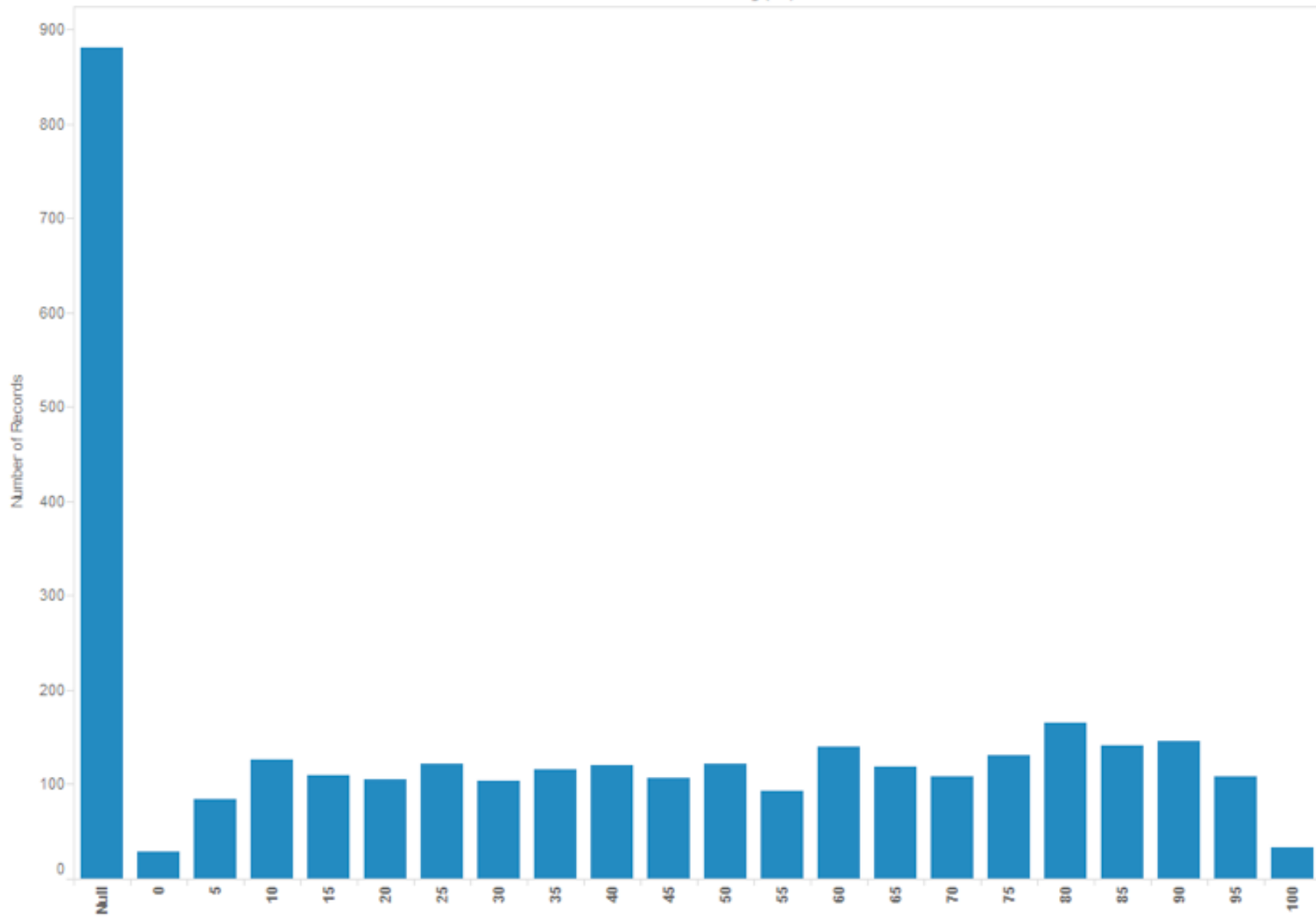
Analysis Example: Motion Pictures Data

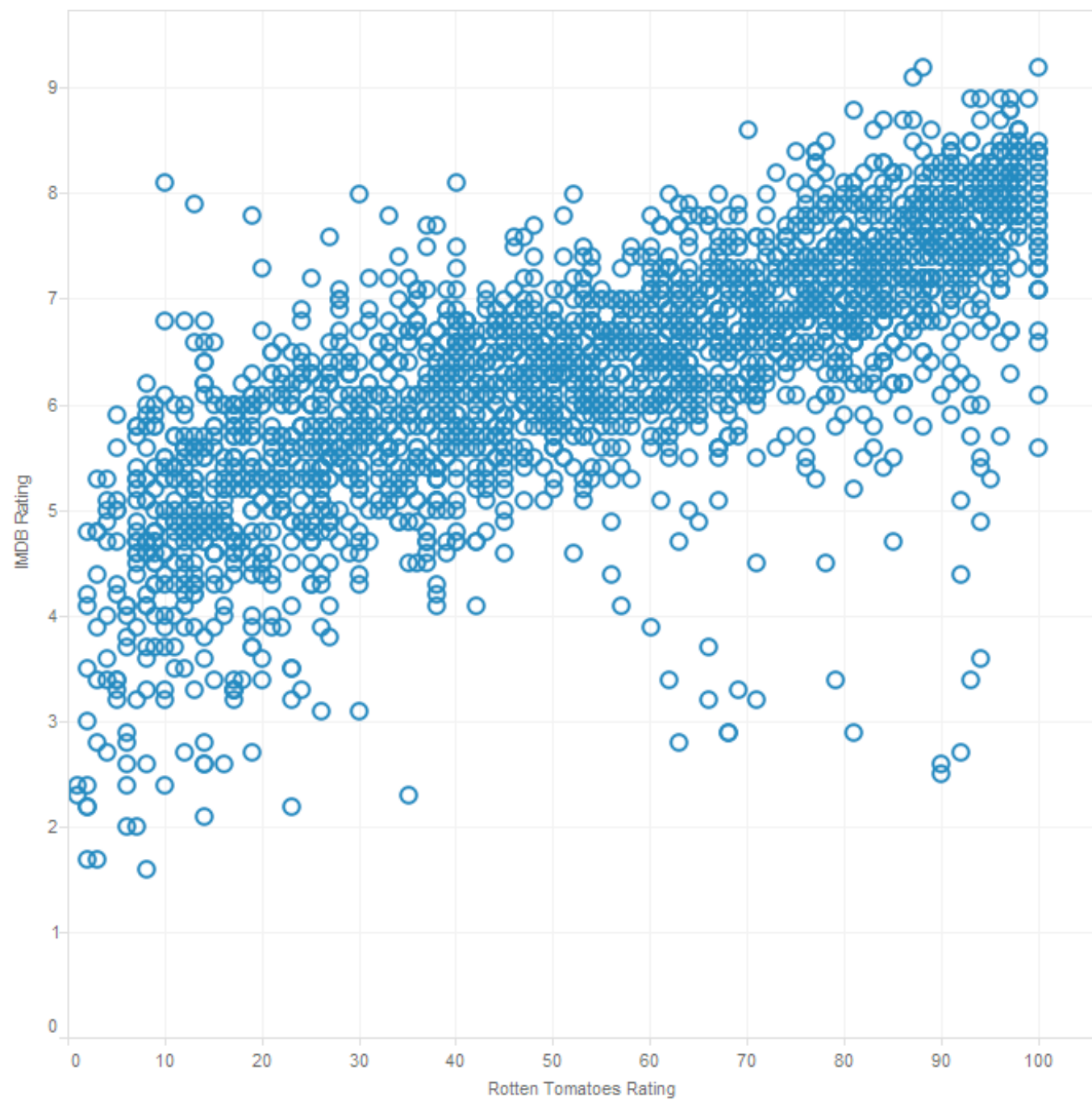
Motion Pictures Data

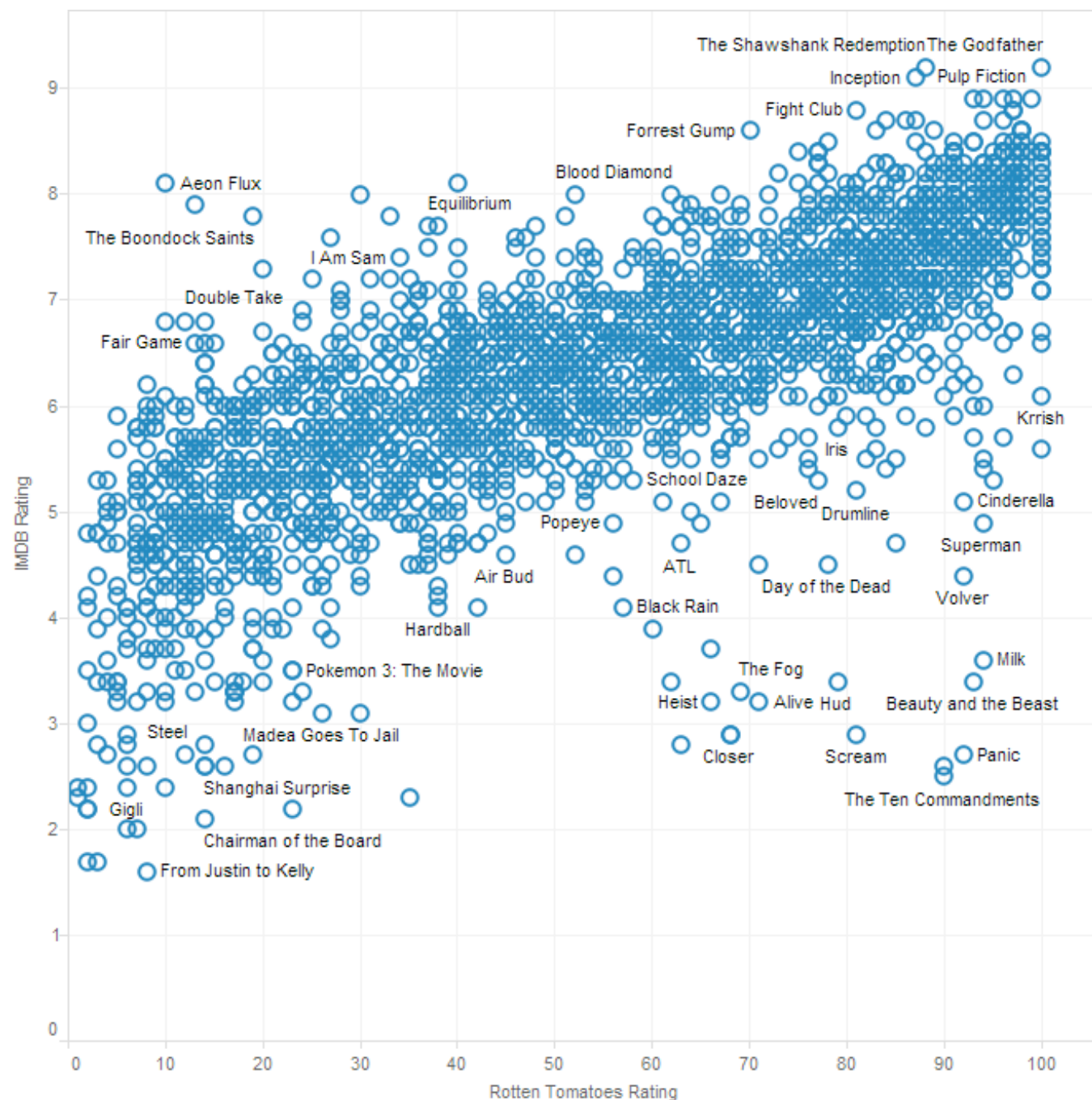
Title	String (N)
IMDB Rating	Number (Q)
Rotten Tomatoes Rating	Number (Q)
MPAA Rating	String (O)
Release Date	Date (T)

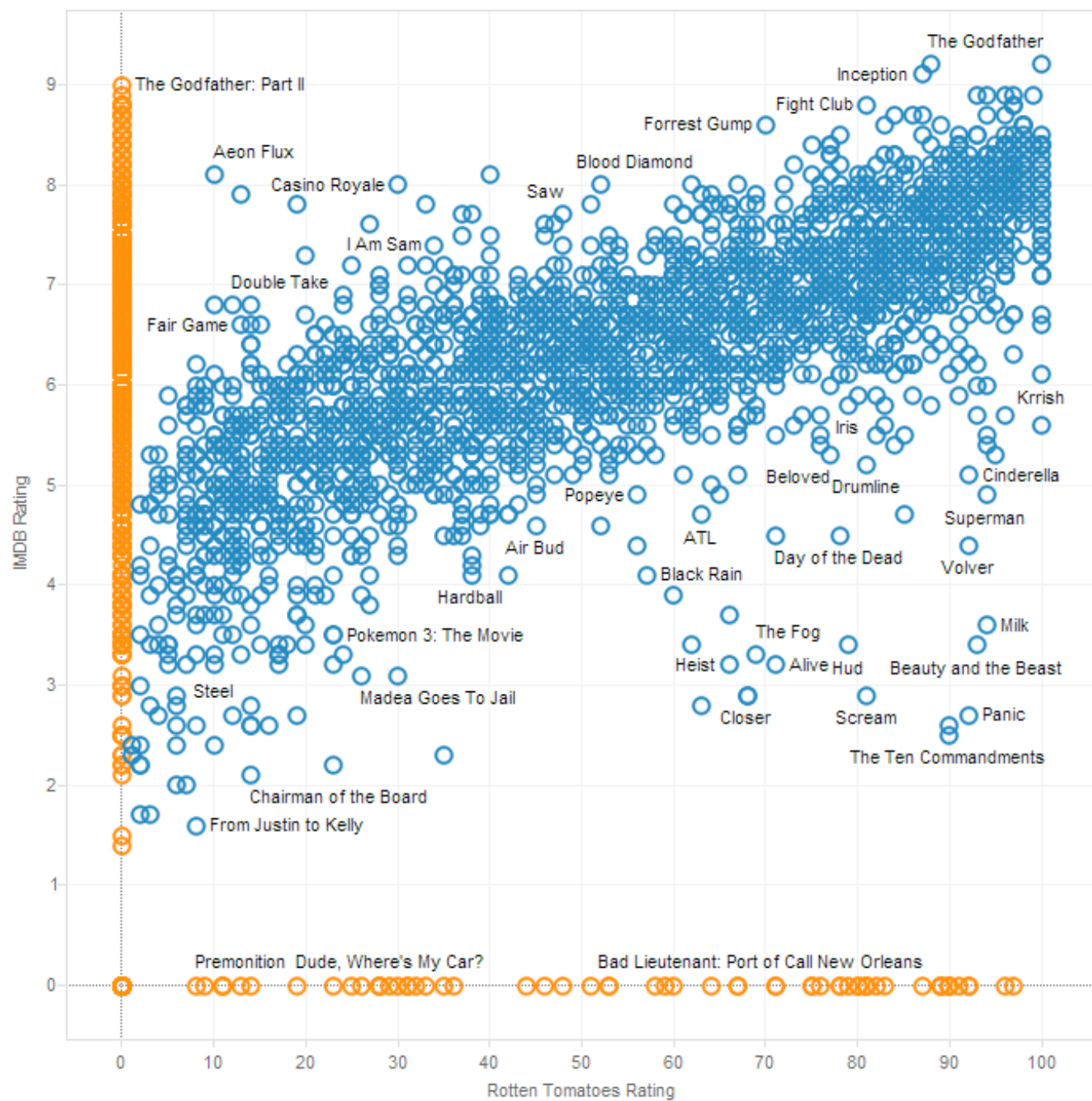


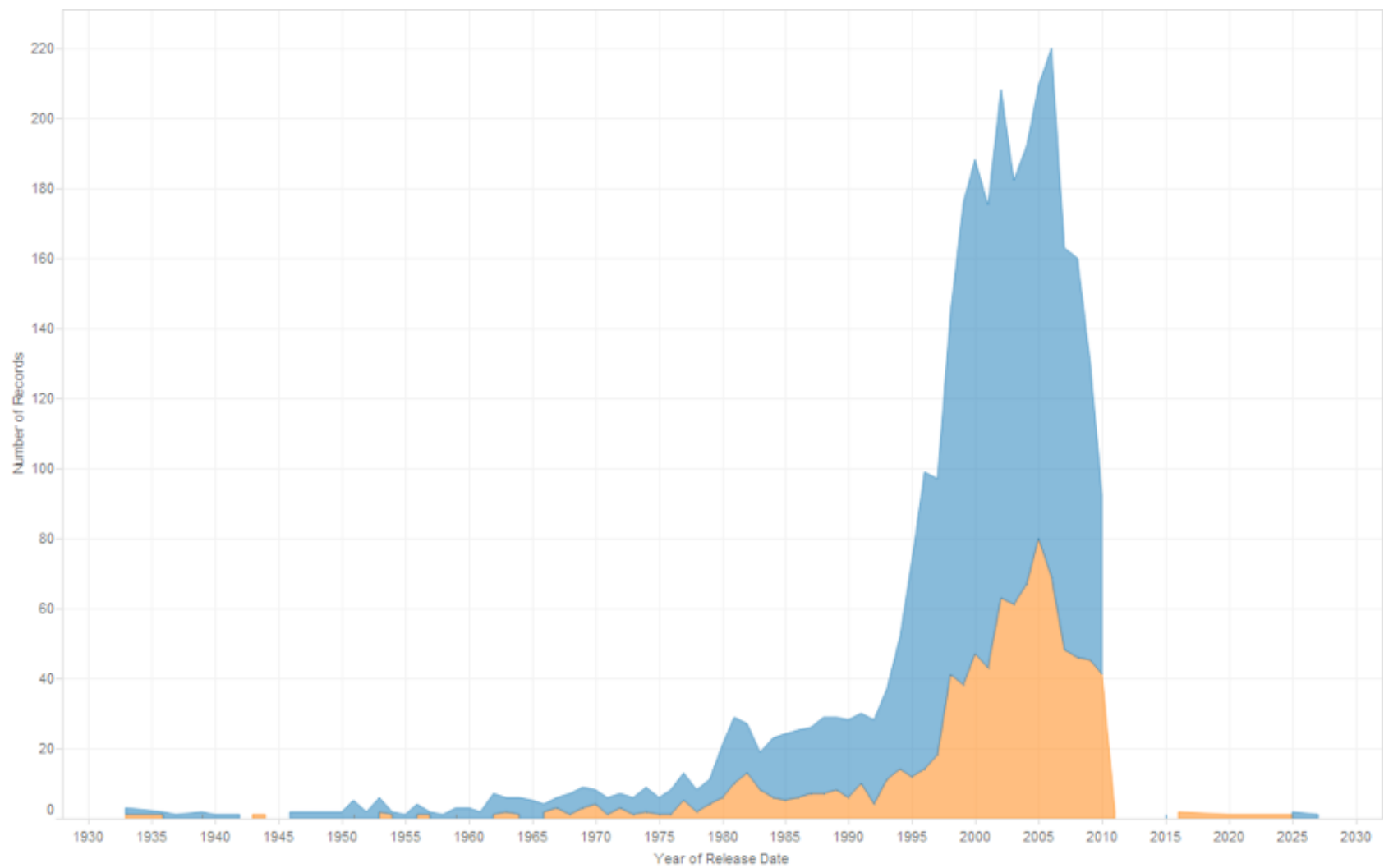
Rotten Tomatoes Rating (bin)











Lesson: Exercise Skepticism

Check **data quality** and your **assumptions**.

Start with **univariate summaries**, then start to consider **relationships among variables**.

Avoid premature fixation!

Analysis Example: Antibiotic Effectiveness

Data Set: Antibiotic Effectiveness

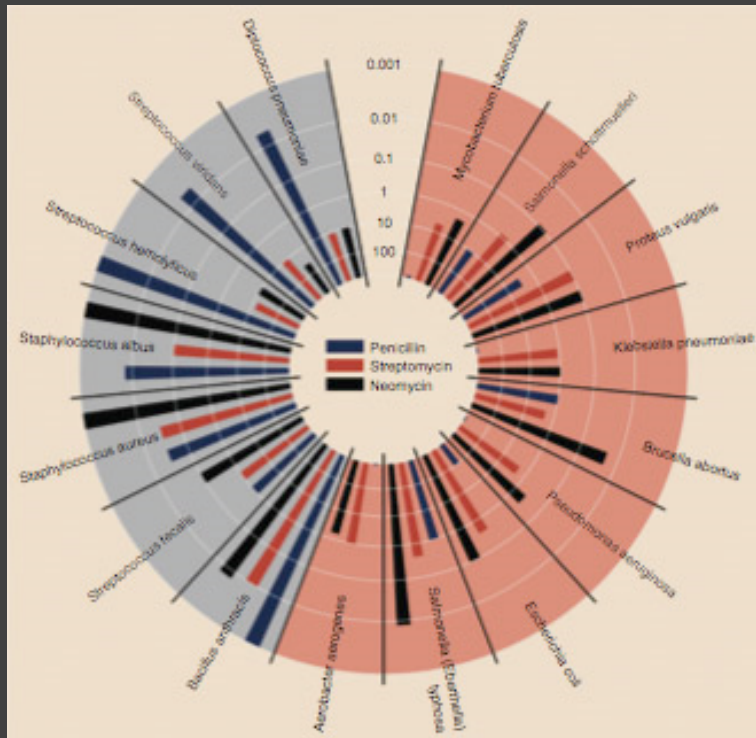
Genus of Bacteria	String (N)
Species of Bacteria	String (N)
Antibiotic Applied	String (N)
Gram-Staining?	Pos / Neg (N)
Min. Inhibitory Concent. (g)	Number (Q)

Collected prior to 1951.

What questions might we ask?

Table 1: Burtin's data.				
Bacteria	Antibiotic			Gram Staining
	Penicillin	Streptomycin	Neomycin	
<i>Aerobacter aerogenes</i>	870	1	1.6	negative
<i>Brucella abortus</i>	1	2	0.02	negative
<i>Brucella anthracis</i>	0.001	0.01	0.007	positive
<i>Diplococcus pneumoniae</i>	0.005	11	10	positive
<i>Escherichia coli</i>	100	0.4	0.1	negative
<i>Klebsiella pneumoniae</i>	850	1.2	1	negative
<i>Mycobacterium tuberculosis</i>	800	5	2	negative
<i>Proteus vulgaris</i>	3	0.1	0.1	negative
<i>Pseudomonas aeruginosa</i>	850	2	0.4	negative
<i>Salmonella (Eberthella) typhosa</i>	1	0.4	0.008	negative
<i>Salmonella schottmuelleri</i>	10	0.8	0.09	negative
<i>Staphylococcus albus</i>	0.007	0.1	0.001	positive
<i>Staphylococcus aureus</i>	0.03	0.03	0.001	positive
<i>Streptococcus fecalis</i>	1	1	0.1	positive
<i>Streptococcus hemolyticus</i>	0.001	14	10	positive
<i>Streptococcus viridans</i>	0.005	10	40	positive

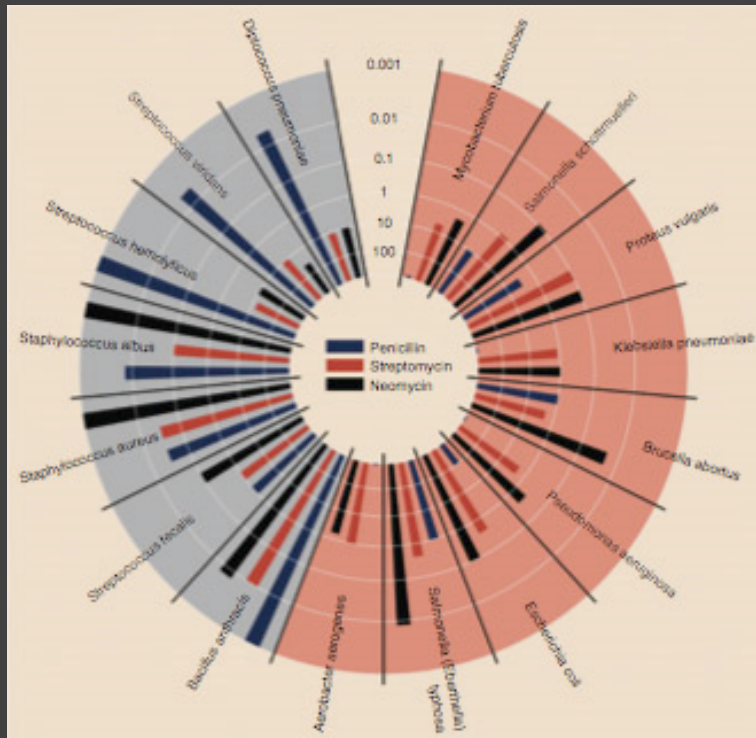
How do the drugs compare?



Bacteria	Penicillin	Antibiotic Streptomycin	Neomycin	Gram stain
<i>Aerobacter aerogenes</i>	870	1	1.6	–
<i>Brucella abortus</i>	1	2	0.02	–
<i>Bacillus anthracis</i>	0.001	0.01	0.007	+
<i>Diplococcus pneumoniae</i>	0.005	11	10	+
<i>Escherichia coli</i>	100	0.4	0.1	–
<i>Klebsiella pneumoniae</i>	850	1.2	1	–
<i>Mycobacterium tuberculosis</i>	800	5	2	–
<i>Proteus vulgaris</i>	3	0.1	0.1	–
<i>Pseudomonas aeruginosa</i>	850	2	0.4	–
<i>Salmonella (Eberthella) typhosa</i>	1	0.4	0.008	–
<i>Salmonella schottmuelleri</i>	10	0.8	0.09	–
<i>Staphylococcus albus</i>	0.007	0.1	0.001	+
<i>Staphylococcus aureus</i>	0.03	0.03	0.001	+
<i>Streptococcus fecalis</i>	1	1	0.1	+
<i>Streptococcus hemolyticus</i>	0.001	14	10	+
<i>Streptococcus viridans</i>	0.005	10	40	+

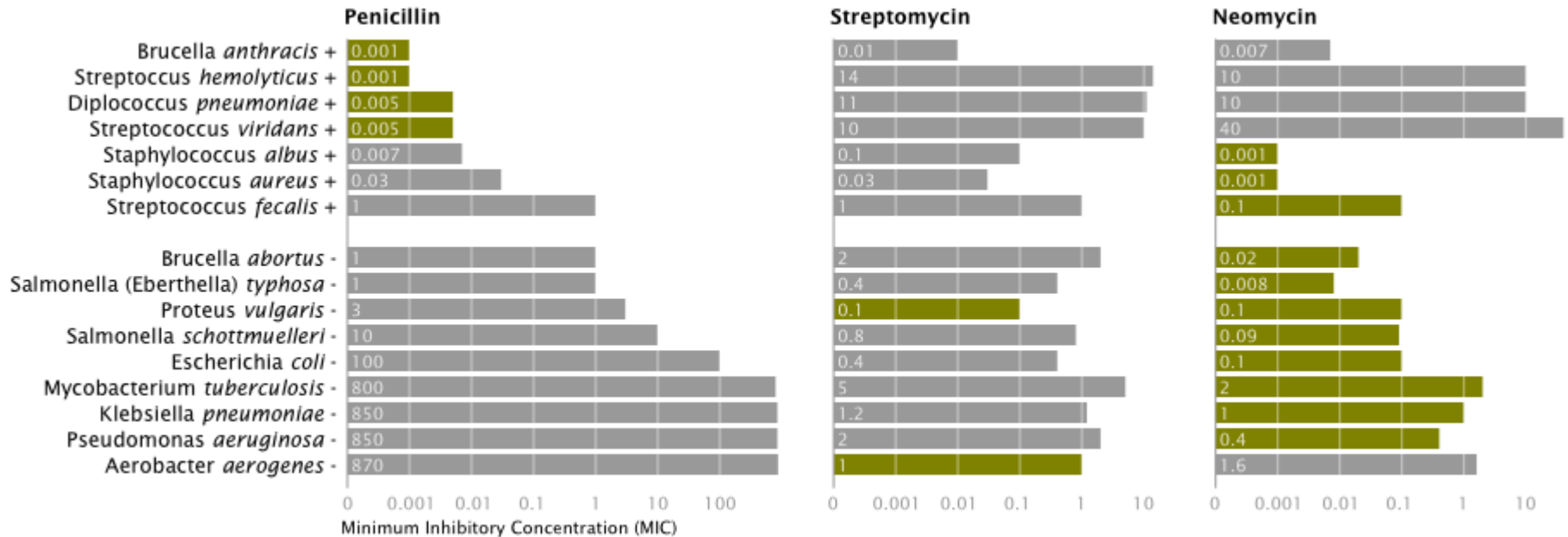
Original graphic by Will Burtin, 1951

How do the drugs compare?

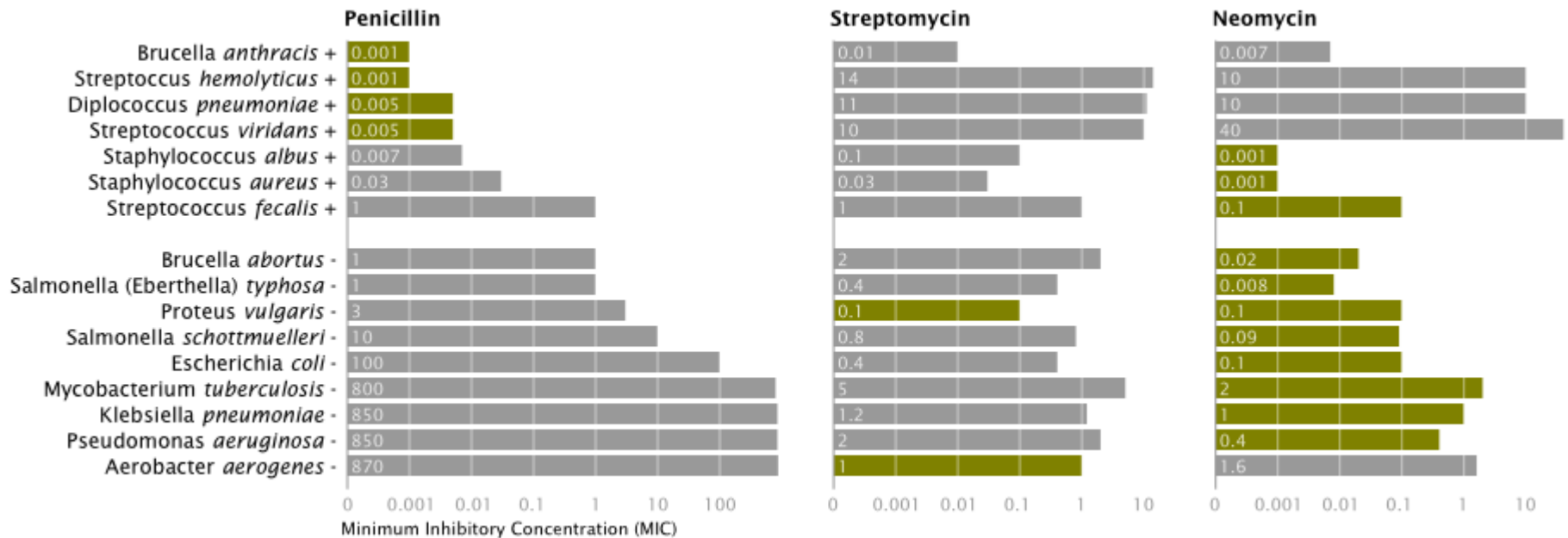


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How do the drugs compare?



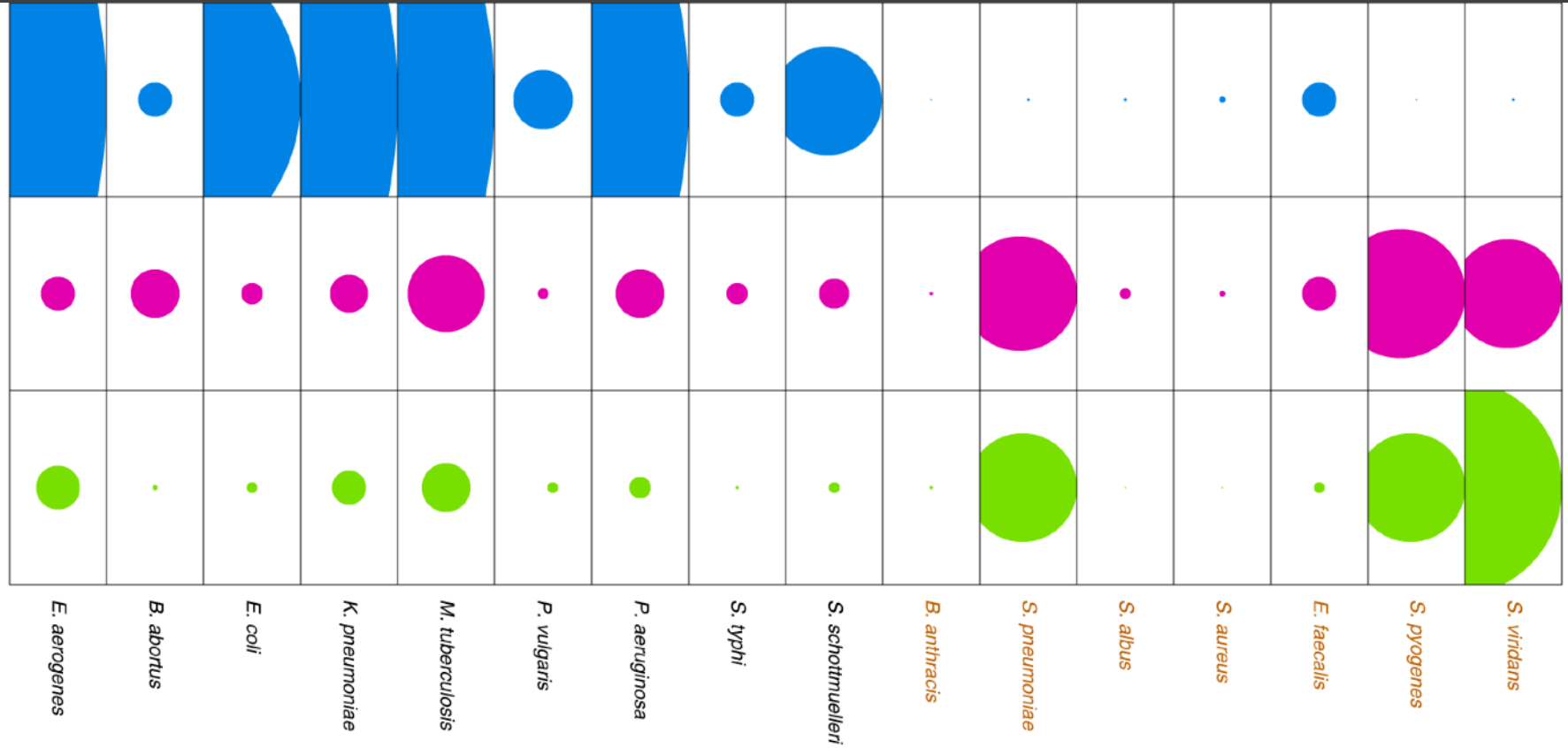
How do the drugs compare?

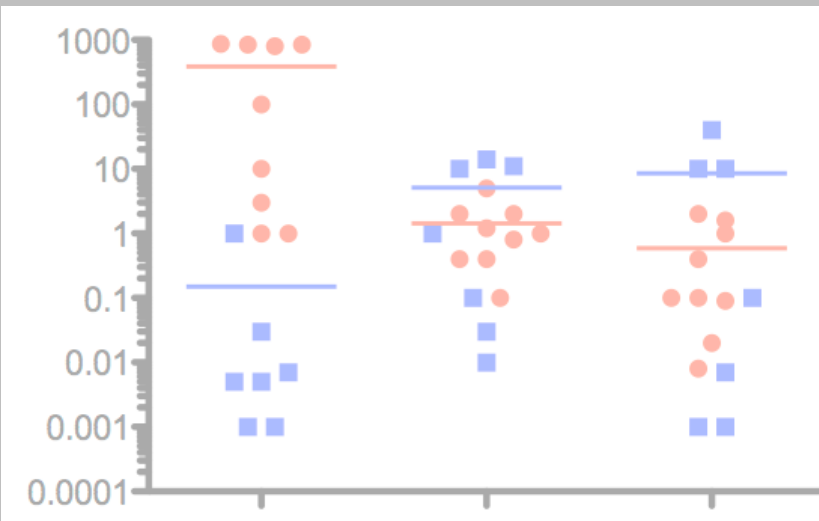
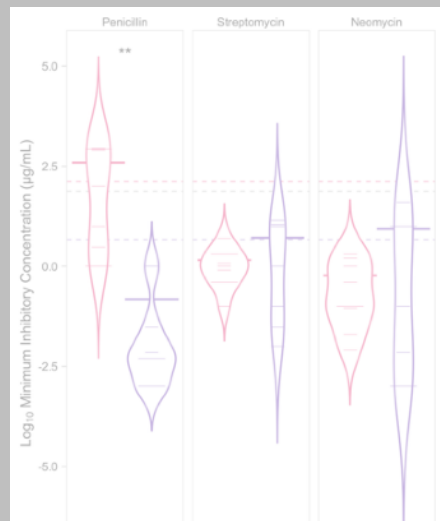
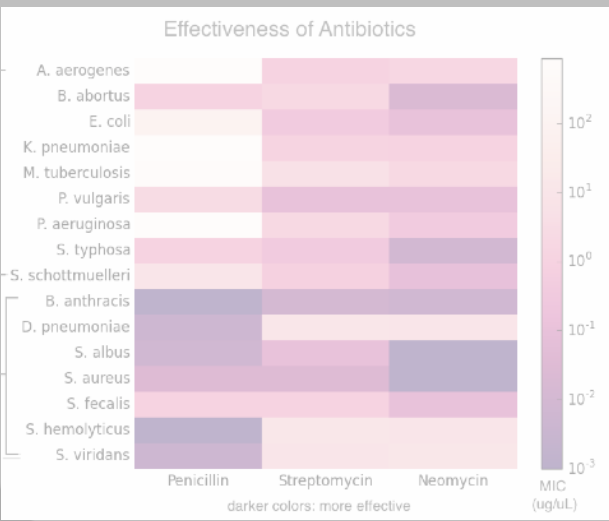
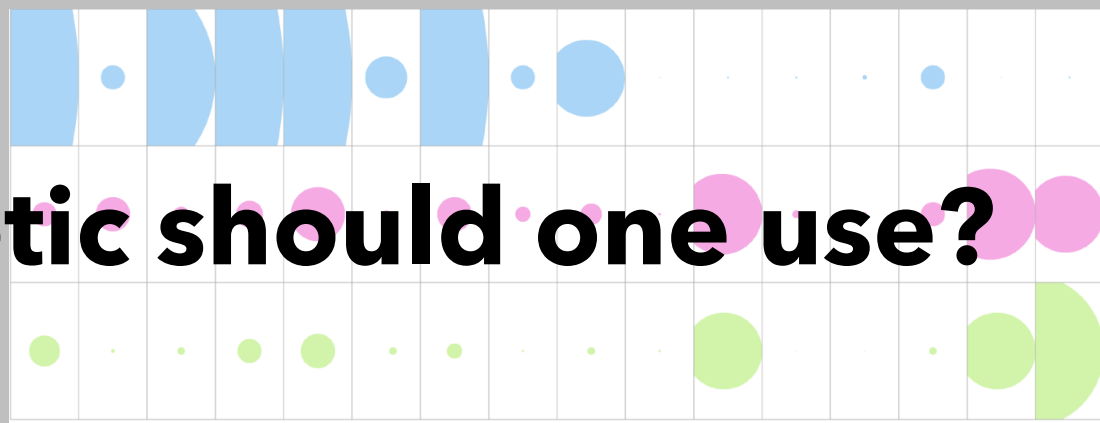
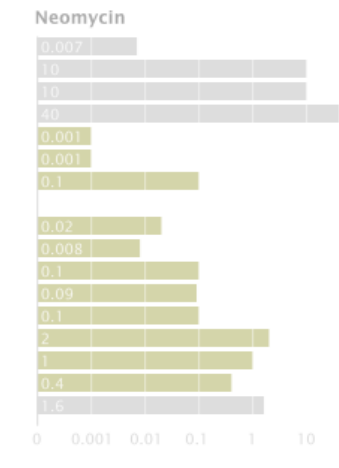
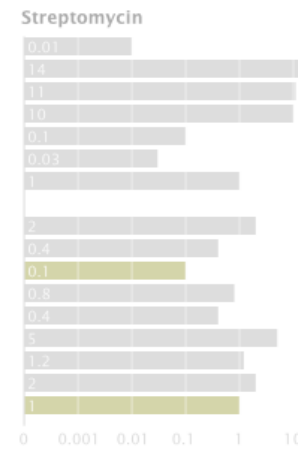
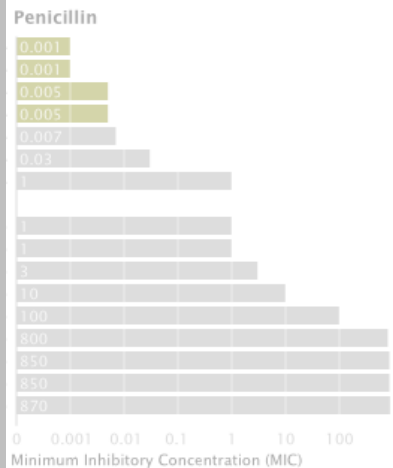
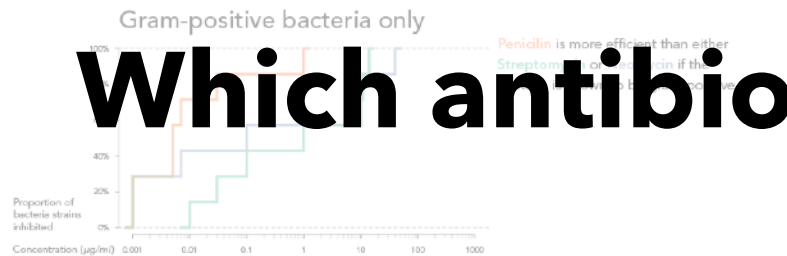
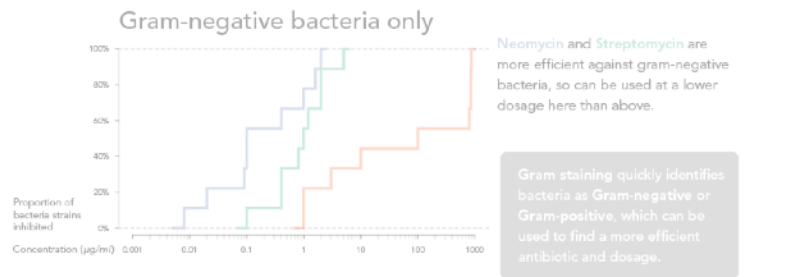
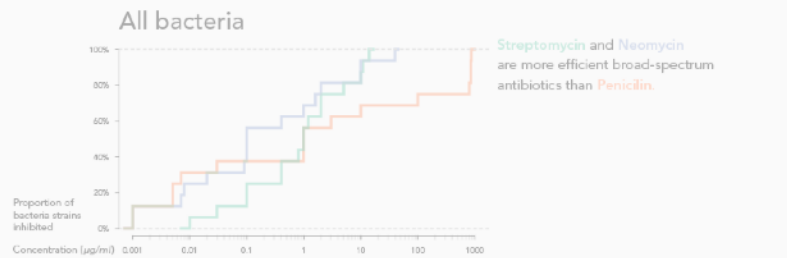


X-axis: Antibiotic | $\log(\text{MIC})$

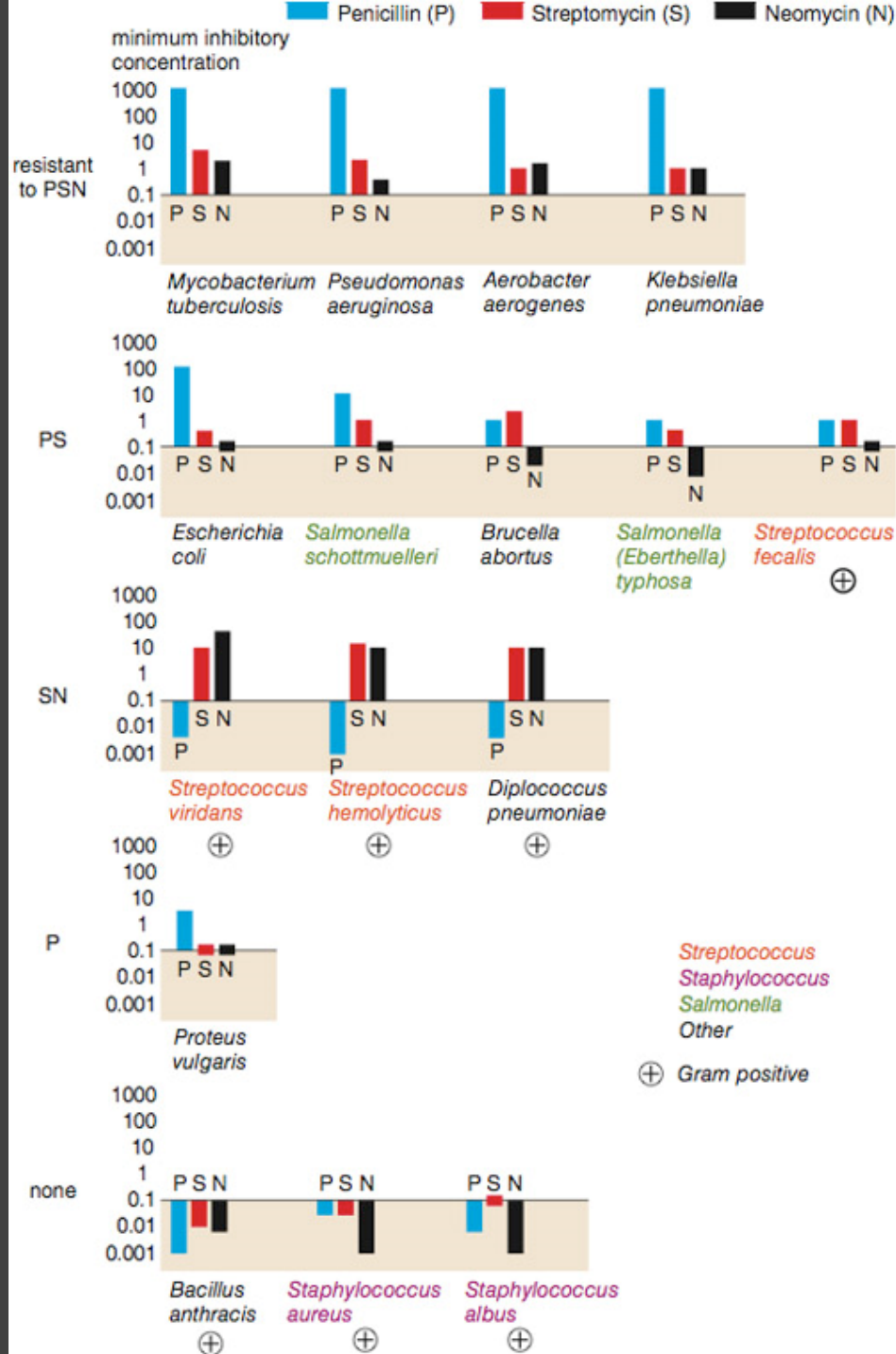
Y-axis: Gram-Staining | Species

Color: Most-Effective?

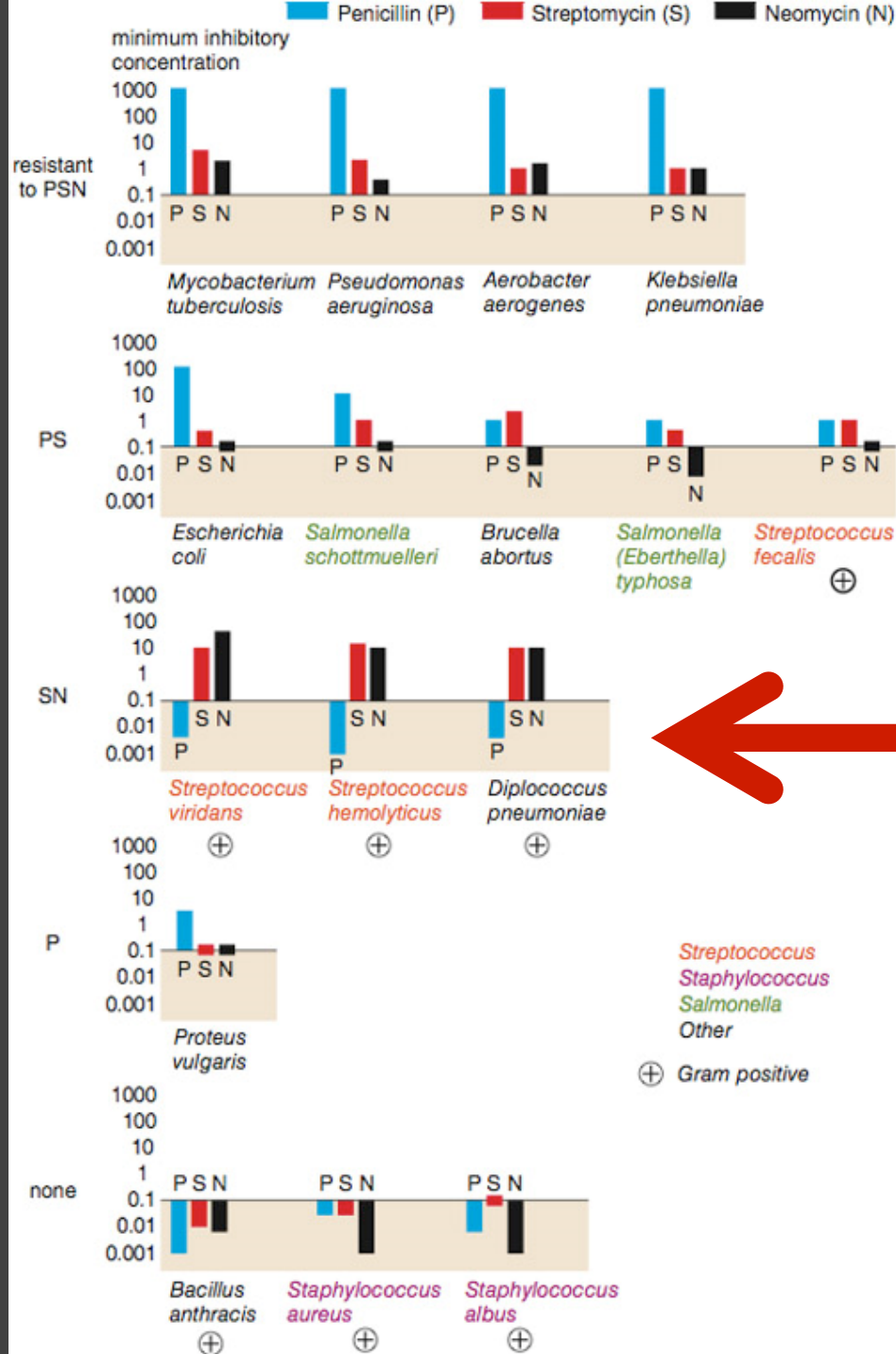




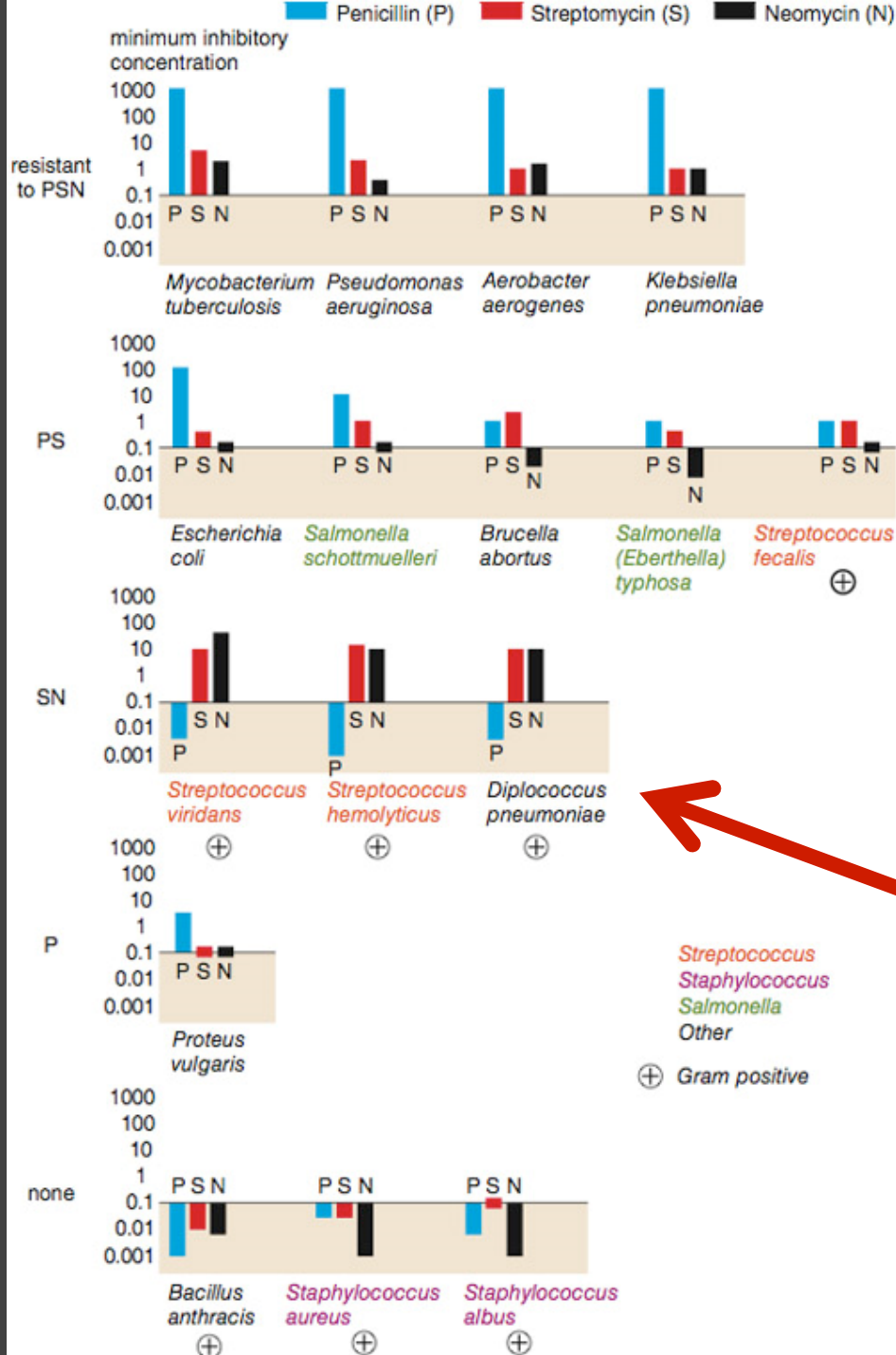
**Do the bacteria
group by antibiotic
resistance?**



Do the bacteria group by antibiotic resistance?



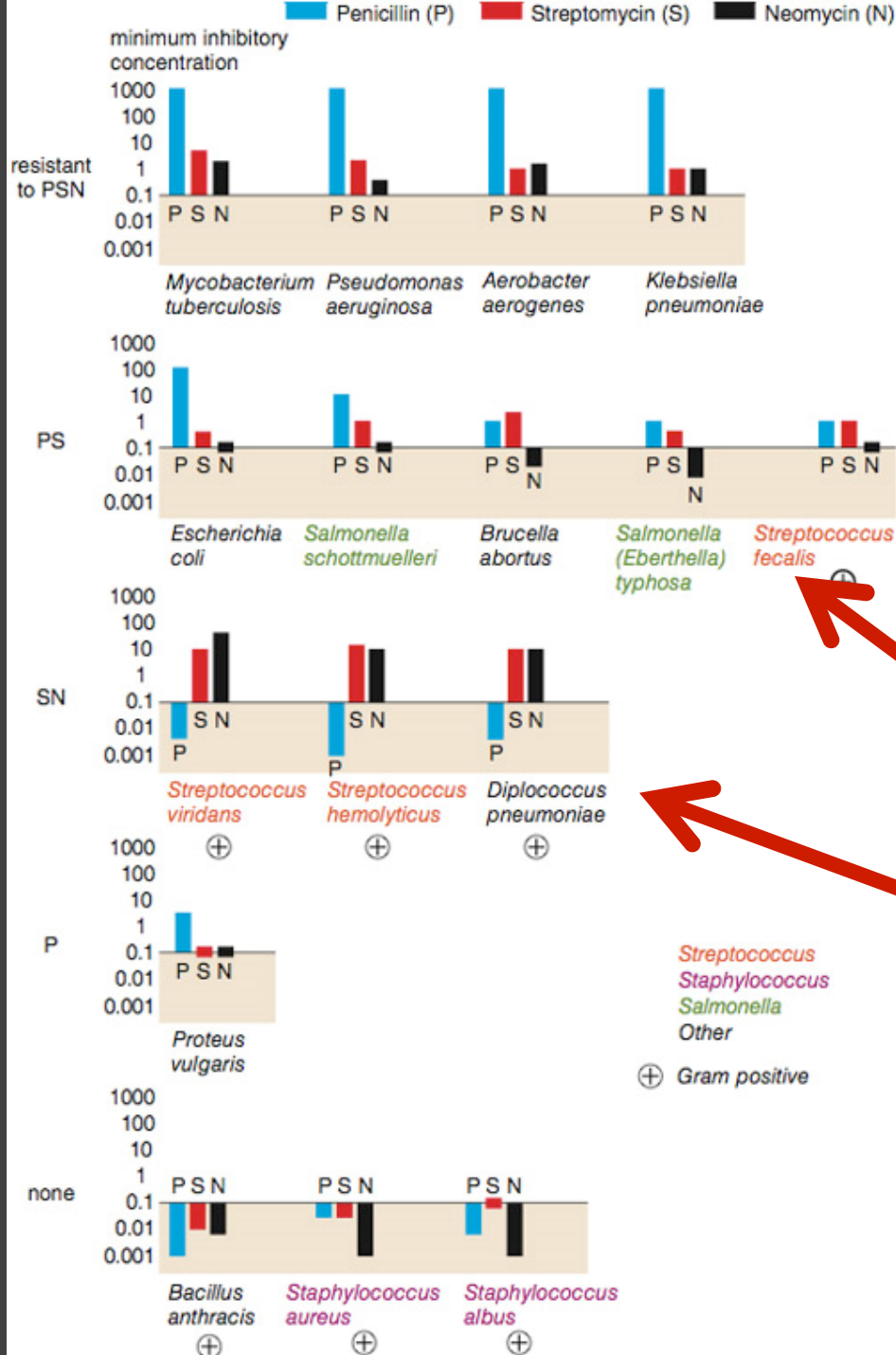
Do the bacteria group by antibiotic resistance?



Do the bacteria group by antibiotic resistance?

Really a streptococcus!
(realized ~20 yrs later)

Wainer & Lysen
American Scientist, 2009

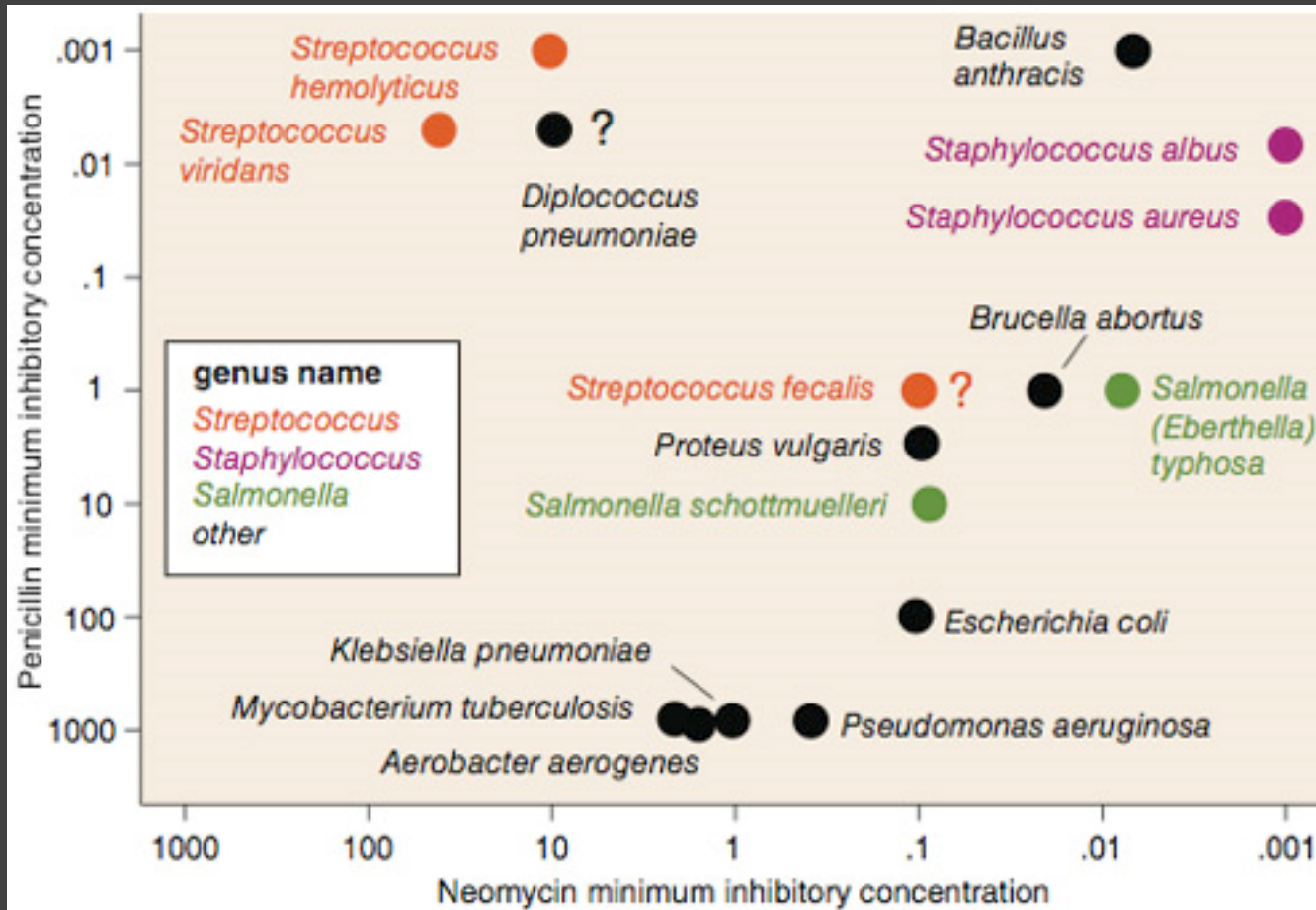


Do the bacteria group by antibiotic resistance?

Not a streptococcus!
(realized ~30 yrs later)

Really a streptococcus!
(realized ~20 yrs later)

Do the bacteria group by resistance?
Do different drugs correlate?



Do the bacteria group by resistance?
Do different drugs correlate?

Lesson: Iterative Exploration

Exploratory Process

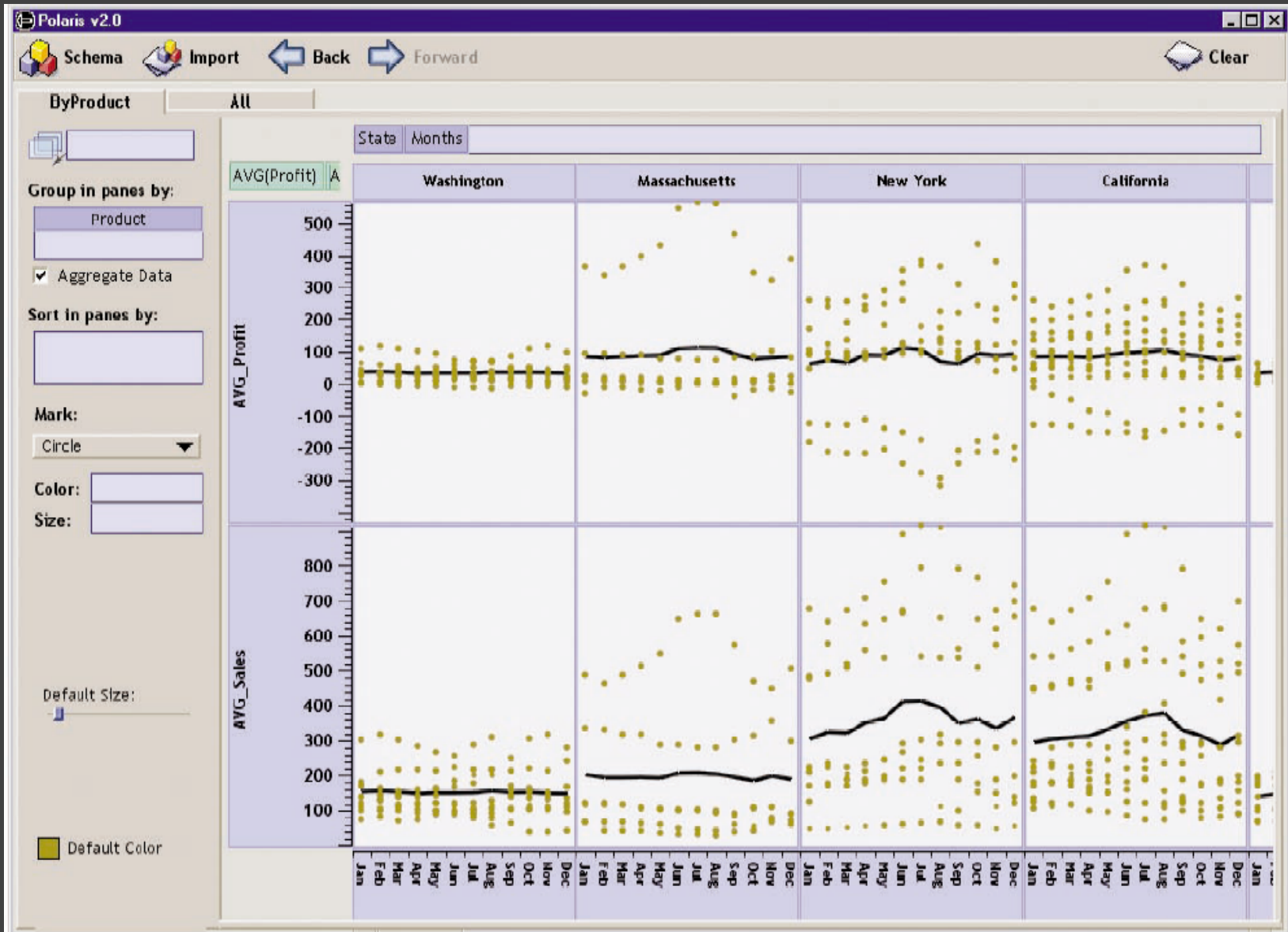
- 1 Construct graphics to address questions
- 2 Inspect “answer” and assess new questions
- 3 Repeat...

Transform data appropriately (e.g., invert, log)

Show data variation, not design variation [Tufte]

Tableau / Polaris

Polaris [Stolte et al.]



Tableau

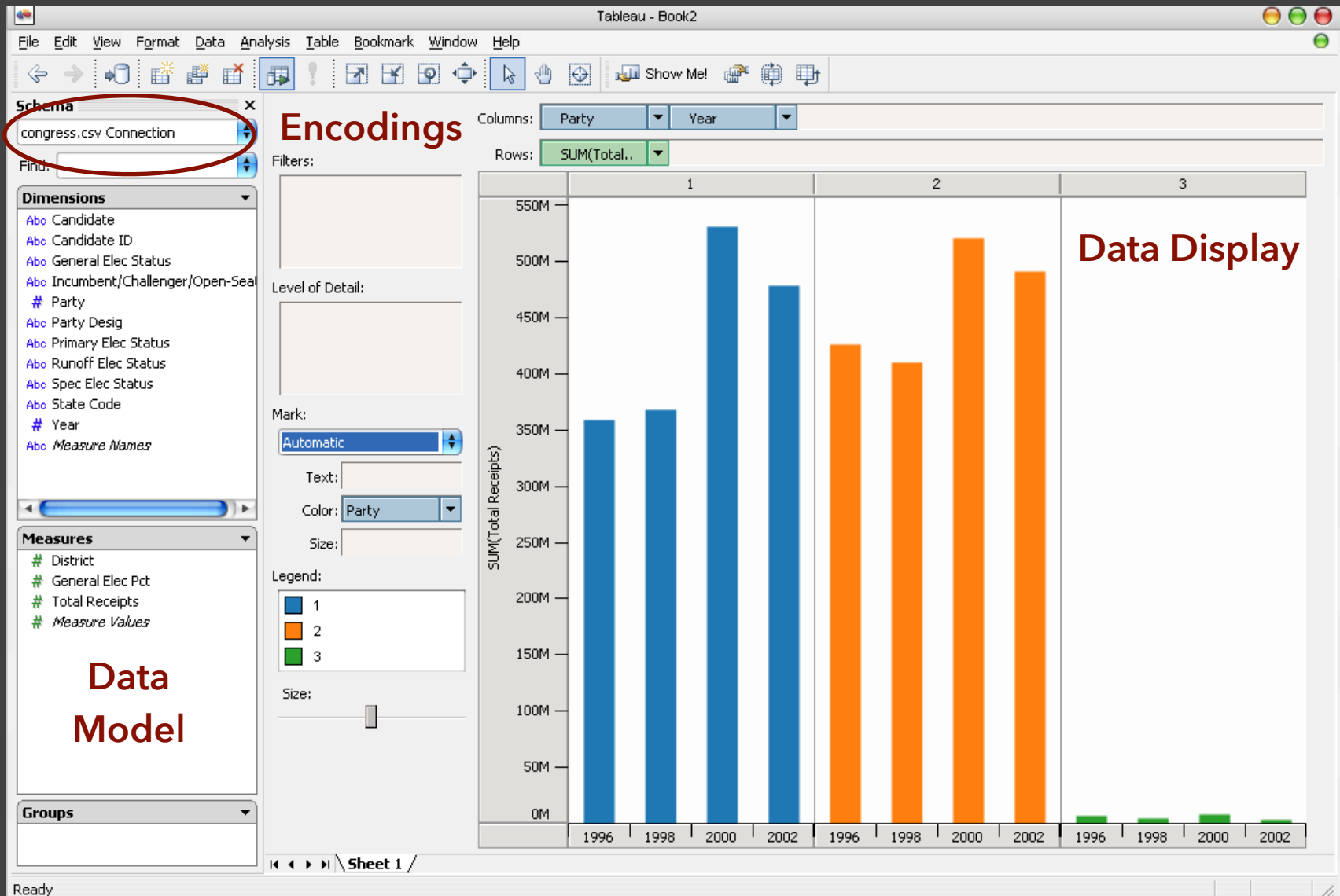


Tableau / Polaris Approach

Insight: can simultaneously specify both
database queries and visualization

Choose data, then visualization, not vice versa

Use smart defaults for visual encodings

Can also suggest encodings upon request

Tableau Demo

The dataset:

Federal Elections Commission Receipts

Every Congressional Candidate from 1996 to 2002

4 Election Cycles

9216 Candidacies

Dataset Schema

Year (Qi)

Candidate Code (N)

Candidate Name (N)

Incumbent / Challenger / Open-Seat (N)

Party Code (N) [1=Dem,2=Rep,3=Other]

Party Name (N)

Total Receipts (Qr)

State (N)

District (N)

This is a subset of the larger data set available from the FEC.

Hypotheses?

What might we learn from this data?

Hypotheses?

What might we learn from this data?

Correlation between receipts and winners?

Do receipts increase over time?

Which states spend the most?

Which party spends the most?

Margin of victory vs. amount spent?

Amount spent between competitors?

Tableau Demo

Specifying Table Configurations

Operands are the database fields

Each operand interpreted as a set {...}

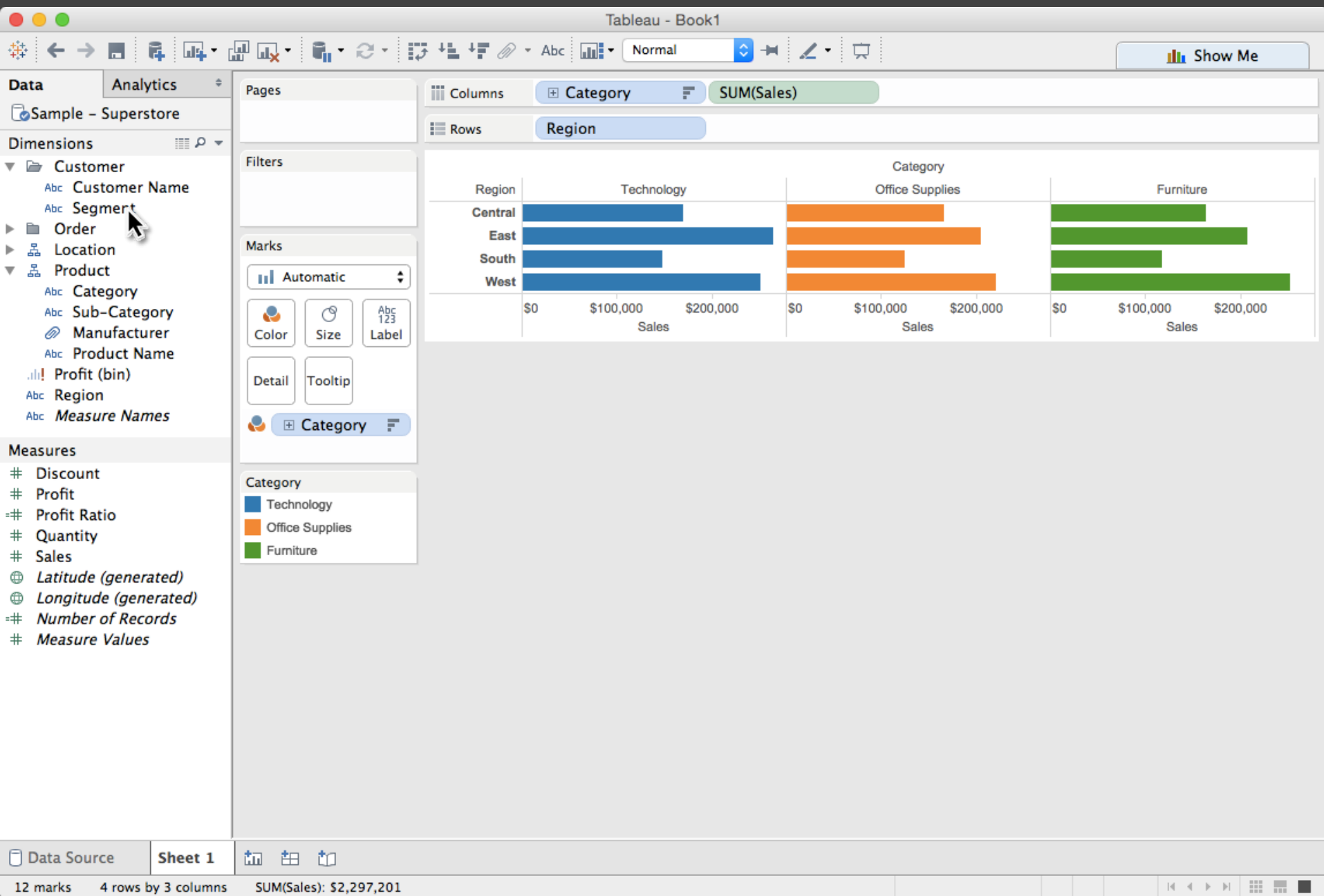
Quantitative and Ordinal fields treated differently

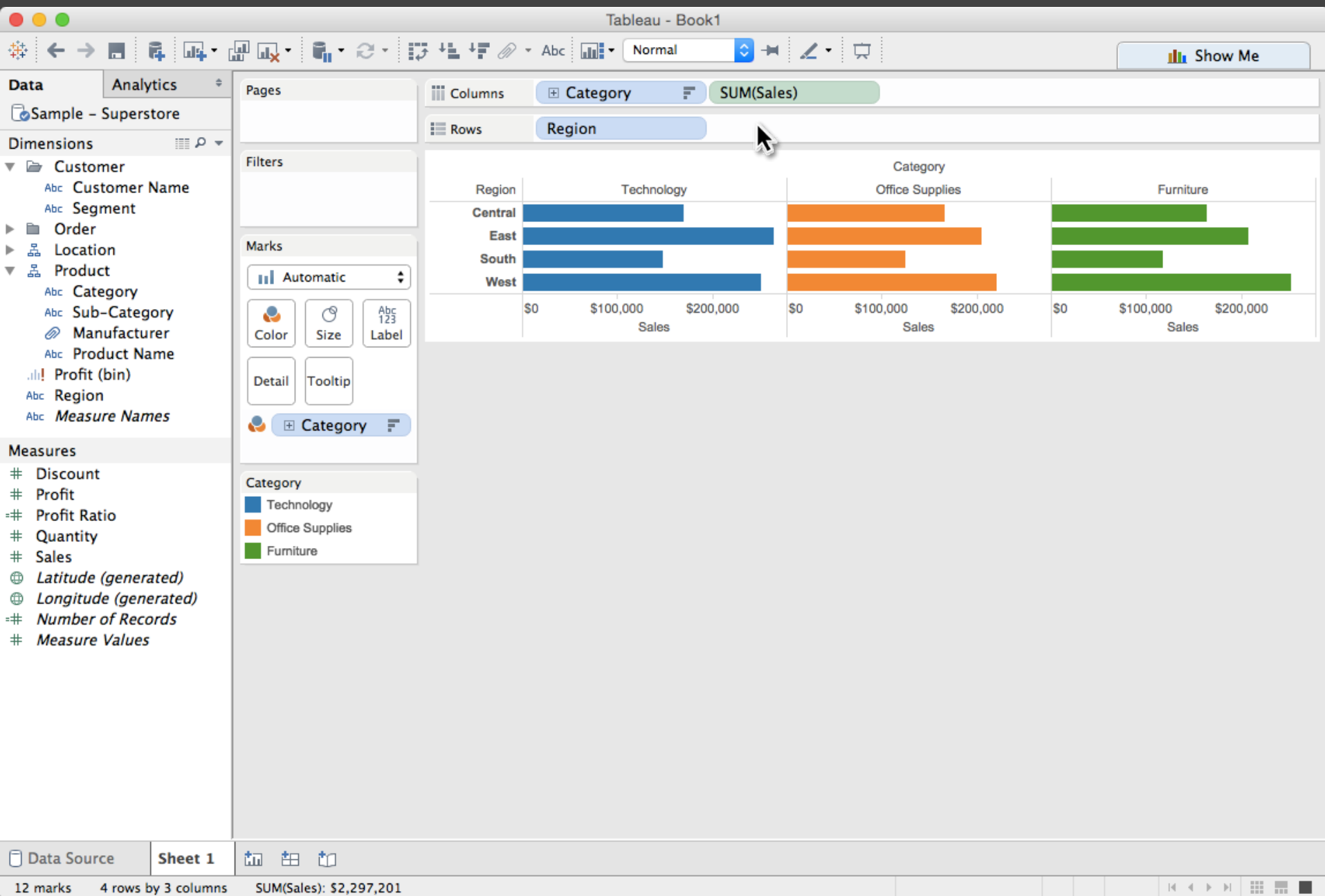
Three operators:

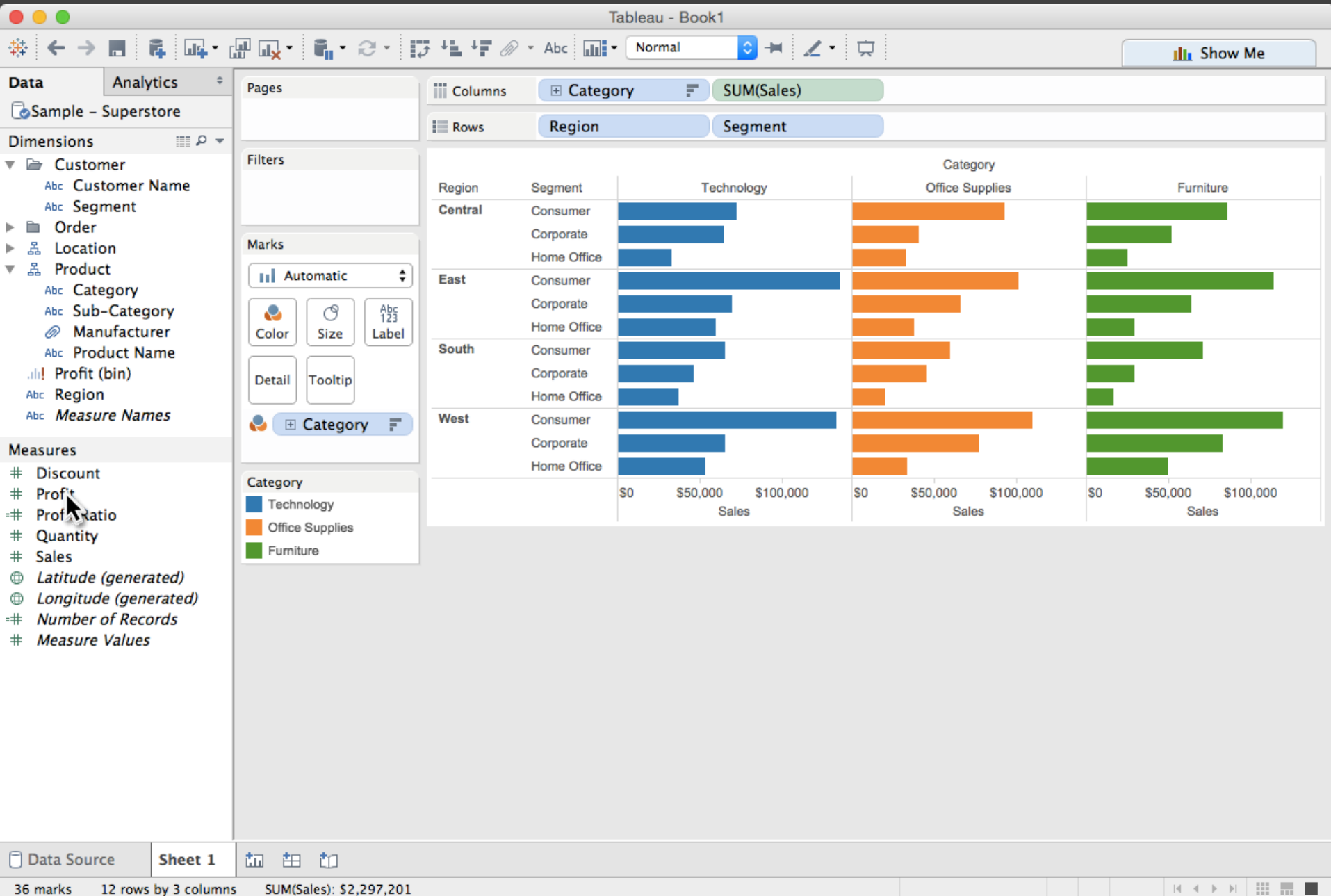
concatenation (+)

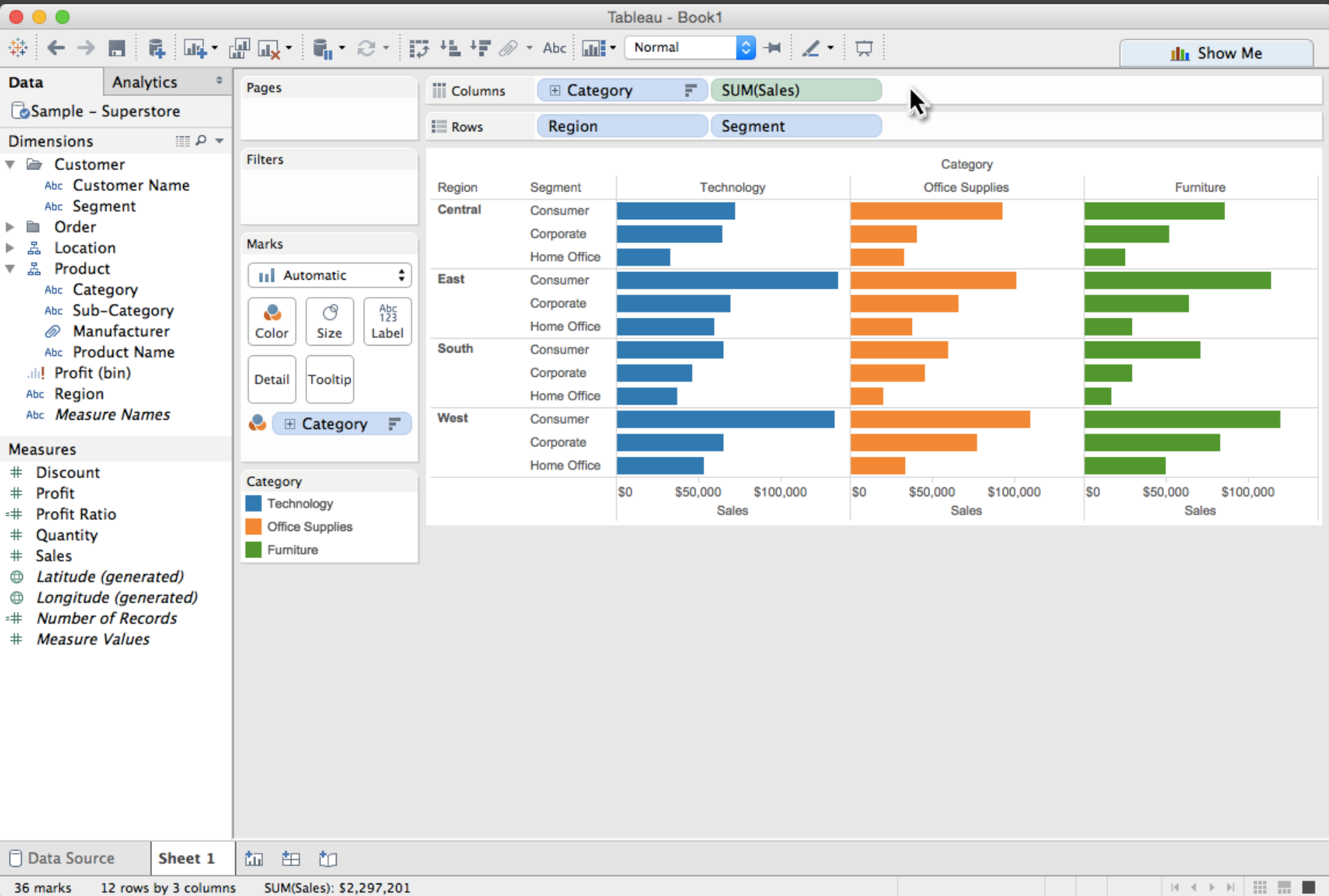
cross product (x)

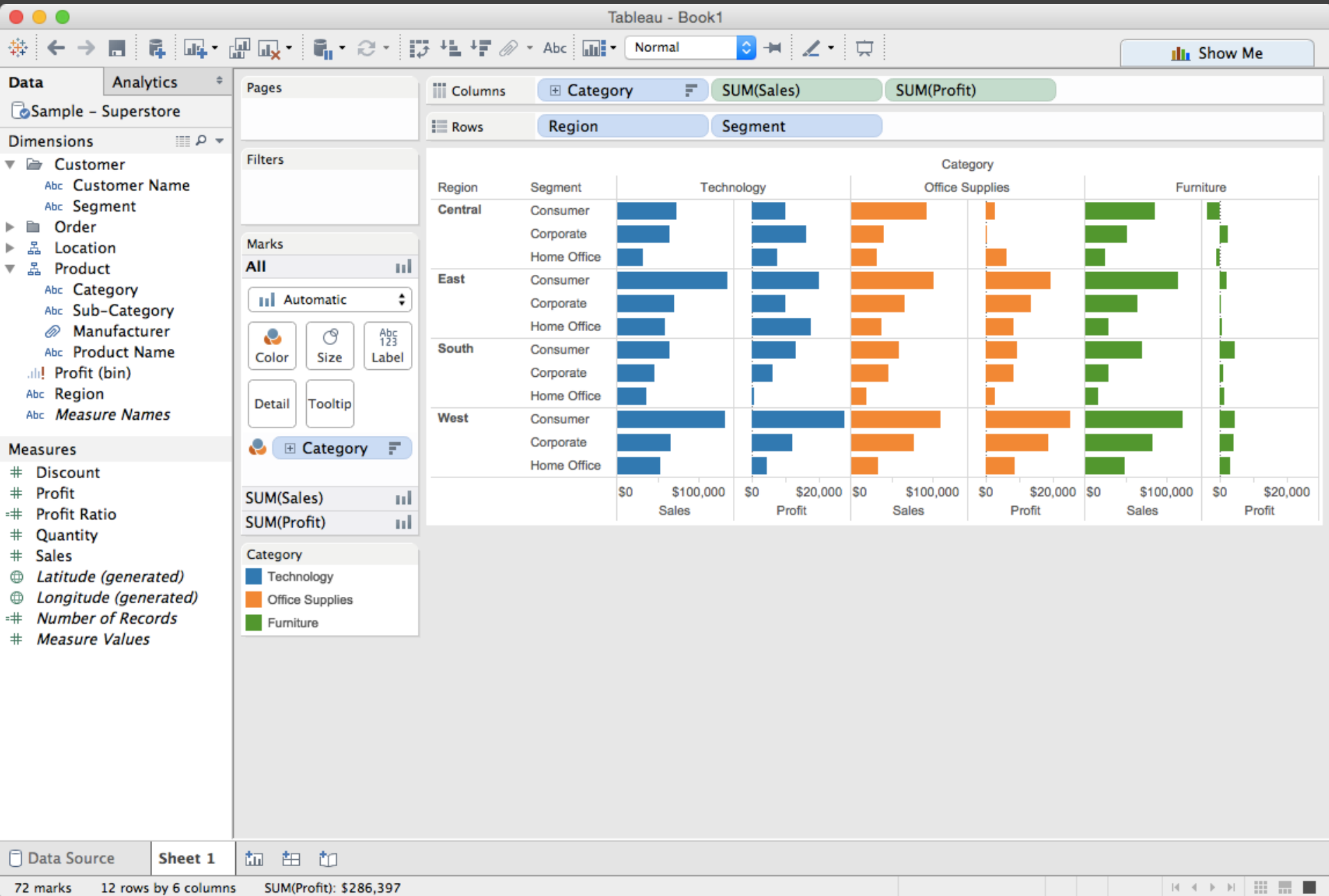
nest (/)











Columns **Category** ~~SUM(Sales)~~ **SUM(Profit)**

Rows **Region** **Segment** **GROUP BY Category, Region, Segment**

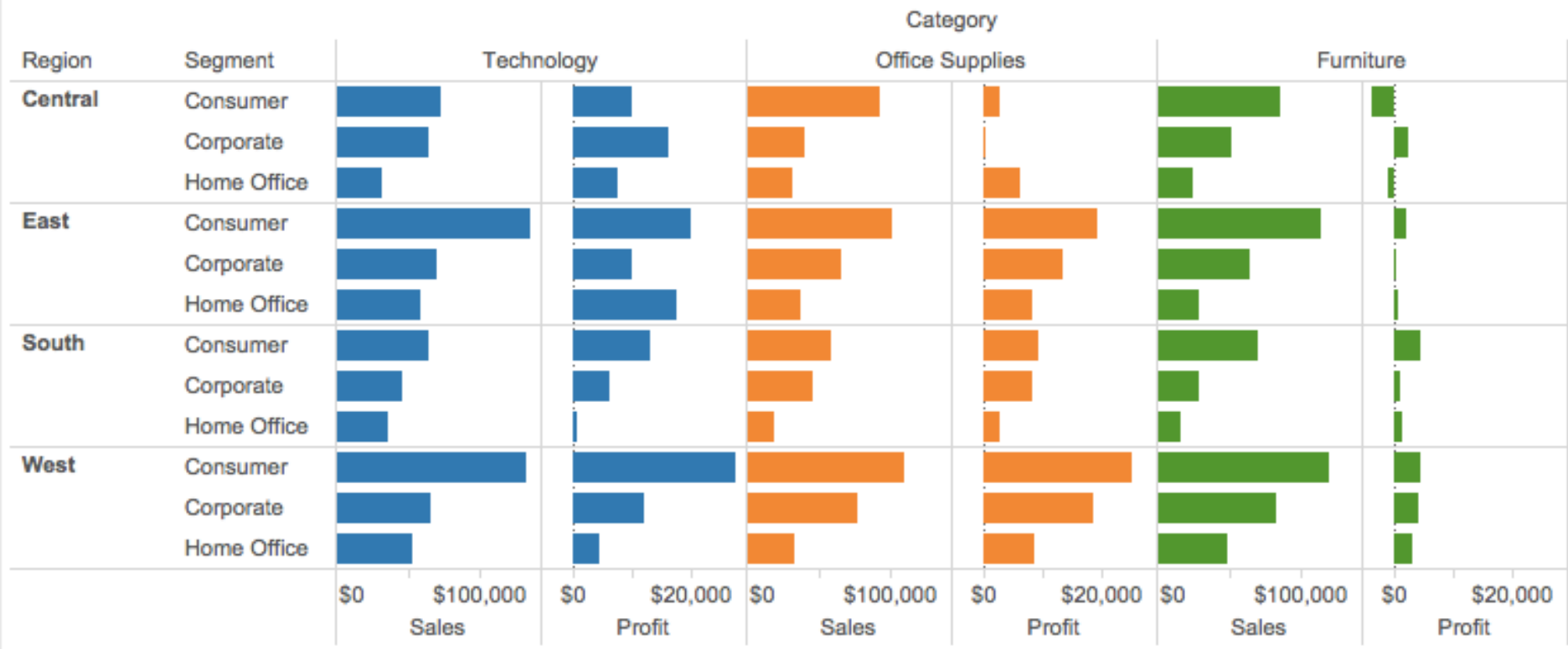


Table Algebra

The operators (+, x, /) and operands (O, Q) provide an *algebra* for tabular visualization.

Algebraic statements are then mapped to:

Visualizations - trellis plot partitions, visual encodings

Queries - selection, projection, group-by aggregation

In Tableau, users make statements via drag-and-drop

Note that this specifies operands *NOT* operators!

Operators are inferred by data type (O, Q)

Table Algebra: Operands

Ordinal fields: interpret domain as a set that partitions table into rows and columns.

Quarter = {(Qtr1),(Qtr2),(Qtr3),(Qtr4)} ->

Qtr1	Qtr2	Qtr3	Qtr4
95892	101760	105282	98225

Quantitative fields: treat domain as single element set and encode spatially as axes.

Profit = {(Profit[-410,650])} ->



Concatenation (+) Operator

Ordered union of set interpretations

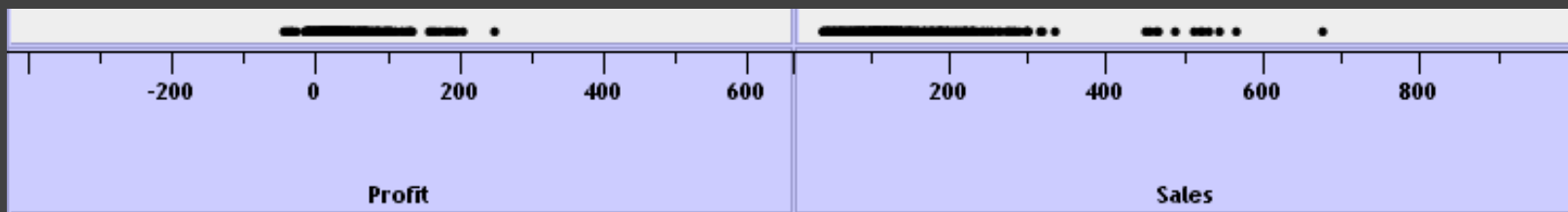
Quarter + Product Type

$= \{(Qtr1), (Qtr2), (Qtr3), (Qtr4)\} + \{(Coffee), (Espresso)\}$

$= \{(Qtr1), (Qtr2), (Qtr3), (Qtr4), (Coffee), (Espresso)\}$

Qtr1	Qtr2	Qtr3	Qtr4	Coffee	Espresso
48	59	57	53	151	21

Profit + Sales = $\{(Profit[-310,620]), (Sales[0,1000])\}$



Cross (x) Operator

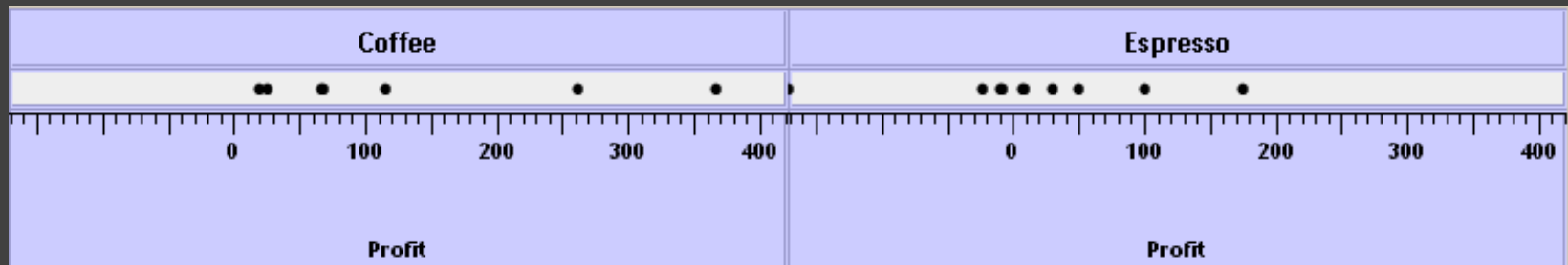
Cross-product of set interpretations

Quarter x Product Type =

{(Qtr1,Coffee), (Qtr1, Tea), (Qtr2, Coffee), (Qtr2, Tea), (Qtr3, Coffee), (Qtr3, Tea), (Qtr4, Coffee), (Qtr4,Tea)}

Qtr1		Qtr2		Qtr3		Qtr4	
Coffee	Espresso	Coffee	Espresso	Coffee	Espresso	Coffee	Espresso
131	19	160	20	178	12	134	33

Product Type x Profit =



Nest (/) Operator

Cross-product filtered by existing records

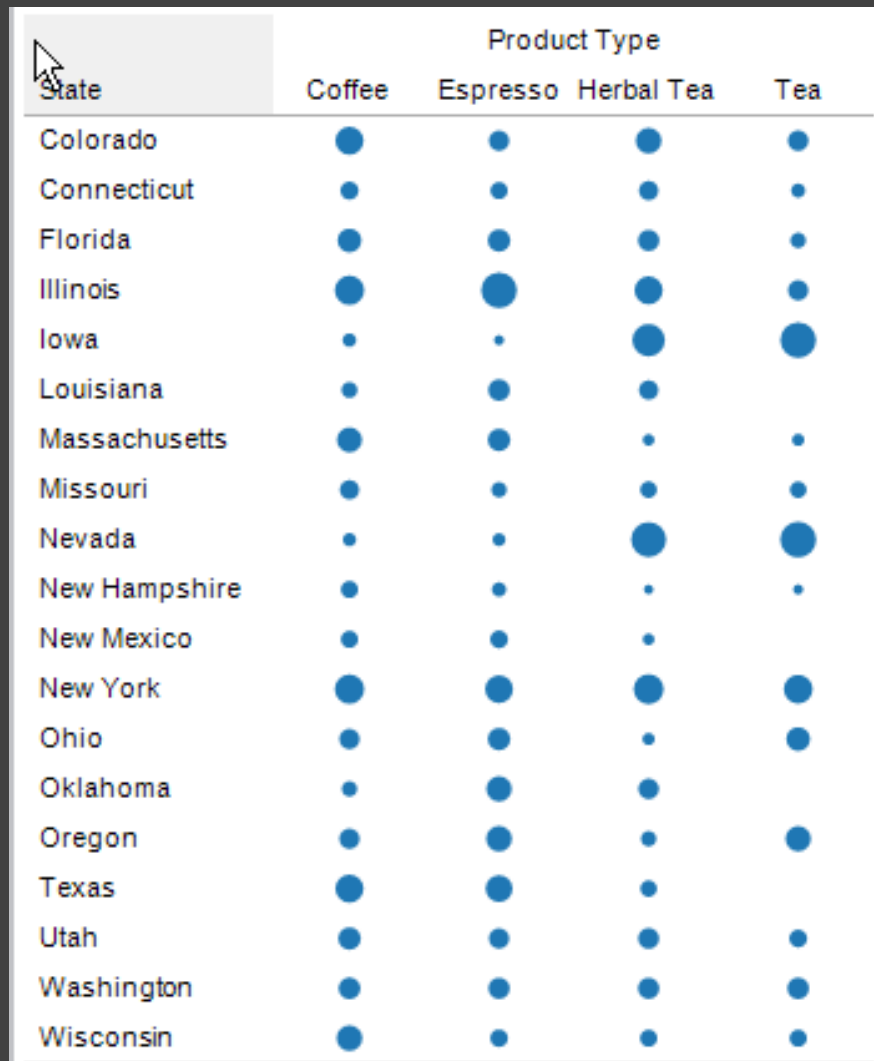
Quarter x Month ->

creates twelve entries for each quarter. i.e.,
(Qtr1, December)

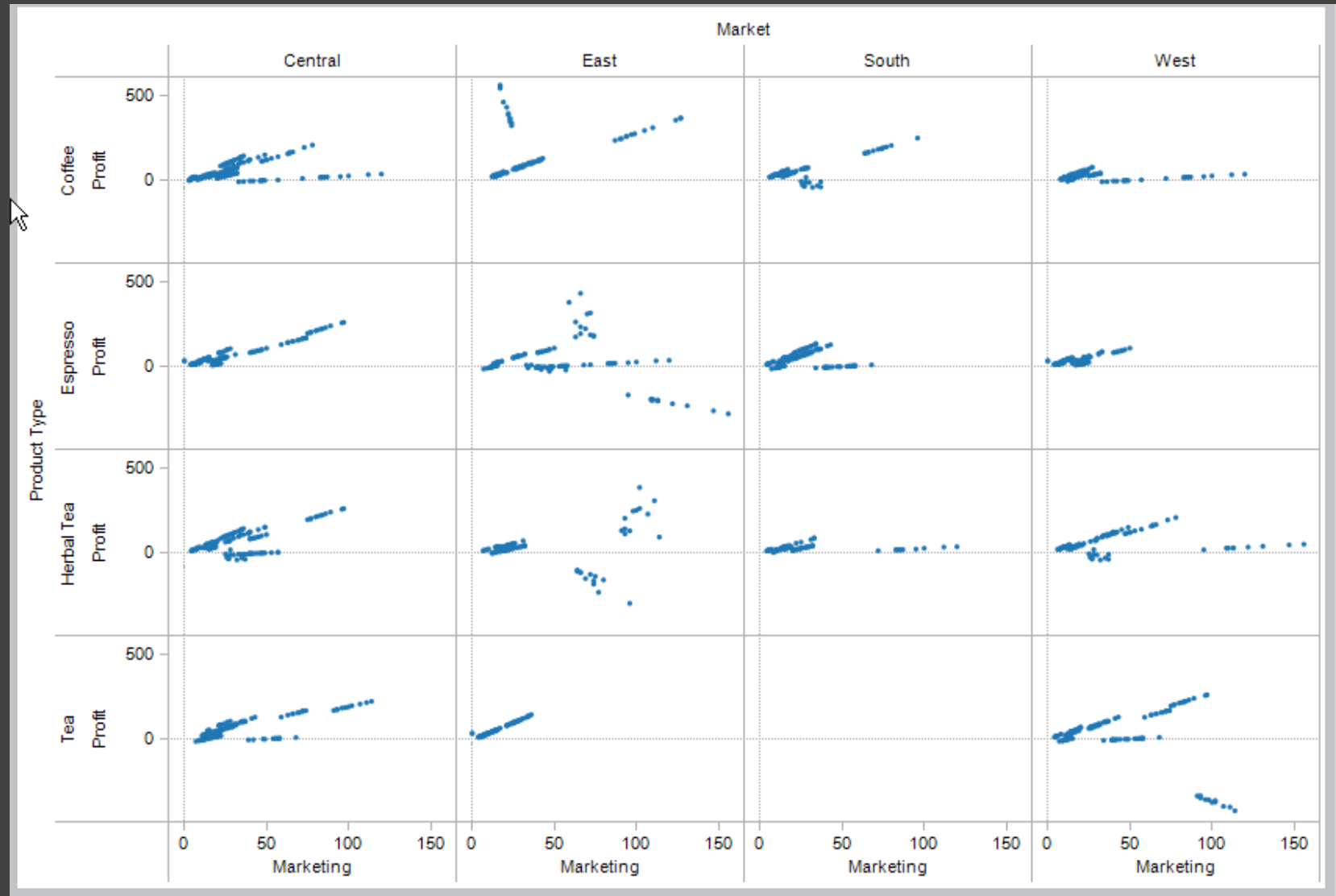
Quarter / Month ->

creates three entries per quarter based on
tuples in database (not semantics)

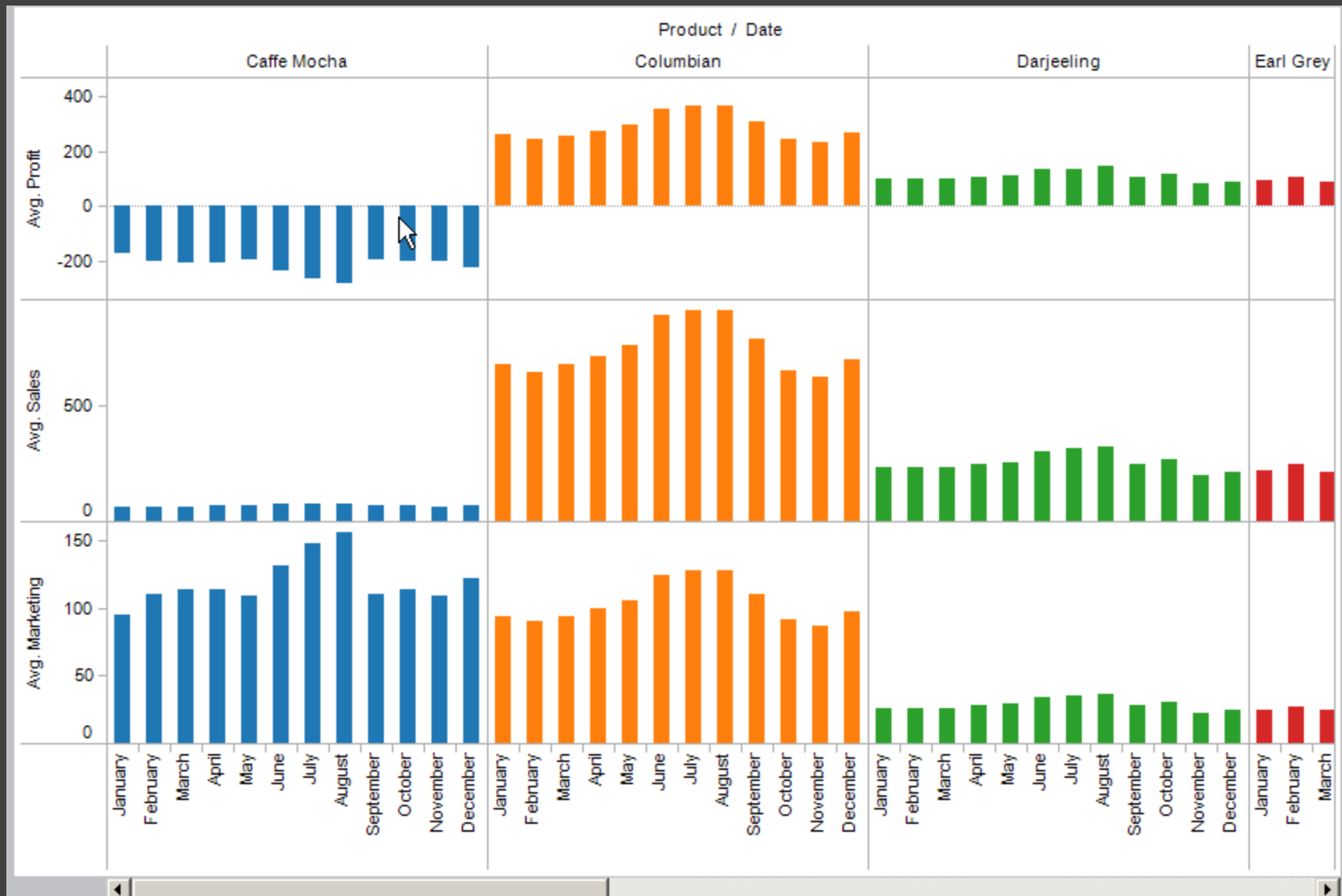
Ordinal-Ordinal



Quantitative-Quantitative



Ordinal-Quantitative



Querying the Database

