Representation of programs

Primary goals:

- analysis is easy & effective
  - just a few cases to handle
  - provide support for linking things of interest
- transformations are easy
- general, across input languages & target machines

Additional goals:

- compact in memory
- easy to translate to and from
- tracks info for source-level debugging, profiling, etc.
- extensible (new optimizations, targets, language features)
- displayable

Example IRs:

- C?
- Java bytecode?
- ...

High-level syntax-based representation

Represent source-level control structures & expressions directly

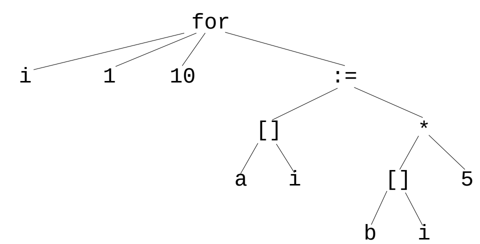
Examples

- (Attributed) AST
- Lisp S-expressions
- lambda calculus? Java bytecode?

Source:

```
for i := 1 to 10 do
  a[i] := b[i] * 5;
end
```

AST:



Low-level representation

Translate input programs into low-level primitive chunks, often close to the target machine

Examples

- assembly code, virtual machine code (e.g. stack machine)
- three address code, register transfer language (RTLs)
- lambda calculus? Java bytecode?

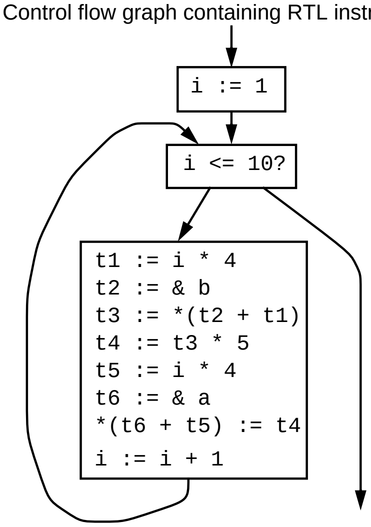
Standard RTL operators:

assignment	`x := y;`
unary op	`x :=` *`op`* `y;`
binary op	`x := y` *`op`* `z;`
address-of	`p := &y;`
load	`x := *(p + o);`
store	`*(p + o) := x;`
call	`x := f(...);`
unary compare	*`op`* `x ?`
binary compare	`x` *`op`* `y ?`

Source:

```
for i := 1 to 10 do
  a[i] := b[i] * 5;
end
```

Control flow graph containing RTL instructions:



Comparison

Advantages of high-level rep:

- analysis can exploit high-level knowledge of constructs
  - probably faster to analyze
- supports semantics-based reasoning about correctness etc. of analysis
- easy to map to source code terms for debugging, profiling
- may be more compact

Advantages of low-level rep:

- can do low-level, machine-specific optimizations (if target-based representation)
  - high-level rep may not be able to express some transformations
- can have relatively few kinds of instructions to analyze
- can be language-independent

High-level rep suitable for a source-to-source or special-purpose optimizer, e.g. inliner, parallelizer

Can mix multiple representations in single compiler

Can sequence compilers using different reps

Components of representation

Operations

Dependences between operations

- **control** dependences: sequencing of operations
  - evaluation of then & else arms depends on result of test
  - side-effects of statements occur in right order
- **data** dependences: flow of values from **definitions** to **uses**
  - operands computed before operation
  - values read from variable before being overwritten

Ideal: represent just those dependences that matter

- dependences constrain transformations
- fewest dependences ⇒ most flexibility in implementation

Representing control dependences

Option 1: **high-level representation**

- control flow implicit in semantics of AST nodes

Option 2: **control flow graph**

- nodes are **basic blocks**
  - instructions in basic block sequence side-effects
- edges represent branches (control flow between basic blocks)

Some fancier options:

- **control dependence graph**, part of **program dependence graph** (PDG) [Ferrante *et al.* 87]
- convert into data dependences on a memory state, in **value dependence graph** (VDG) [Weise *et al. 94*]

Kinds of data dependences

read-after-write (RAW): **true/flow dependence**

- reflects real data flow, operands to operation

write-after-read (WAR): **anti-dependence**

write-after-write (WAW): **output dependence**

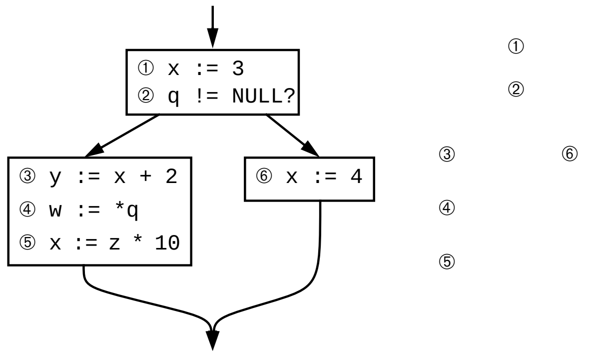
- reflects overwriting of memory, not real data flow
  ⇒ can sometimes be eliminated by optimization

read-after-read (RAR): no dependence

- can occur in any order

Example

```
x := 3
if q != NULL then
  y := x + 2
  w := *q
  x := z * 10
else
  x := 4
endif
```



Representing data dependences

Option 1: implicitly through variable defs/uses in CFG

+ simple, source-like
– may overconstrain order of operations
– analysis wants important things explicit ⇒ analysis can be slow

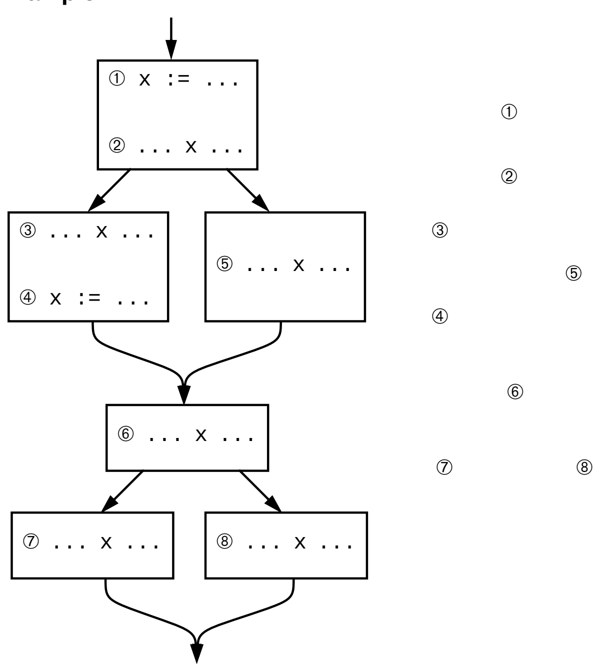
Option 2: def/use chains, linking each def to each use

+ explicit ⇒ analysis can be fast
– must be computed, maintained after transformations
– may be space-consuming

Fancier options:

- **static single assignment** (SSA) form [Alpern *et al.* 88]
- value dependence graphs (VDGs)
- **dependence flow graphs** (DFGs)
- ...

Example



Data flow analysis

Want to compute some info about program

- at **program points**
- to identify opportunities for improving transformations

Can model data flow analysis as solving system of **constraints**

- each node in CFG imposes a constraint relating info at predecessor and successor points
- solution to constraints is result of analysis

Solution must be **safe**/**sound**
Solution can be **conservative**

Key issues:

- how to represent info efficiently?
- how to represent & solve constraints efficiently?
  - how long does constraint solving take? does it terminate?
- what if multiple solutions are possible?
- how to synchronize transformations with analysis?
- how to know if analysis & transformations we've defined is semantics-preserving?

Example: reaching definitions

For each program point,
want to compute set of definitions (statements) that *may reach* that point

- reach: are the last definition of some variable

Info ≡ set of *var*→*stmt* bindings

E.g.:
$\{x \rightarrow s_1, y \rightarrow s_5, y \rightarrow s_8\}$

Can use reaching definition info to:

- build def-use chains
- do constant & copy propagation
- detect references to undefined variables
- present use/def info to programmer
- ...

Safety rule (for these intended uses of this info):
can have more bindings than the "true" answer,
but can't miss any

Constraints for reaching definitions

Main constraints:

A simple assignment removes any old reaching defs for the lhs and replaces them with this stmt:

- **strong update**

s: `x := ...:`

$$info_{succ} = info_{pred} - \{x \rightarrow s' \mid \forall s'\} \cup \{x \rightarrow s\}$$

A pointer assignment may modify anything, but doesn't definitely replace anything

- **weak update**

s: `*p := ...:`

$$info_{succ} = info_{pred} \cup \{x \rightarrow s \mid \forall x \in \text{may-point-to}(p)\}$$

Other statements: do nothing

$$info_{succ} = info_{pred}$$

Constraints for reaching definitions, continued

Branches pass through reaching defs to both successors

$$info_{succ[i]} = info_{pred}$$

Merges take the union of all incoming reaching defs

- we don't know which path is being taken at run-time
  ⇒ be conservative

$$info_{succ} = \bigcup_i info_{pred[i]}$$

Conditions at entry to CFG: definitions of formals

$$info_{entry} = \{x \rightarrow \mathit{entry} \mid \forall x \in \text{formals}\}$$

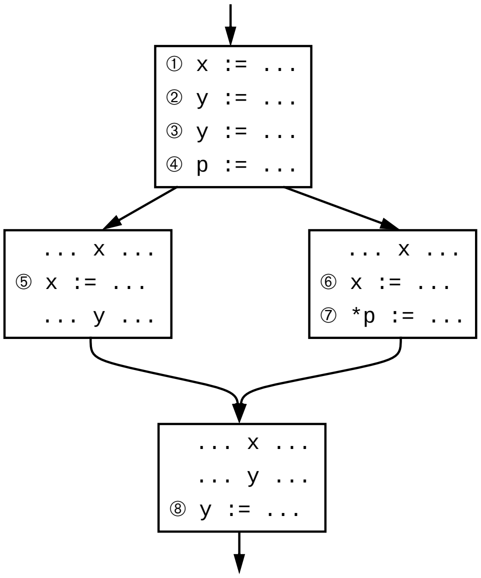
Solving constraints

A given program yields a system of constraints

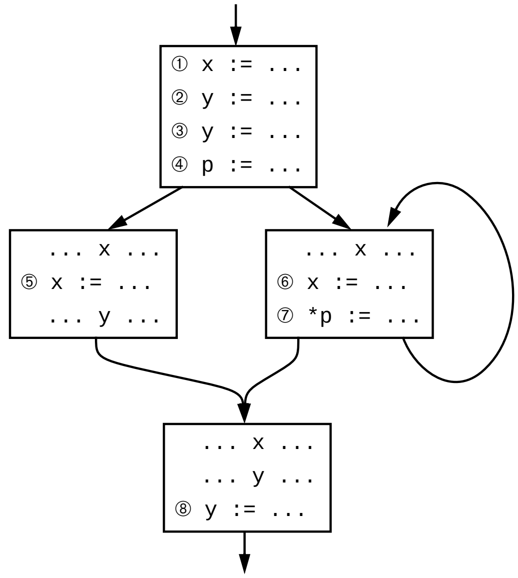
Need to solve constraints

For reaching definitions,
can traverse instructions in forward topological order,
computing successor info from predecessor info

Example



Another example



Topological order not defined!

Loop terminology

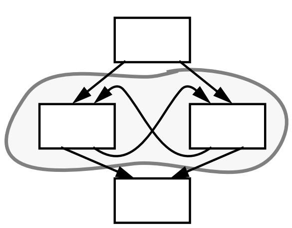
loop: strongly-connected component in CFG with single entry

loop entry edge: source not in loop, target in loop
loop exit edge: the reverse
back edge: target is loop head node

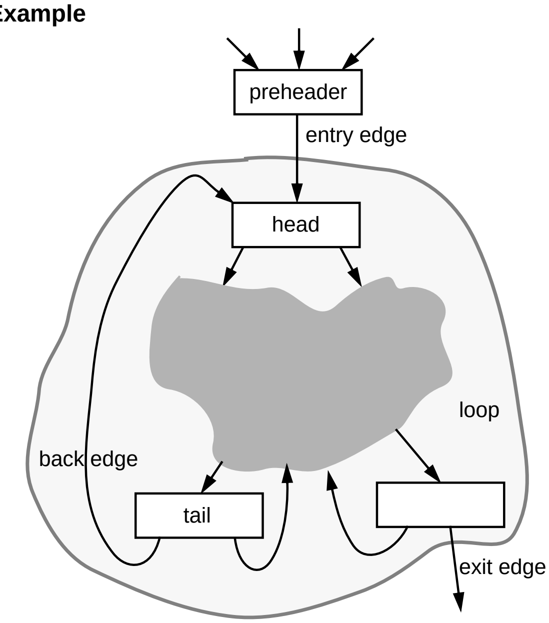
loop head node: target of loop entry edge
loop tail node: source of back edge
loop preheader node:
single node that's source of loop entry edge

nested loop: loop whose head is inside another loop

reducible flow graph: all SCC's have single entry



Example



Analysis of loops

If CFG has a loop, data flow constraints are recursively defined:
$info_{loop\text{-}head} = info_{loop\text{-}entry} \cup info_{back\text{-}edge}$
$info_{back\text{-}edge} = \ldots\ info_{loop\text{-}head} \cdots$

Substituting definition of $info_{back\text{-}edge}$:
$info_{loop\text{-}head} = info_{loop\text{-}entry} \cup (\ldots\ info_{loop\text{-}head} \ldots)$

Summarizing r.h.s. as F:
$info_{loop\text{-}head} = F(info_{loop\text{-}head})$

A legal solution to constraints is a **fixed-point** of F

Recursive constraints can have many solutions

- want **least** or **greatest** fixed-point, whichever corresponds to the most precise answer

How to find least/greatest fixed-point of F?

- for restricted CFGs can use specialized methods
  - e.g. **interval analysis** for **reducible** CFGs
- for arbitrary CFGs, can use **iterative** approximation

Iterative data flow analysis

1. Start with initial guess of info at loop head:
   $info_{loop\text{-}head} = guess$

2. Solve equations for loop body:
   $info_{back\text{-}edge} = F_{body}\,(info_{loop\text{-}head})$
   $info_{loop\text{-}head}' = info_{loop\text{-}entry} \cup info_{back\text{-}edge}$

3. Test if found fixed-point:
   $info_{loop\text{-}head}' = info_{loop\text{-}head}$?

   A. if same, then done

   B. if not, then adopt result as (better) guess and repeat:

   $info_{back\text{-}edge}' = F_{body}\,(info_{loop\text{-}head}')$
   $info_{loop\text{-}head}'' = info_{loop\text{-}entry} \cup info_{back\text{-}edge}'$
   $info_{loop\text{-}head}'' = info_{loop\text{-}head}'$?
   ...

When does iterating work?

1. need to be able to make an initial guess
2. $info^{n+1}$ must be closer to the fixed-point than $info^{n}$
   (true if F_{body} is **monotonic**)
3. must eventually reach the fixed-point
   in a finite number of iterations
   (true if info drawn from a **finite-height domain**)

To reach best fixed-point, initial guess for loop head should be **optimistic**

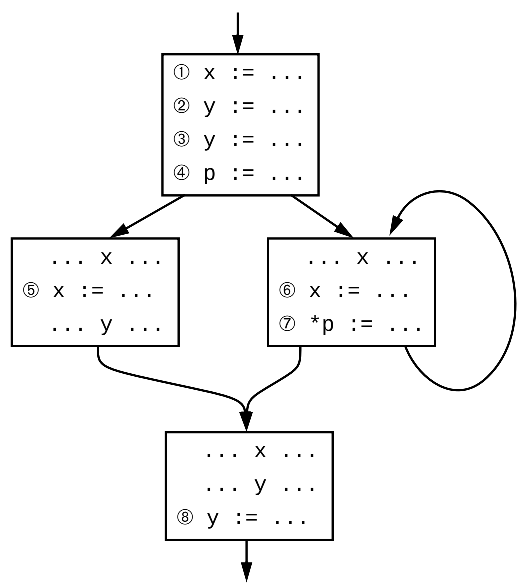
- easy choice: $info_{loop\text{-}head} = info_{loop\text{-}entry}$

(Even if guess is overly optimistic, iteration will ensure we won't stop analysis until the answer is safe.)

To speed iterative analysis, want to test guess ASAP

- avoid solving constraints outside of loop until fixed-point is reached within loop

The example, again



Direction of dataflow analysis

In what order are constraints solved, in general?

Constraints are declarative, not directional/procedural, so may require mixing forward & backward solving, or other more global solution methods

But often constraints can be solved by (directional) propagation & iteration
- may be **forward** or **backward** propagation of info
- topological traversals of acyclic subgraphs minimize analysis time

Directional constraints often called **flow functions**
- often written as functions on input info to compute output

$RD_{s:\ \texttt{x := ...}}(in) = in - \{x \to s' \mid \forall s'\} \cup \{x \to s\}$
$RD_{s:\ \texttt{*p := ...}}(in) = in \cup \{x \to s \mid \forall x \in \text{may-point-to}(p)\}$

GEN and KILL sets

For even more structure,
can often think of flow functions in terms of each's GEN set and KILL set
- GEN = new information added
- KILL = old information removed

Then
$F_{instr}(in) = in - KILL_{instr} \cup GEN_{instr}$

E.g., for reaching defs:
$RD_{s:\ \texttt{x := ...}}(in) = in - \{x \to s' \mid \forall s'\} \cup \{x \to s\}$
$RD_{s:\ \texttt{*p := ...}}(in) = in \qquad \cup \{x \to s \mid \forall x \in \text{mpt}(p)\}$

Bit vectors

Can sometimes represent info/KILL/GEN sets as **bit vectors**
- if can express abstractly as set of things
  (e.g. statements, vars),
  drawn from a statically known set of things,
  each thing getting a statically determined bit position
- bitvector encodes **characteristic function** of set

E.g., for reaching defs:
info = bitvector over statements,
each stmt getting a distinct bit position
- statement implies which variable is defined

Bit vectors compactly represent sets
Bit-vector operations efficiently perform set difference & union

Flow function may be able to be represented simply by a pair of bit vectors, if they don't depend on input bit vector
- can merge the KILL and GEN bit vectors of a whole basic block of instructions into a single overall KILL and GEN set, for faster iterating

Another example: constant propagation

Goal: data flow analysis that implements constant propagation

What info computed for each program point?

I is a conservative approximation to true info I_{true} iff:

$CP_{\texttt{x := }N}$:

$CP_{\texttt{x := y + z}}$:

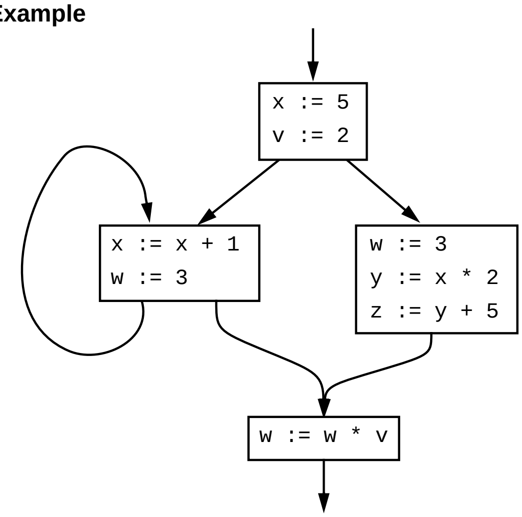
$CP_{\texttt{*p := *q + *r}}$:

Merge function?

Direction of analysis?

Initial info, at what program point(s)?
Can use bit vectors?

Example



May vs. must info

Some kinds of info imply guarantees: **must** info

Some kinds of info imply possibilities: **may** info

- the complement of **may** info is **must not** info

	May	**Must**
desired info	small set	big set
safe	overly big set	overly small set
GEN	add everything that might be true	add only if guaranteed true
KILL	remove only if guaranteed wrong	remove everything possibly wrong
MERGE	$\cup$	$\cap$

Another example: live variables

Want the set of variables that are **live** at each pt. in program

- live: might be used later in the program

Supports dead assignment elimination, register allocation

What info computed for each program point?

May or must info?

I is a conservative approximation to true info I_{true} iff:

$LV_{\texttt{x := y + z}}$:

$LV_{\texttt{*p := *q + *r}}$:

Merge function?

Direction of analysis?

Initial info, at what program point(s)?

Can use bit vectors?

Example