First a type system

(In this example, exceptions raise constant strings $s$)

$$\begin{align*}
\tau & ::= \text{bool} \mid \tau \to \tau \mid \tau \star \tau \\
e & ::= x \mid \text{true} \mid \text{false} \mid \lambda x. \ e \mid e \ e \mid (e, e) \mid e. 1 \mid e. 2 \\
& \quad \mid \text{if } e e \ e \mid \text{raise } s \mid \text{try } e \ \text{handle } s \ e
\end{align*}$$

Type-and-effect systems

New topic: An elegant framework to extend type systems to track “things that may happen” (effects) during evaluation

Plain-old type systems have judgments like $\Gamma \vdash e : \tau$ to mean:

- $e$ won’t get stuck
- If $e$ produces a value, that value has type $\tau$

Adding effects reuses the “plumbing” of typing rules to compute something about “how $e$ executes”

- There are many things we may want to conservatively approximate
- Example: What exceptions might get thrown
- All effect systems are very similar, especially treatment of functions
- Example: All values have no effect since their “computation” does nothing

Add effects

$$\begin{align*}
\tau & ::= \text{...sets of strings...} \\
e & ::= x \mid \text{true} \mid \text{false} \mid \lambda x. \ e \mid e \ e \mid (e, e) \mid e. 1 \mid e. 2 \\
& \quad \mid \text{if } e e e \mid \text{raise } s \mid \text{try } e \ \text{handle } s \ e
\end{align*}$$

$$\begin{align*}
\Gamma \vdash e : \tau; e \\
\Gamma \vdash x : \Gamma(x); \emptyset \\
\Gamma \vdash \text{true} : \text{bool}; \emptyset \\
\Gamma \vdash \text{false} : \text{bool}; \emptyset \\
\end{align*}$$
Key facts

Soundness: If $\vdash e : \tau; \epsilon$ and $e$ raises uncaught exception $s$, then $s \in \epsilon$

- Corollary to Preservation and Progress (once you define the operational semantics for exceptions)

All effect systems work this way:

- Values effectless
- Functions have latent effects
- Conservative due to control-flow (if and try/handle)
- Often some way to mask effects (here, catch an exception)

Only a couple rules special to this effect system

- Also, not always sets and $\cup$

More general rules

Every effect system also substantially more expressive via appropriate subsumption:

- Typing rule for subeffecting (also useful for Preservation)
- Subtyping of function types is covariant in latent effects

\[ \frac{\Gamma \vdash \tau : e; \epsilon \subseteq \epsilon'}{\Gamma \vdash \tau : e; \epsilon'} \quad \frac{\tau_3 \leq \tau_1 \quad \tau_2 \leq \tau_4 \quad \epsilon \subseteq \epsilon'}{\tau_1 \rightarrow \tau_2 \leq \tau_3 \rightarrow \tau_4} \]

Not shown: Also want effect polymorphism (type variables ranging over effects) for higher-order functions like map

Other examples

- Definitely terminates (true) or possibly diverges (false)
  - Give fix $e$ effect false
  - Give values effect true
  - Treat $\cup$ as and
  - No change to rules for functions, pairs, conditionals, etc.
- What type casts might occur
- Are certain variables always accessed in critical sections
- Does code obey a locking protocol
- Does code only access memory regions that haven’t been deallocated
- ...

Really a general way to lift static analysis to higher-order functions

- Key is recognizing “from a mile away” when an effect system is the right tool