Where are we

Done:

- Formal definition of evaluation contexts and first-class continuations
- Continuation-passing style as a programming idiom
- The CPS transform

Now:

- Implement an efficient lambda-calculus interpreter using little more than malloc and a single while-loop
  - Explicit evaluation contexts (i.e., continuations) is essential
  - Key novelty is maintaining the current context incrementally
  - letcc and throw can be $O(1)$ operations (homework problem)
See the code

See lec14code.ml for four interpreters where each is:

- More efficient than the previous one and relies on less from the meta-language
- Close enough to the previous one that equivalence among them is tractable to prove

The interpreters:

1. Plain-old small-step with substitution
2. Evaluation contexts, re-decomposing at each step
3. Incremental decomposition, made efficient by representing evaluation contexts (i.e., continuations) as a linked list with “shallow end” of the stack at the beginning of the list
4. Replacing substitution with environments

The last interpreter is trivial to port to assembly or C
Example

Small-step (first interpreter):

Decomposition (second interpreter):
Example

Decomposition (second interpreter):

\[
\begin{align*}
E &= \lambda a. a \\
L &= \lambda b. b \quad \lambda c. c \\
R &= \lambda d. d \quad \lambda e. e
\end{align*}
\]

Decomposition rewritten with linked list (hole implicit at \textit{front}):

\[
\begin{align*}
c &= L(A(\lambda d. d, \lambda e. e)) :: R(\lambda a. a) :: [] \\
e &= A(\lambda b. b, \lambda c. c) \\
c &= R(\lambda c. c) :: R(\lambda a. a) :: [] \\
e &= A(\lambda d. d, \lambda e. e)
\end{align*}
\]
Example

Decomposition rewritten with linked list (hole implicit at *front*):

\[
\begin{align*}
  c &= L(A(\lambda d. d, \lambda e. e)) :: R(\lambda a. a) :: [] \\
  e &= A(\lambda b. b, \lambda c. c)
\end{align*}
\]

\[
\begin{align*}
  c &= R(\lambda c. c) :: R(\lambda a. a) :: [] \\
  e &= A(\lambda d. d, \lambda e. e)
\end{align*}
\]

Some loop iterations of third interpreter:

\[
\begin{align*}
  e &= A(\lambda b. b, \lambda c. c) \\
  c &= L(A(\lambda d. d, \lambda e. e)) :: R(\lambda a. a) :: []
\end{align*}
\]

\[
\begin{align*}
  e &= \lambda b. b \\
  c &= L(\lambda c. c) :: L(A(\lambda d. d, \lambda e. e)) :: R(\lambda a. a) :: []
\end{align*}
\]

\[
\begin{align*}
  e &= \lambda c. c \\
  c &= R(\lambda b. b) :: L(A(\lambda d. d, \lambda e. e)) :: R(\lambda a. a) :: []
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  e &= \lambda c. c \\
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\[
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  c &= R(\lambda c. c) :: R(\lambda a. a) :: []
\end{align*}
\]

Fourth interpreter: replace substitution with environment/closures
The end result

The last interpreter needs just:

- A loop
- Lists for contexts and environments
- Tag tests

Moreover:

- Function calls execute in $O(1)$ time
- Variable look-ups don’t, but that’s fixable
  - (e.g., de Bruijn indices and arrays for environments)
- Other operations, including pairs, conditionals, letcc, and throw also all work in $O(1)$ time
  - Need new kinds of contexts and values
  - Left as a homework exercise as a way to understand the code

Making evaluation contexts explicit data structures was key