Polymorphic Type Inference Review Problems

CSE505

November 28, 2001

Determine the (principal) types of the following expressions (using the Hindley-Milner algorithm), or say how the algorithm fails:

a. $\lambda x.\lambda y.\lambda z.((x\ z)\ (y\ z))$

   $(\alpha \to \beta \to \gamma) \to (\alpha \to \beta) \to \alpha \to \gamma$

b. let f = $\lambda x.$ (if x then x else f(1)) in f(0)

   The type inferred for $\lambda x.$ (if x then x else f(1)) is Bool → Bool. In the process, the type of f is inferred to be Int → Bool because of the call to f(1) in the body of the function (since f is treated as monomorphic within its own definition). These two types must be unified to get the final type of f, and this unification fails because Int and Bool cannot be unified.

c. $\lambda x.(x\ x)$

   When inferring the type of $(x\ x)$, we attempt to unify α (the assumed type of x) with $\alpha \to \beta$, for some fresh β. This unification fails because of the occurs check.

d. λ f. $\lambda x.$ if x then f(x) else f(0)+1

   After inferring the type of f(x), we have learned that f has a type of the form Bool → α. When inferring the type of f(0), we attempt to unify Bool → α with Int → β, for some fresh type variable β, which fails because Bool and Int cannot be unified.

e. let f = $\lambda z.0$ in $\lambda x.$ if x then f(x) else f(0)+1

   Bool → Int