Natural Language Processing (CSE 490U): Featurized Language Models

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What's wrong with n-grams?

Data sparseness: most histories and most words will be seen only rarely (if at all).

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Next central idea: teach histories and words how to share.

Log-Linear Models: Definitions

We define a conditional log-linear model $p(Y \mid X)$ as:

- $\mathcal{Y}$ is the set of events/outputs (☺ for language modeling, $\mathcal{V}$)
- $\mathcal{X}$ is the set of contexts/inputs (☺ for n-gram language modeling, $\mathcal{V}^{\mathsf{n}-1}$)
- $\boldsymbol{\phi} : \mathcal{X} \times \mathcal{Y} \rightarrow \mathbb{R}^d$ is a feature vector function
- $\mathbf{w} \in \mathbb{R}^d$ are the model parameters

$$p_{\mathbf{w}}(Y = y \mid X = x) = \frac{\exp \mathbf{w} \cdot \boldsymbol{\phi}(x, y)}{\displaystyle\sum_{y' \in \mathcal{Y}} \exp \mathbf{w} \cdot \boldsymbol{\phi}(x, y')}$$

Breaking It Down

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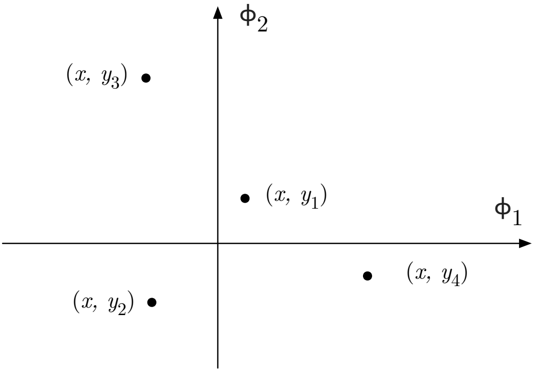
"Log-linear" comes from the fact that:

$$\log p_{\mathbf{w}}(Y = y \mid X = x) = \mathbf{w} \cdot \boldsymbol{\phi}(x, y) - \underbrace{\log Z_{\mathbf{w}}(x)}_{\text{constant in } y}$$

This is an instance of the family of **generalized linear models**.

The Geometric View

Suppose we have instance x, $\mathcal{Y} = \{y_1, y_2, y_3, y_4\}$, and there are only two features, ϕ_1 and ϕ_2.



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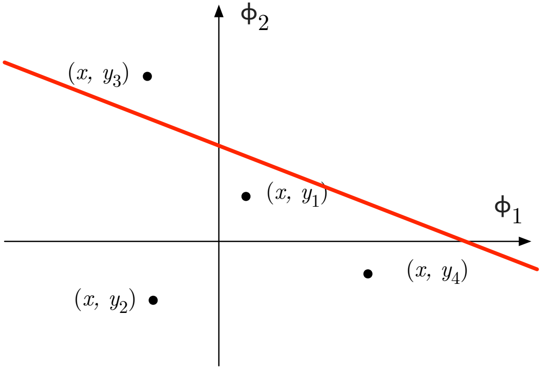

$$\mathbf{w} \cdot \boldsymbol{\phi} = w_1\phi_1 + w_2\phi_2 = 0$$

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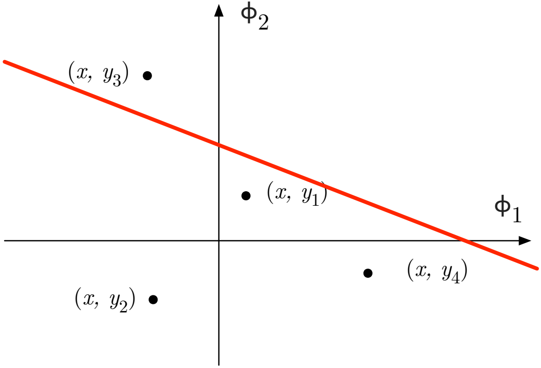

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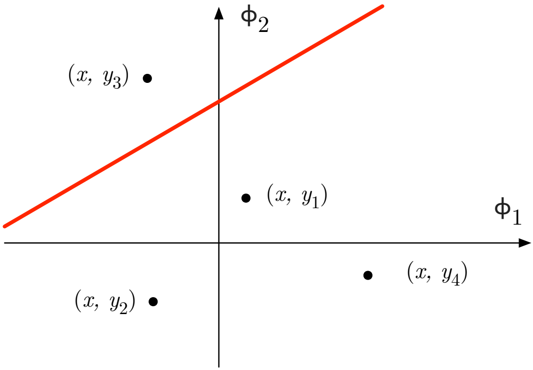

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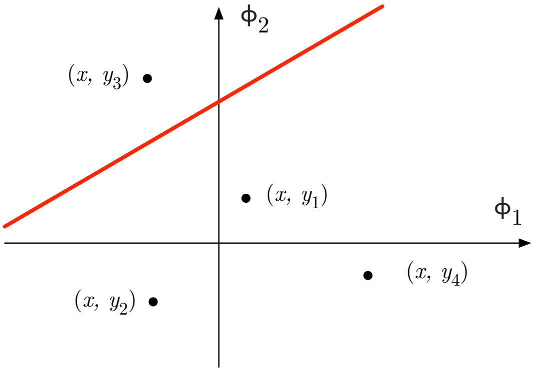

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Why Build Language Models This Way?

- Exploit **features** of histories for sharing of statistical strength and better smoothing (Lau et al., 1993)
- Condition the whole text on more interesting variables like the gender, age, or political affiliation of the author (Eisenstein et al., 2011)
- Interpretability!
  - Each feature ϕ_k controls a factor to the probability (e^{w_k}).
  - If $w_k < 0$ then ϕ_k makes the event less likely by a factor of $\frac{1}{e^{w_k}}$.
  - If $w_k > 0$ then ϕ_k makes the event more likely by a factor of e^{w_k}.
  - If $w_k = 0$ then ϕ_k has no effect.

Log-Linear n-Gram Models

$$
\begin{aligned}
p_{\mathbf{w}}(\boldsymbol{X} = \boldsymbol{x}) &= \prod_{j=1}^{\ell} p_{\mathbf{w}}(X_j = x_j \mid \boldsymbol{X}_{1:j-1} = \boldsymbol{x}_{1:j-1}) \\
&= \prod_{j=1}^{\ell} \frac{\exp \mathbf{w} \cdot \boldsymbol{\phi}(\boldsymbol{x}_{1:j-1}, x_j)}{Z_{\mathbf{w}}(\boldsymbol{x}_{1:j-1})} \\
&\overset{\text{assumption}}{=} \prod_{j-1}^{\ell} \frac{\exp \mathbf{w} \cdot \boldsymbol{\phi}(\boldsymbol{x}_{j-\mathsf{n}+1:j-1}, x_j)}{Z_{\mathbf{w}}(\boldsymbol{x}_{j-\mathsf{n}+1:j-1})} \\
&= \prod_{j=1}^{\ell} \frac{\exp \mathbf{w} \cdot \boldsymbol{\phi}(\boldsymbol{h}_j, x_j)}{Z_{\mathbf{w}}(\boldsymbol{h}_j)}
\end{aligned}
$$

Example

The man who knew too $\boxed{\begin{array}{c}\text{much}\\ \text{many}\\ \text{little}\\ \text{few}\\ \vdots\\ \text{hippopotamus}\end{array}}$

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  - "Feature selection" methods, e.g., ignoring features with very low counts, can help.
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- More recent work in neural networks can be seen as *discovering* features (instead of engineering them).
- But in much of NLP, there's a strong preference for *interpretable* features.

How to Estimate $\mathbf{w}$?

	n-gram	log-linear n-gram
$p_{\boldsymbol{\theta}}(\boldsymbol{x}) =$	$\prod_{j=1}^{\ell} \theta_{x_j \mid \boldsymbol{h}_j}$	$\prod_{j-1}^{\ell} \frac{\exp \mathbf{w} \cdot \boldsymbol{\phi}(\boldsymbol{h}_j, x_j)}{Z_{\mathbf{w}}(\boldsymbol{h}_j)}$
Parameters:	$\theta_{v \mid \boldsymbol{h}}$ $\forall v \in \mathcal{V}, \boldsymbol{h} \in (\mathcal{V} \cup \{\bigcirc\})^{\mathsf{n}-1}$	w_k $\forall k \in \{1, \ldots, d\}$
MLE:	$\hat{\theta}_{v \mid \boldsymbol{h}} = \frac{c(\boldsymbol{h}v)}{c(\boldsymbol{h})}$	no closed form

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$$= \max_{\mathbf{w}\in\mathbb{R}^d} \sum_{i=1}^{N} \mathbf{w} \cdot \boldsymbol{\phi}(\boldsymbol{h}_i, x_i) - \log \underbrace{\sum_{v\in\mathcal{V}} \exp \mathbf{w} \cdot \boldsymbol{\phi}(\boldsymbol{h}_i, v)}_{Z_{\mathbf{w}}(\boldsymbol{h}_i)}$$

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- This is *concave* in $\mathbf{w}$.
- $Z_{\mathbf{w}}(\boldsymbol{h}_i)$ involves a sum over V terms.

MLE for $\mathbf{w}$

$$\max_{\mathbf{w} \in \mathbb{R}^d} \sum_{i=1}^{N} \underbrace{\mathbf{w} \cdot \phi(\boldsymbol{h}_i, x_i) - \log Z_{\mathbf{w}}(\boldsymbol{h}_i)}_{f_i(\mathbf{w})}$$

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Hope/fear view: for each instance i,

- increase the score of the correct output x_i,
  $score(x_i) = \mathbf{w}\cdot\boldsymbol{\phi}(\boldsymbol{h}_i, x_i)$
- decrease the "softened max" score overall,
  $\log\sum_{v\in\mathcal{V}} \exp score(v)$

MLE for $\mathbf{w}$

$$\max_{\mathbf{w}\in\mathbb{R}^d} \sum_{i=1}^{N} \underbrace{\mathbf{w}\cdot\boldsymbol{\phi}(\boldsymbol{h}_i, x_i) - \log Z_{\mathbf{w}}(\boldsymbol{h}_i)}_{f_i(\mathbf{w})}$$

Gradient view:

$$\nabla_{\mathbf{w}} f_i = \underbrace{\boldsymbol{\phi}(\boldsymbol{h}_i, x_i)}_{\text{observed features}} - \underbrace{\sum_{v\in\mathcal{V}} p_{\mathbf{w}}(v \mid \boldsymbol{h}_i)\cdot\boldsymbol{\phi}(\boldsymbol{h}, v)}_{\text{expected features}}$$

Setting this to zero means getting model's expectations to match empirical observations.

MLE for $\mathbf{w}$: Algorithms

- Batch methods (L-BFGS is popular)
- Stochastic gradient ascent/descent more common today, especially with special tricks for adapting the step size over time
- Many specialized methods (e.g., “iterative scaling”)

Stochastic Gradient Descent

Goal: minimize $\sum_{i=1}^{N} f_i(\mathbf{w})$ with respect to $\mathbf{w}$.

Input: initial value $\mathbf{w}$, number of epochs T, learning rate α

For $t \in \{1, \ldots, T\}$:

- Choose a random permutation π of $\{1, \ldots, N\}$.
- For $i \in \{1, \ldots, N\}$:

$$\mathbf{w} \leftarrow \mathbf{w} - \alpha \cdot \nabla_{\mathbf{w}} f_{\pi(i)}$$

Output: $\mathbf{w}$

Avoiding Overfitting

Maximum likelihood estimation:

$$\max_{\mathbf{w} \in \mathbb{R}^d} \sum_{i=1}^{N} \mathbf{w} \cdot \boldsymbol{\phi}(\boldsymbol{h}_i, x_i) - \log Z_{\mathbf{w}}(\boldsymbol{h}_i)$$

- If $\phi_j(\boldsymbol{h}, x)$ is (almost) always positive, we can always increase the objective (a little bit) by increasing w_j toward $+\infty$.

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Standard solution is to add a regularization term:

$$\max_{\mathbf{w} \in \mathbb{R}^d} \sum_{i=1}^{N} \mathbf{w} \cdot \boldsymbol{\phi}(\boldsymbol{h}_i, x_i) - \log \sum_{v \in \mathcal{V}} \exp \mathbf{w} \cdot \boldsymbol{\phi}(\boldsymbol{h}_i, v) - \lambda \|\mathbf{w}\|_p^p$$

where $\lambda > 0$ is a hyperparameter and $p = 2$ or 1.

MLE for $\mathbf{w}$

If we had more time, we'd study this problem more carefully!

Here's what you must remember:

- There is no closed form; you must use a numerical optimization algorithm like stochastic gradient descent.
- Log-linear models are powerful but expensive $(Z_{\mathbf{w}}(\boldsymbol{h}_i))$.
- Regularization is very important; we don't actually do MLE.
  - Just like for n-gram models! Only even more so, since log-linear models are even more expressive.

To-Do List

- Online quiz: due 11:59 pm Tuesday
- Read: Collins (2011) §2
- A1, out today, due January 18

References I

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Extras

Special Case: Logistic Regression

Consider the case where $\mathcal{Y} = \{+1, -1\}$.

$$p_{\mathbf{w}}(Y = +1 \mid x) = \frac{\exp \mathbf{w} \cdot \boldsymbol{\phi}(x, +1)}{\exp \mathbf{w} \cdot \boldsymbol{\phi}(x, +1) + \exp \mathbf{w} \cdot \boldsymbol{\phi}(x, -1)}$$

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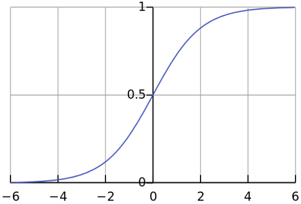
- Should be familiar, if you know about logistic regression.



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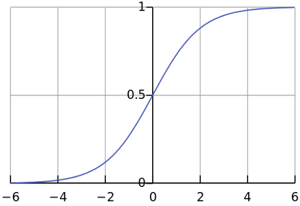

- When $\mathcal{Y} = \{1, 2, \ldots, k\}$, log-linear models are often called **multinomial logistic regression**.

Special Case: Classic n-Gram Language Model

Consider an n-gram language model, where $\mathcal{X} = \mathcal{V}^{\mathsf{n}-1}$ and $\mathcal{Y} = \mathcal{V}$. Let:

- $d = 1$
- $\phi_1(\boldsymbol{h}, v) = \log c(\boldsymbol{h}v)$
- $w_1 = 1$
- $Z(\boldsymbol{h}) = \sum_{v' \in \mathcal{V}} \exp \log c(\boldsymbol{h}v') = \sum_{v' \in \mathcal{V}} c(\boldsymbol{h}v') = c(\boldsymbol{h})$

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Alternately:

- $d = |\mathcal{V}|^{\mathsf{n}}$
- $\phi_{\tilde{\boldsymbol{h}}, \tilde{v}}(\boldsymbol{h}, v) = \begin{cases} 1 & \text{if } \boldsymbol{h} = \tilde{\boldsymbol{h}} \wedge v = \tilde{v} \\ 0 & \text{otherwise} \end{cases}$
- $w_{\tilde{\boldsymbol{h}}, \tilde{v}} = \log \frac{c(\tilde{\boldsymbol{h}}\tilde{v})}{c(\tilde{\boldsymbol{h}})}$
- $Z(\boldsymbol{h}) = 1$

ℓ_1 Regularization

This case warrants a little more discussion:

$$\max_{\mathbf{w} \in \mathbb{R}^d} \sum_{i=1}^{N} \mathbf{w} \cdot \boldsymbol{\phi}(\boldsymbol{h}_i, x_i) - \log \sum_{v \in \mathcal{V}} \exp \mathbf{w} \cdot \boldsymbol{\phi}(\boldsymbol{h}_i, v) - \lambda \|\mathbf{w}\|_1$$

Note that:

$$\|\mathbf{w}\|_1 = \sum_{j=1}^{d} |w_j|$$

- This results in **sparsity** (i.e., many $w_j = 0$).

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- This is not differentiable at $w_j = 0$.
- Optimization: special solutions for batch (e.g., Andrew and Gao, 2007) and stochastic (e.g., Langford et al., 2009) settings.

Maximum Entropy

Consider a distribution p over events in $\mathcal{X}$. The Shannon entropy (in bits) of p is defined as:

$$H(p) = -\sum_{x \in \mathcal{X}} p(X = x) \begin{cases} 0 & \text{if } p(X = x) = 0 \\ \log_2 p(X = x) & \text{otherwise} \end{cases}$$

This is a measure of "randomness"; entropy is zero when p is deterministic and $\log |\mathcal{X}|$ when p is uniform.

Maximum entropy principle: among distributions that fit the data, pick the one with the greatest entropy.

Maximum Entropy

If "fit the data" is taken to mean

$$\forall k \in \{1, \ldots, d\}, \mathbb{E}_p[\phi_k] = \tilde{\mathbb{E}}[\phi_k]$$

then the MLE of the log-linear family with features ϕ is the maximum entropy solution.

This is why log-linear models are sometimes called "maxent" models (e.g., Berger et al., 1996)

"Whole Sentence" Log-Linear Models

(Rosenfeld, 1994)

Instead of a log-linear model for each word-given-history, define a single log-linear model over event space $\mathcal{V}^\dagger$:

$$p_{\mathbf{w}}(\boldsymbol{x}) = \frac{\exp \mathbf{w} \cdot \boldsymbol{\phi}(\boldsymbol{x})}{Z_{\mathbf{w}}}$$

- Any feature of the sentence could be included in this model!
- $Z_{\mathbf{w}}$ is deceptively simple-looking!

$$Z_{\mathbf{w}} = \sum_{\boldsymbol{x} \in \mathcal{V}^\dagger} \exp \mathbf{w} \cdot \boldsymbol{\phi}(\boldsymbol{x})$$