Where are we

Done:
- Formal definition of evaluation contexts and first-class continuations
- Continuation-passing style as a programming idiom
- The CPS transform

Now:
- Implement an efficient lambda-calculus interpreter using little more than malloc and a single while-loop
  - Explicit evaluation contexts (i.e., continuations) is essential
  - Key novelty is maintaining the current context incrementally
  - letcc and throw can be $O(1)$ operations (homework problem)

See the code

See lec14code.ml for four interpreters where each is:
- More efficient than the previous one and relies on less from the meta-language
- Close enough to the previous one that equivalence among them is tractable to prove

The interpreters:
1. Plain-old small-step with substitution
2. Evaluation contexts, re-decomposing at each step
3. Incremental decomposition, made efficient by representing evaluation contexts (i.e., continuations) as a linked list with “shallow end” of the stack at the beginning of the list
4. Replacing substitution with environments

The last interpreter is trivial to port to assembly or C

Example

Small-step (first interpreter):

Decomposition (second interpreter):

$e =$ \lambda.a \ a
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$E =$ \lambda.a \ a
$D =$ \lambda.b \ b
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$D =$ \lambda.b \ b
$H =$ \lambda.e \ e
$H =$ \lambda.e \ e
$H =$ \lambda.e \ e
$H =$ \lambda.e \ e
Example

Decomposition (second interpreter):

Decomposition rewritten with linked list (hole implicit at front):

\[
\begin{align*}
E &= \lambda.a. a \\
R &= \lambda.d. d \\
H &= \lambda.e. e \\
e &= \lambda.b. b \\
A &= \lambda.c. c \\
\end{align*}
\]

Decomposition rewritten with linked list (hole implicit at front):

\[
\begin{align*}
c &= L(A(\lambda.d. d, \lambda.e. e)) :: R(\lambda.a. a) :: [] \\
e &= A(\lambda.b. b, \lambda.c. c) \\
\end{align*}
\]

The last interpreter needs just:

- A loop
- Lists for contexts and environments
- Tag tests

Moreover:

- Function calls execute in \(O(1)\) time
- Variable look-ups don’t, but that’s fixable
  - (e.g., de Bruijn indices and arrays for environments)
- Other operations, including pairs, conditionals, letcc, and throw also all work in \(O(1)\) time
  - Need new kinds of contexts and values
  - Left as a homework exercise as a way to understand the code

Making evaluation contexts explicit data structures was key

Example

Decomposition rewritten with linked list (hole implicit at front):

\[
\begin{align*}
c &= R(\lambda.c. c) :: R(\lambda.a. a) :: [] \\
e &= A(\lambda.d. d, \lambda.e. e) \\
\end{align*}
\]

Some loop iterations of third interpreter:

\[
\begin{align*}
e &= A(\lambda.b. b, \lambda.c. c) \\
c &= L(A(\lambda.d. d, \lambda.e. e)) :: R(\lambda.a. a) :: [] \\
e &= \lambda.b. b \\
c &= L(\lambda.c. c) :: L(A(\lambda.d. d, \lambda.e. e)) :: R(\lambda.a. a) :: [] \\
e &= \lambda.c. c \\
c &= R(\lambda.b. b) :: L(A(\lambda.d. d, \lambda.e. e)) :: R(\lambda.a. a) :: [] \\
e &= \lambda.c. c \\
c &= L(A(\lambda.d. d, \lambda.e. e)) :: R(\lambda.a. a) :: [] \\
e &= A(\lambda.d. d, \lambda.e. e) \\
c &= R(\lambda.c. c) :: R(\lambda.a. a) :: [] \\
\end{align*}
\]

Fourth interpreter: replace substitution with environment/closures