# CSE 490 U: Deep Learning
## Spring 2016

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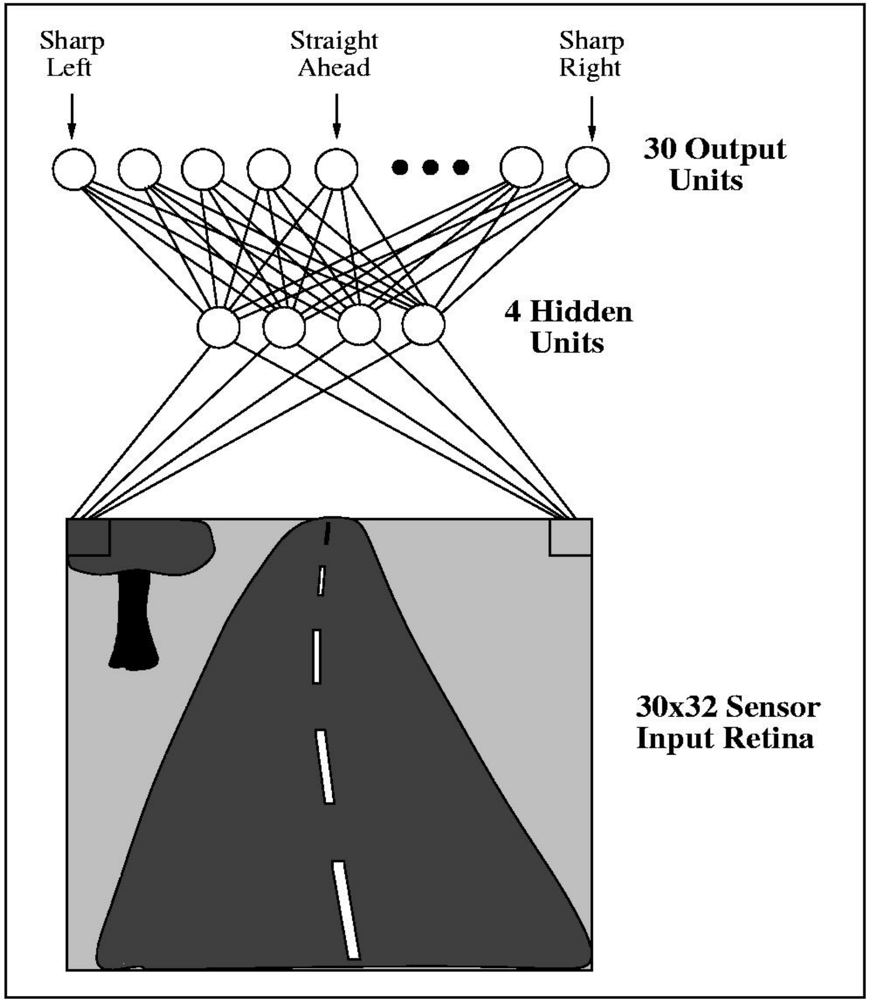
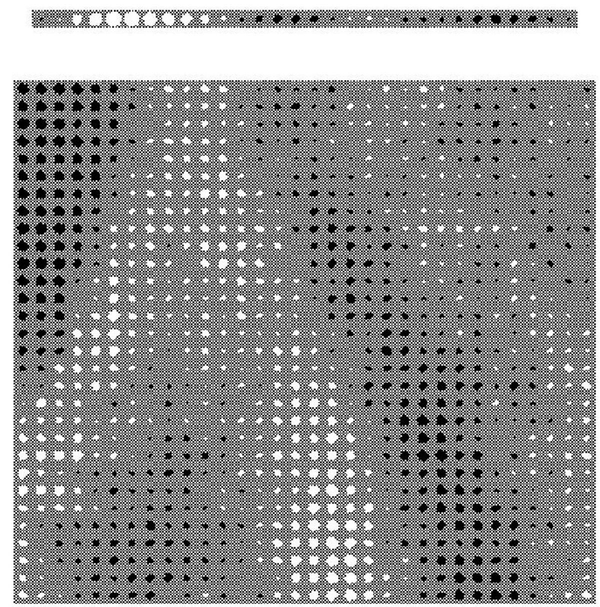
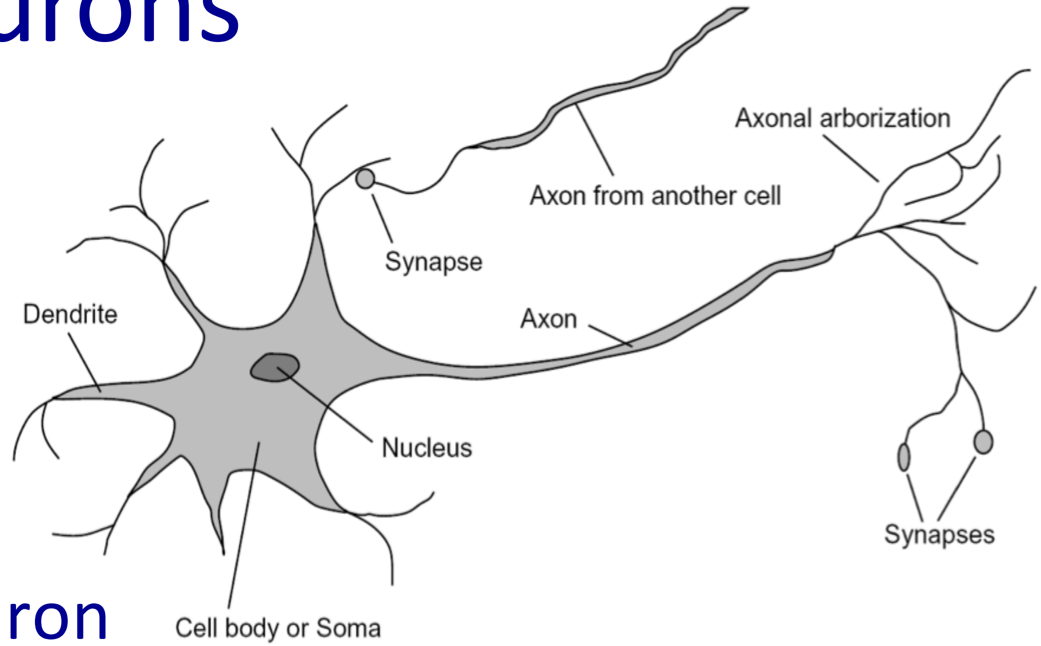
Some slides from Carlos Guestrin, Andrew Rosenberg, Luke Zettlemoyer


Sharp
Left
Straight
Ahead
Sharp
Right
30 Output
Units
4 Hidden
Units
30x32 Sensor
Input Retina

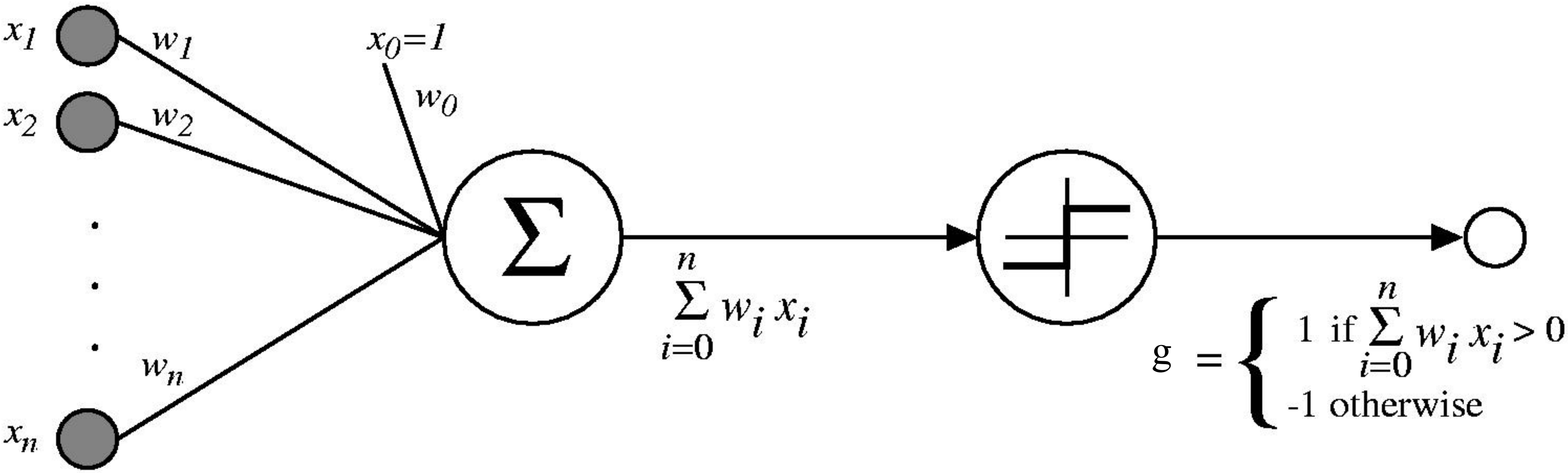


# Human Neurons




- Switching time
  - ~ 0.001 second
- Number of neurons
  - $10^{10}$
- Connections per neuron
  - $10^{4-5}$
- Scene recognition time
  - 0.1 seconds
- Number of cycles per scene recognition?
  - 100 → much parallel computation!

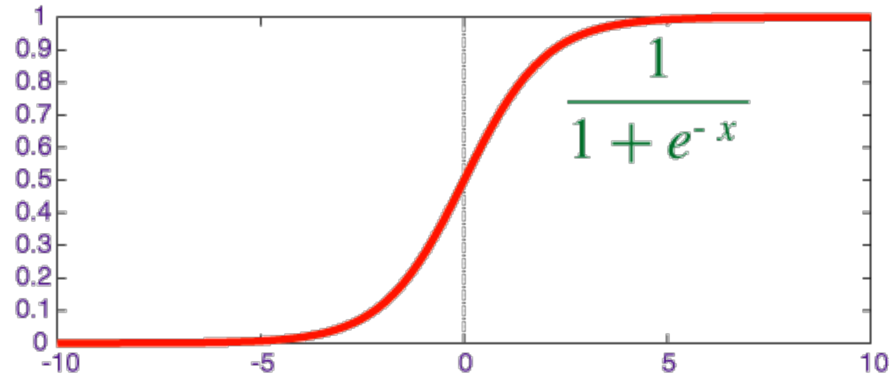
# Perceptron as a Neural Network



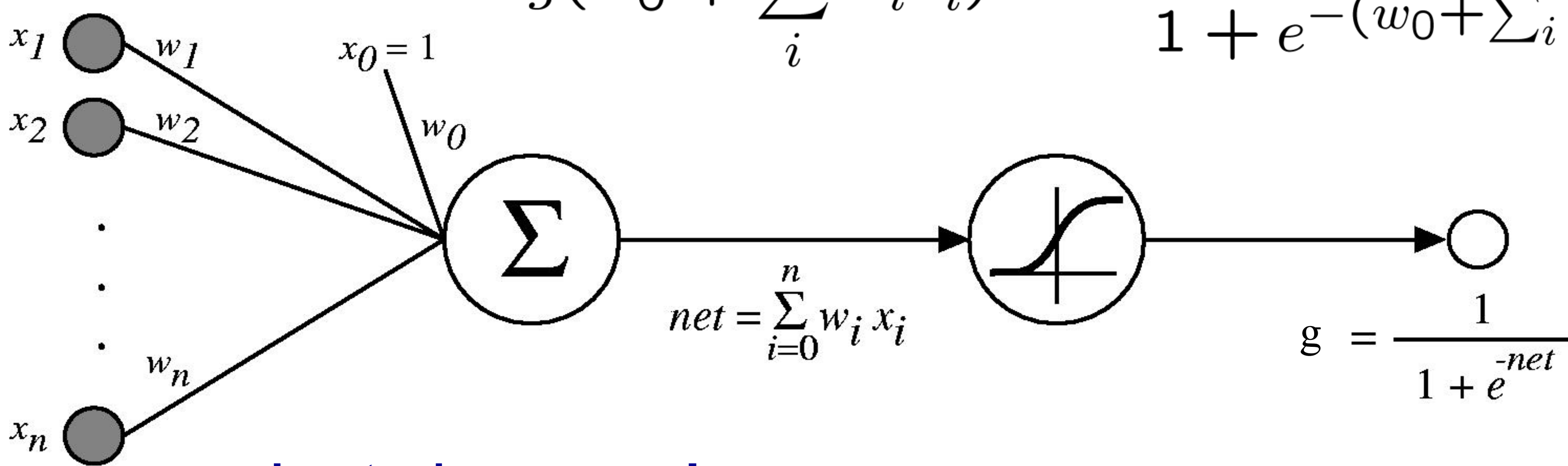

## This is one neuron:

- Input edges $x_1 \ldots x_n$, along with basis
- The sum is represented graphically
- Sum passed through an activation function g

# Sigmoid Neuron




$$g(w_0 + \sum_i w_i x_i) = \frac{1}{1+e^{-(w_0+\sum_i w_i x_i)}}$$




## Just change g!

- Why would we want to do this?
- Notice new output range [0,1]. What was it before?
- Look familiar?

# Optimizing a neuron

$$\boxed{\frac{\partial}{\partial x} f(g(x)) = f'(g(x))g'(x)}$$

We train to minimize sum-squared error

$$\ell(W) = \frac{1}{2}\sum_j [y^j - g(w_0 + \sum_i w_i x_i^j)]^2$$

$$\frac{\partial l}{\partial w_i} = -\sum_j [y_j - g(w_0 + \sum_i w_i x_i^j)] \frac{\partial}{\partial w_i} g(w_0 + \sum_i w_i x_i^j)$$

$$\frac{\partial}{\partial w_i} g(w_0 + \sum_i w_i x_i^j) = x_i^j g'(w_0 + \sum_i w_i x_i^j)$$

$$\boxed{\frac{\partial \ell(W)}{\partial w_i} = -\sum_j [y^j - g(w_0 + \sum_i w_i x_i^j)]\ x_i^j\ g'(w_0 + \sum_i w_i x_i^j)}$$

Solution just depends on g’: derivative of activation function!

# Sigmoid units: have to differentiate g

$$\frac{\partial \ell(W)}{\partial w_i} = -\sum_j [y^j - g(w_0 + \sum_i w_i x_i^j)]\ x_i^j\ g'(w_0 + \sum_i w_i x_i^j)$$

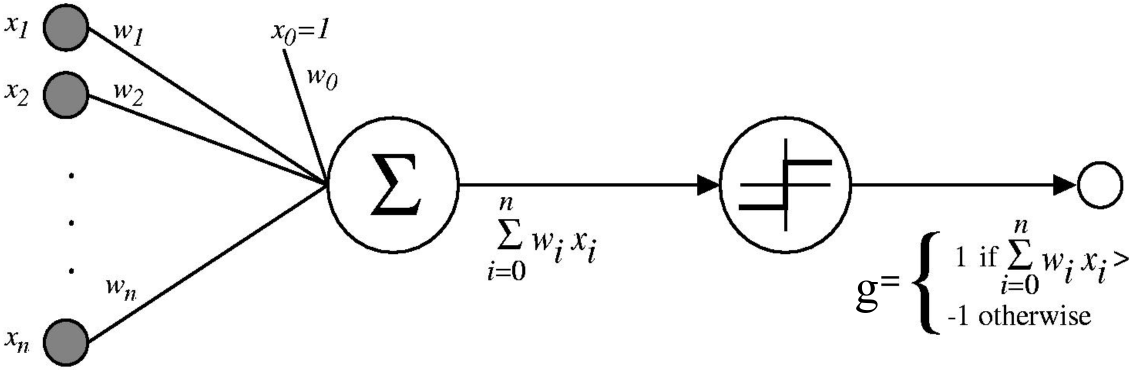
$$g(x) = \frac{1}{1+e^{-x}} \qquad g'(x) = g(x)(1-g(x))$$

$$\boxed{w_i \leftarrow w_i + \eta \sum_j x_i^j \delta^j}$$

$$\delta^j = [y^j - g(w_0 + \sum_i w_i x_i^j)] g^j (1 - g^j)$$

$$g^j = g(w_0 + \sum_i w_i x_i^j)$$

# Perceptron, linear classification, Boolean functions: $x_i \in \{0,1\}$




- Can learn $x_1 \vee x_2$?
  - $-0.5 + x_1 + x_2$
- Can learn $x_1 \wedge x_2$?
  - $-1.5 + x_1 + x_2$
- Can learn any conjunction or disjunction?
  - $0.5 + x_1 + \ldots + x_n$
  - $(-n+0.5) + x_1 + \ldots + x_n$
- Can learn majority?
  - $(-0.5^*n) + x_1 + \ldots + x_n$
- What are we missing? The dreaded XOR!, etc.

# Going beyond linear classification

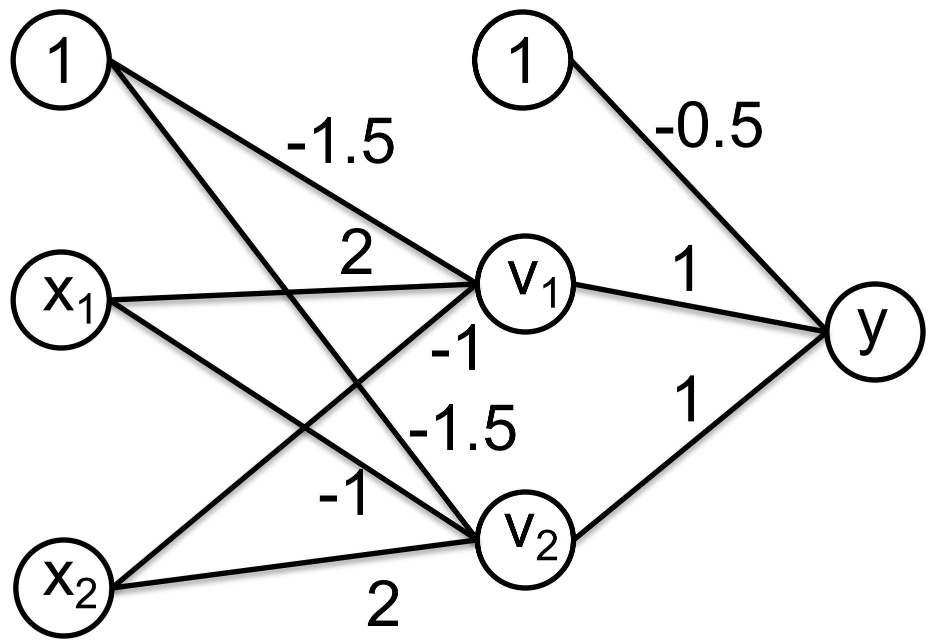
Solving the XOR problem

$$y = x_1 \text{ XOR } x_2 = (x_1 \wedge \neg x_2) \vee (x_2 \wedge \neg x_1)$$

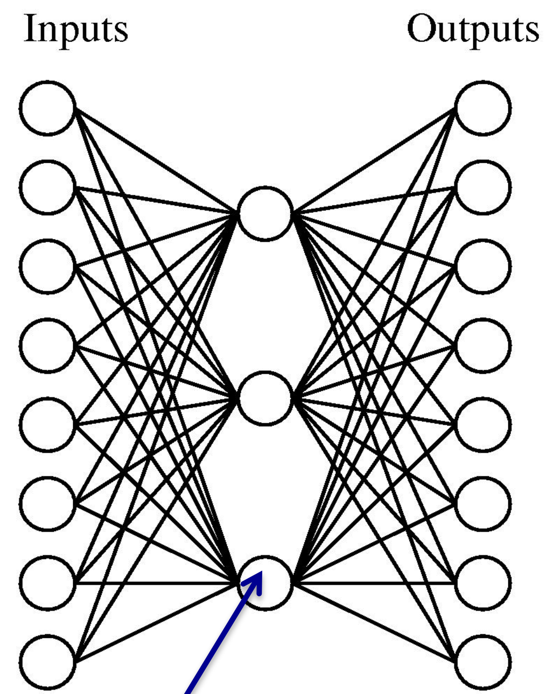
$$v_1 = (x_1 \wedge \neg x_2)$$
$$= -1.5 + 2x_1 - x_2$$

$$v_2 = (x_2 \wedge \neg x_1)$$
$$= -1.5 + 2x_2 - x_1$$

$$y = v_1 \vee v_2$$
$$= -0.5 + v_1 + v_2$$



# Hidden layer




- Single unit:

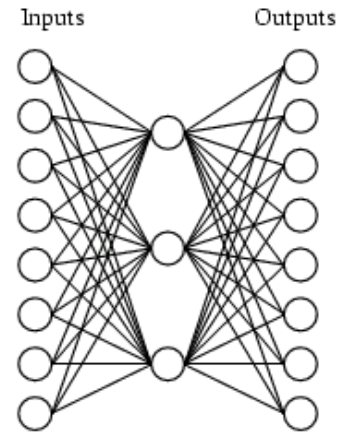
$$out(\mathbf{x}) = g(w_0 + \sum_i w_i x_i)$$

- 1-hidden layer:

$$out(\mathbf{x}) = g\left(w_0 + \sum_k w_k \overbrace{g(w_0^k + \sum_i w_i^k x_i)}\right)$$

- No longer convex function!

# Example data for NN with hidden layer



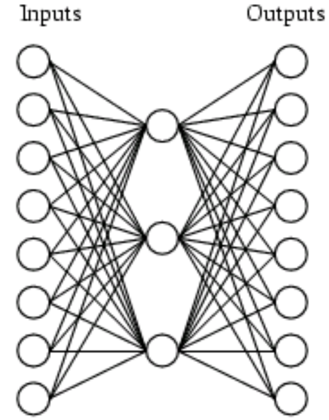

A target function:

| Input | | Output |
|---|---|---|
| 10000000 | $\rightarrow$ | 10000000 |
| 01000000 | $\rightarrow$ | 01000000 |
| 00100000 | $\rightarrow$ | 00100000 |
| 00010000 | $\rightarrow$ | 00010000 |
| 00001000 | $\rightarrow$ | 00001000 |
| 00000100 | $\rightarrow$ | 00000100 |
| 00000010 | $\rightarrow$ | 00000010 |
| 00000001 | $\rightarrow$ | 00000001 |

Can this be learned??

# Learned weights for hidden layer

A network:




Learned hidden layer representation:

| Input | | Hidden Values | | | | Output |
|---|---|---|---|---|---|---|
| 10000000 | → | .89 | .04 | .08 | → | 10000000 |
| 01000000 | → | .01 | .11 | .88 | → | 01000000 |
| 00100000 | → | .01 | .97 | .27 | → | 00100000 |
| 00010000 | → | .99 | .97 | .71 | → | 00010000 |
| 00001000 | → | .03 | .05 | .02 | → | 00001000 |
| 00000100 | → | .22 | .99 | .99 | → | 00000100 |
| 00000010 | → | .80 | .01 | .98 | → | 00000010 |
| 00000001 | → | .60 | .94 | .01 | → | 00000001 |

# Why “representation learning”?

- MaxEnt (multinomial logistic regression):

$$y = \mathrm{softmax}(w \cdot \underline{f(x, y)})$$

You design the feature vector

- NNs:

$$y = \mathrm{softmax}(w \cdot \underline{\sigma(Ux)})$$

$$y = \mathrm{softmax}(w \cdot \underline{\sigma(U^{(n)}(\ldots\sigma(U^{(2)}\sigma(U^{(1)}x))))})$$

Feature representations are “learned” through hidden layers

# Very deep models in computer vision

[1]Inception 5 (GoogLeNet)

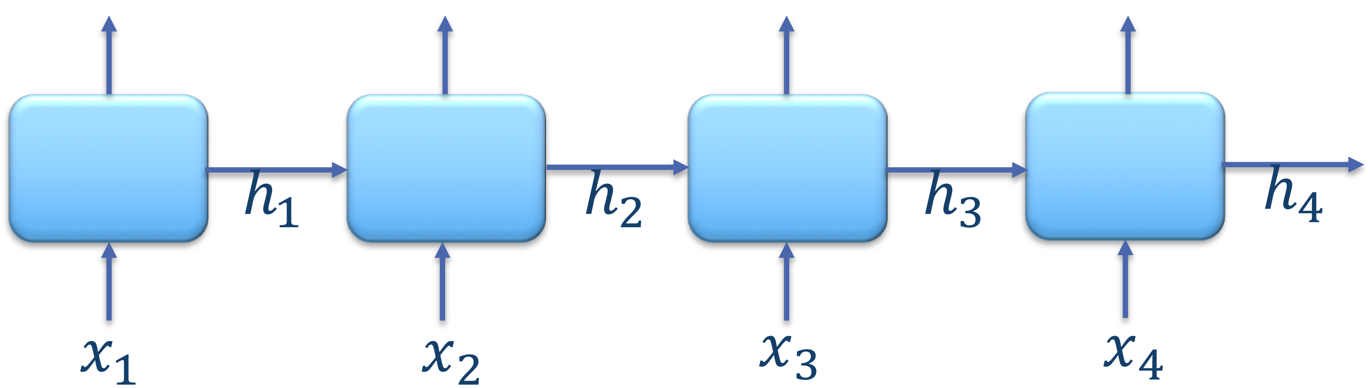
Inception 7a

[1]Going Deeper with Convolutions, [C. Szegedy et al, CVPR 2015]

# RECURRENT NEURAL NETWORKS

# Recurrent Neural Networks (RNNs)

- Each RNN unit computes a new hidden state using the previous state and a new input
$$h_t = f(x_t, h_{t-1})$$
- Each RNN unit (optionally) makes an output using the current hidden state
$$y_t = \mathrm{softmax}(V h_t)$$
- Hidden states $h_t \in R^D$ are continuous vectors
  - Can represent very rich information
  - Possibly the entire history from the beginning
- Parameters are shared (tied) across all RNN units (unlike feedforward NNs)



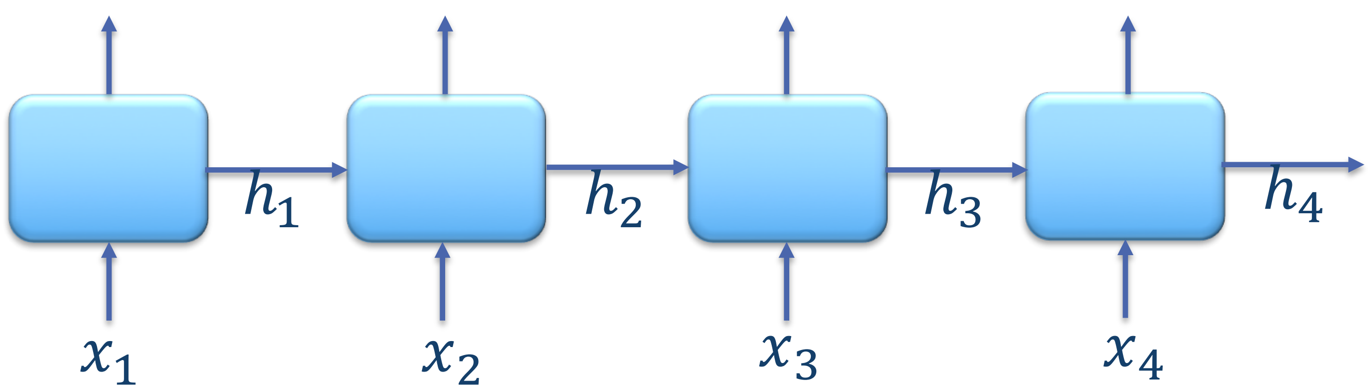
# Recurrent Neural Networks (RNNs)

- Generic RNNs:

$$h_t = f(x_t, h_{t-1})$$
$$y_t = \text{softmax}(V h_t)$$

- Vanilla RNN:

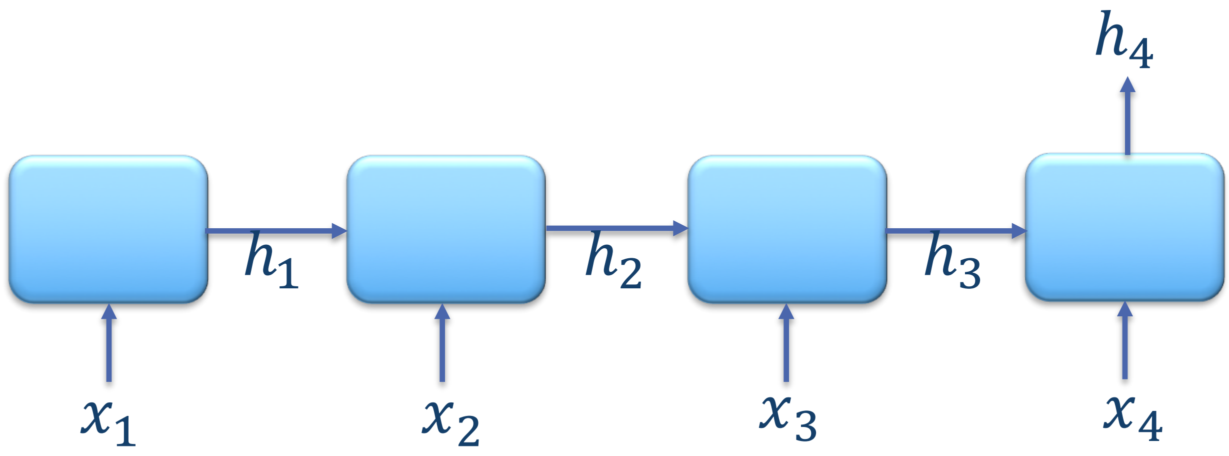
$$h_t = \tanh(U x_t + W h_{t-1} + b)$$
$$y_t = \text{softmax}(V h_t)$$



# Many uses of RNNs
# 1. Classification (seq to one)

- Input: a sequence
- Output: one label (classification)
- Example: sentiment classification

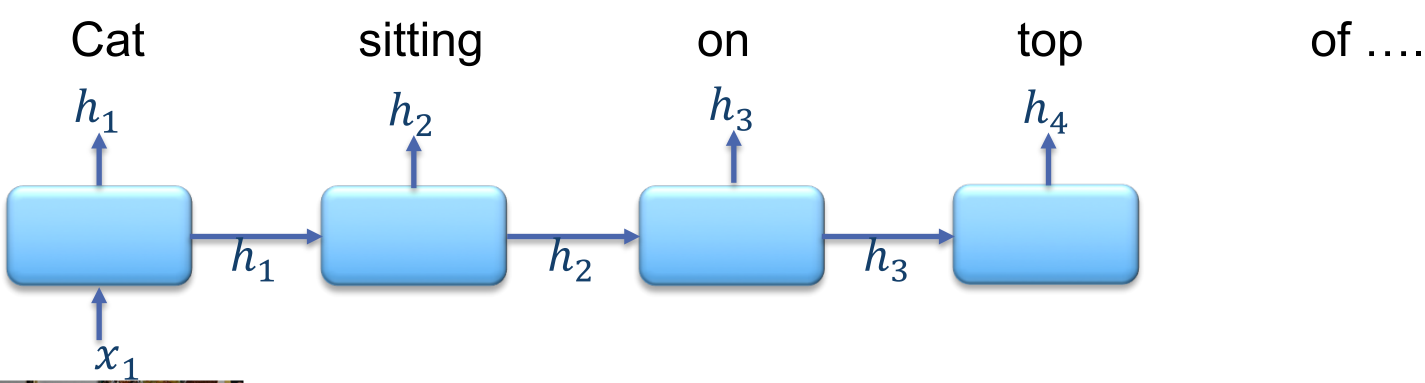
$$h_t = f(x_t, h_{t-1})$$
$$y = \mathrm{softmax}(V h_n)$$

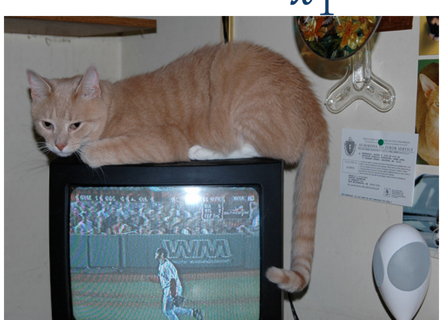


# Many uses of RNNs
# 2. one to seq

- Input: one item
- Output: a sequence
- Example: Image captioning

$$h_t = f(x_t, h_{t-1})$$
$$y_t = \mathrm{softmax}(V h_t)$$

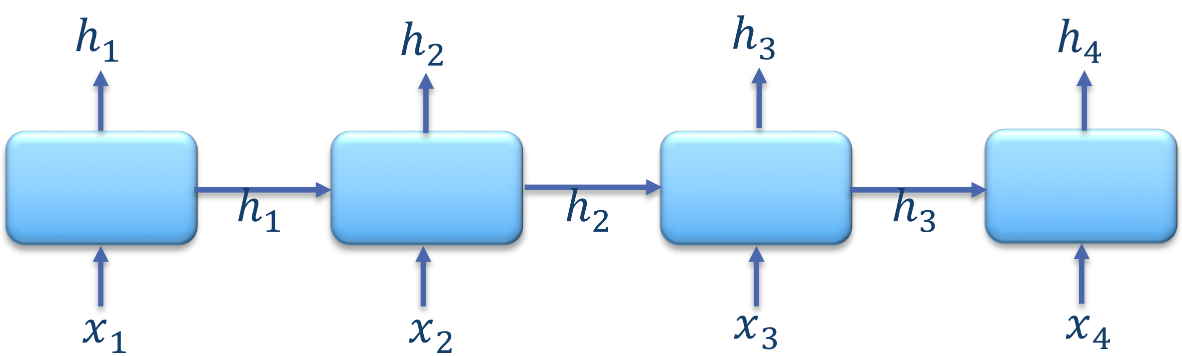


# Many uses of RNNs
# 3. sequence tagging

- Input: a sequence
- Output: a sequence (of the same length)
- Example: POS tagging, Named Entity Recognition
- How about Language Models?
  - Yes! RNNs can be used as LMs!
  - RNNs make markov assumption: T/F?

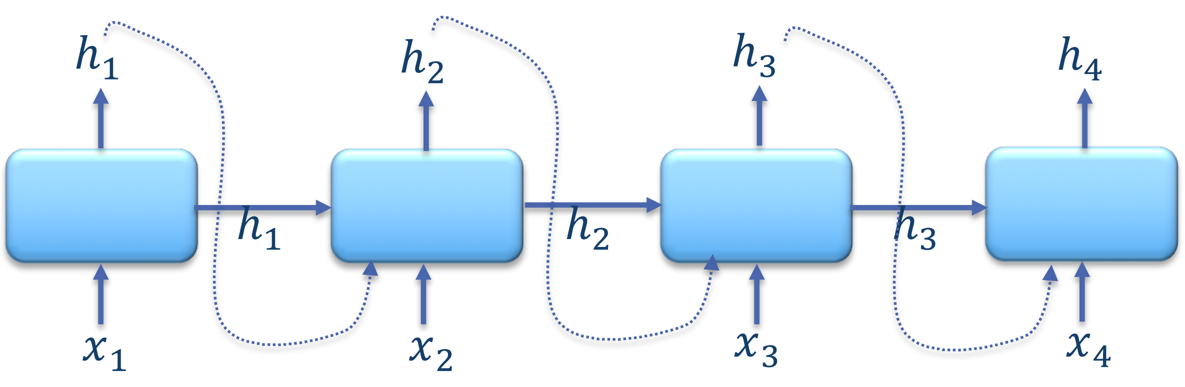
$$h_t = f(x_t, h_{t-1})$$
$$y_t = \text{softmax}(V h_t)$$



# Many uses of RNNs
# 4. Language models

- Input: a sequence of words
- Output: one next word
- Output: or a sequence of next words
- During training, x_t is the actual word in the training sentence.
- During testing, x_t is the word predicted from the previous time step.
- Does RNN LMs make Markov assumption?
  - i.e., the next word depends only on the previous N words

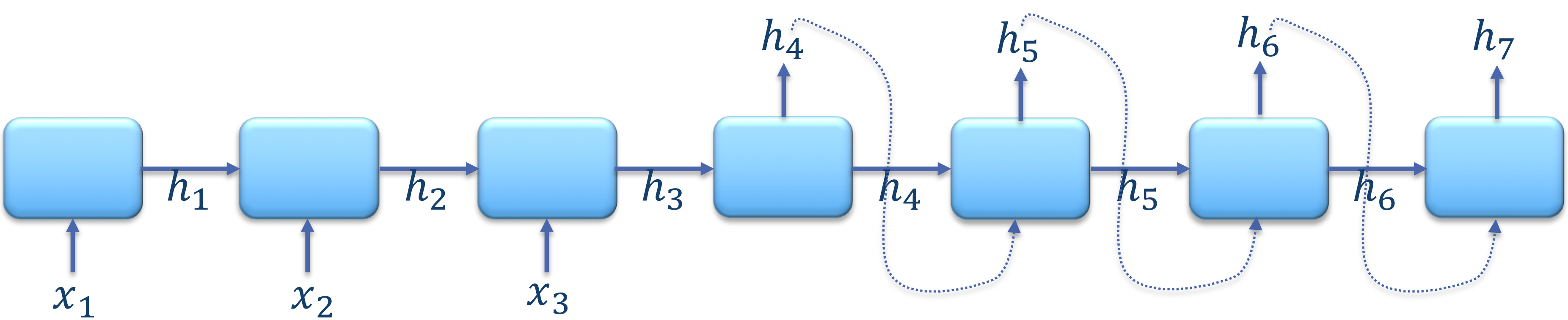
$$h_t = f(x_t, h_{t-1})$$
$$y_t = \mathrm{softmax}(V h_t)$$



# Many uses of RNNs
# 5. seq2seq (aka "encoder-decoder")

- Input: a sequence
- Output: a sequence (of *different* length)
- Examples?

$$h_t = f(x_t, h_{t-1})$$
$$y_t = \mathrm{softmax}(V h_t)$$



# Many uses of RNNs

# 4. seq2seq (aka “encoder-decoder”)

- Conversation and Dialogue
- Machine Translation

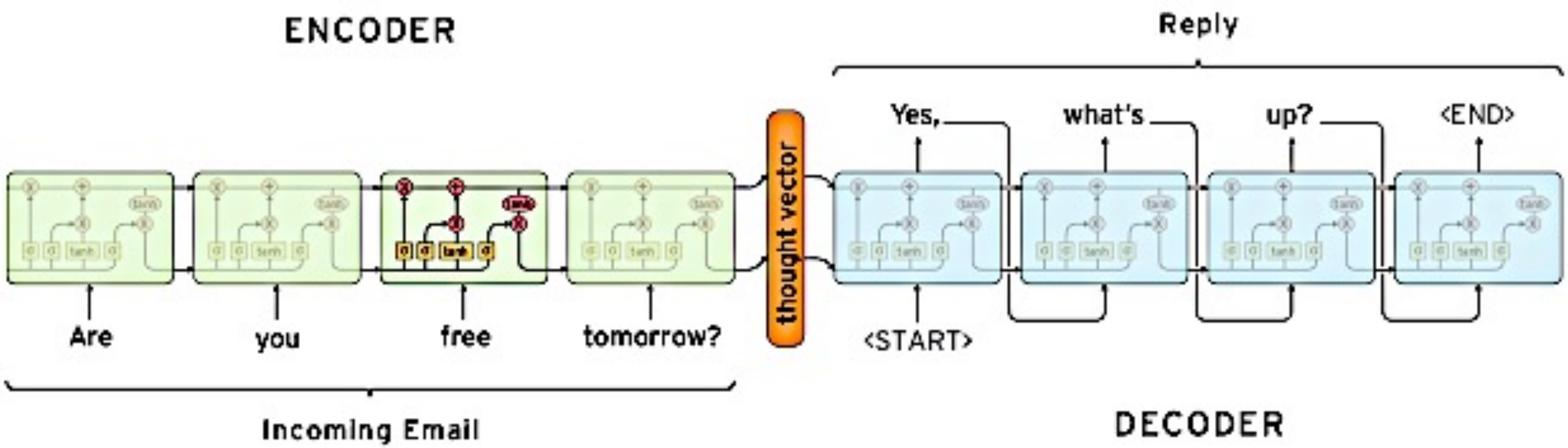
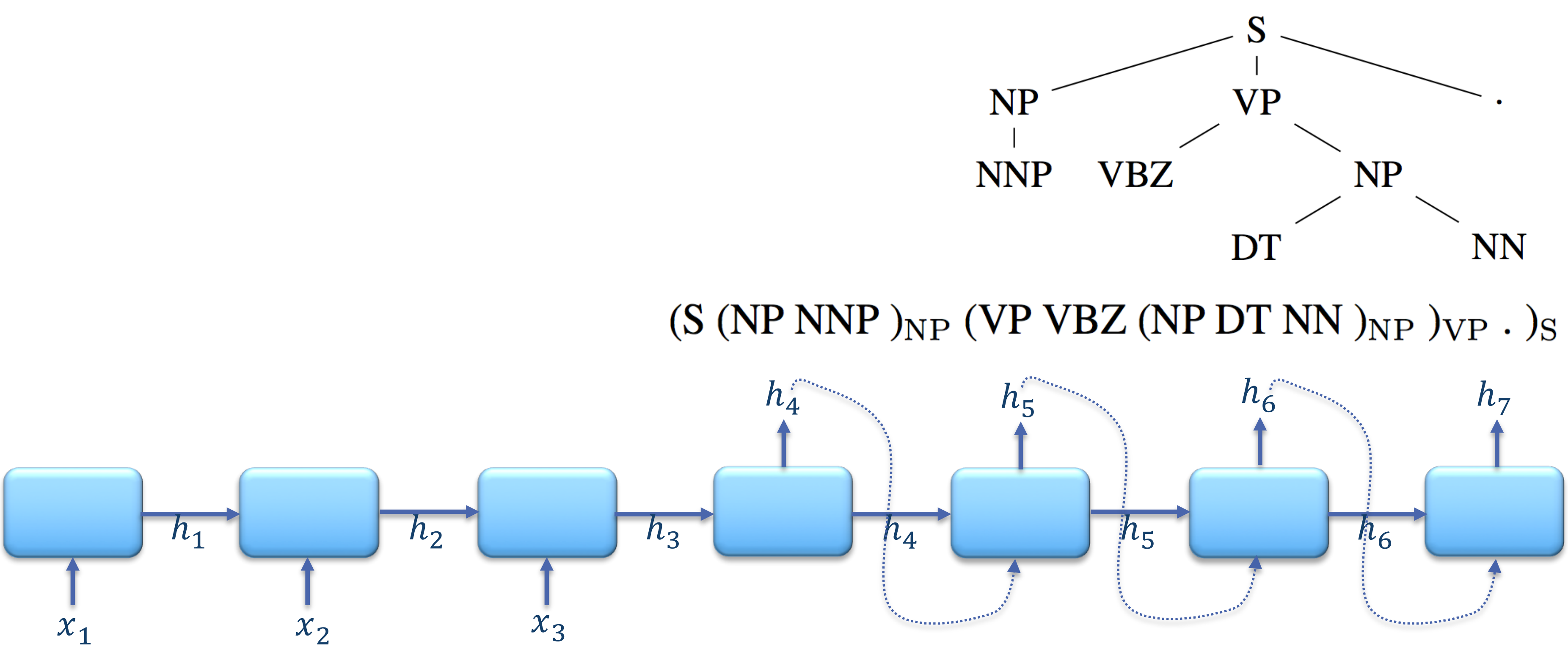



Figure from http://www.wildml.com/category/conversational-agents/

# Many uses of RNNs
# 4. seq2seq (aka "encoder-decoder")

Parsing!

- *"Grammar as Foreign Language"* (Vinyals et al., 2015)




John has a dog

# Recurrent Neural Networks (RNNs)

- Generic RNNs:

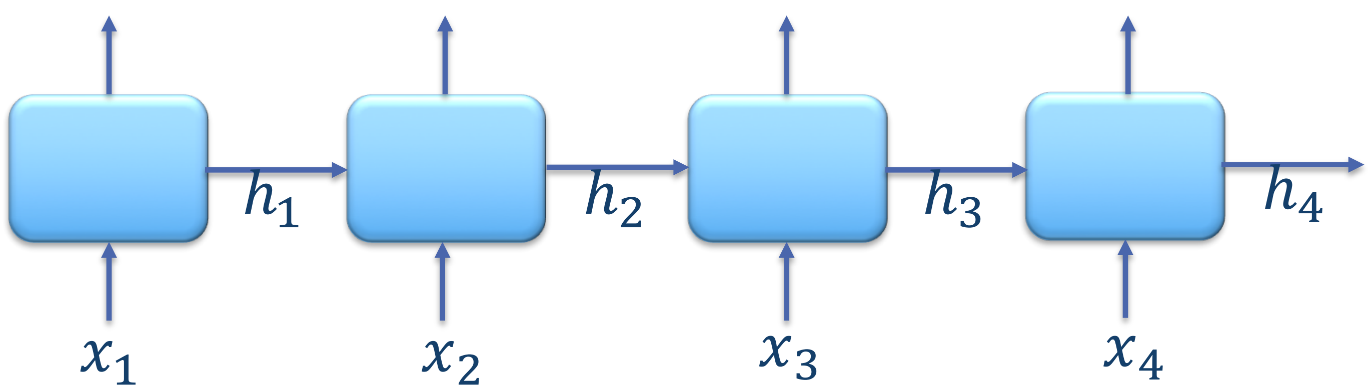
$$h_t = f(x_t, h_{t-1})$$

$$y_t = \text{softmax}(V h_t)$$

- Vanilla RNN:

$$h_t = \tanh(U x_t + W h_{t-1} + b)$$

$$y_t = \text{softmax}(V h_t)$$

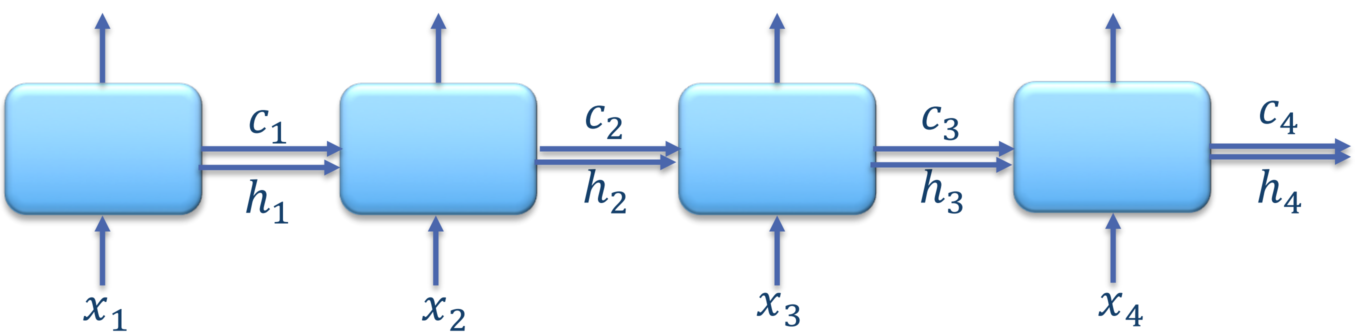


# Recurrent Neural Networks (RNNs)

- Generic RNNs: $h_t = f(x_t, h_{t-1})$
- Vanilla RNNs: $h_t = \tanh(Ux_t + Wh_{t-1} + b)$
- LSTMs (**L**ong **S**hort-**t**erm **M**emory Networks):

$$
\begin{aligned}
i_t &= \sigma(U^{(i)}x_t + W^{(i)}h_{t-1} + b^{(i)}) \\
f_t &= \sigma(U^{(f)}x_t + W^{(f)}h_{t-1} + b^{(f)}) \\
o_t &= \sigma(U^{(o)}x_t + W^{(o)}h_{t-1} + b^{(o)}) \\
\tilde{c}_t &= \tanh(U^{(c)}x_t + W^{(c)}h_{t-1} + b^{(c)}) \\
c_t &= f_t \circ c_{t-1} + i_t \circ \tilde{c}_t \\
h_t &= o_t \circ \tanh(c_t)
\end{aligned}
$$

There are many known variations to this set of equations!




$c_t$ : cell state

$h_t$ : hidden state

# LSTMS (LONG SHORT-TERM MEMORY NETWORKS

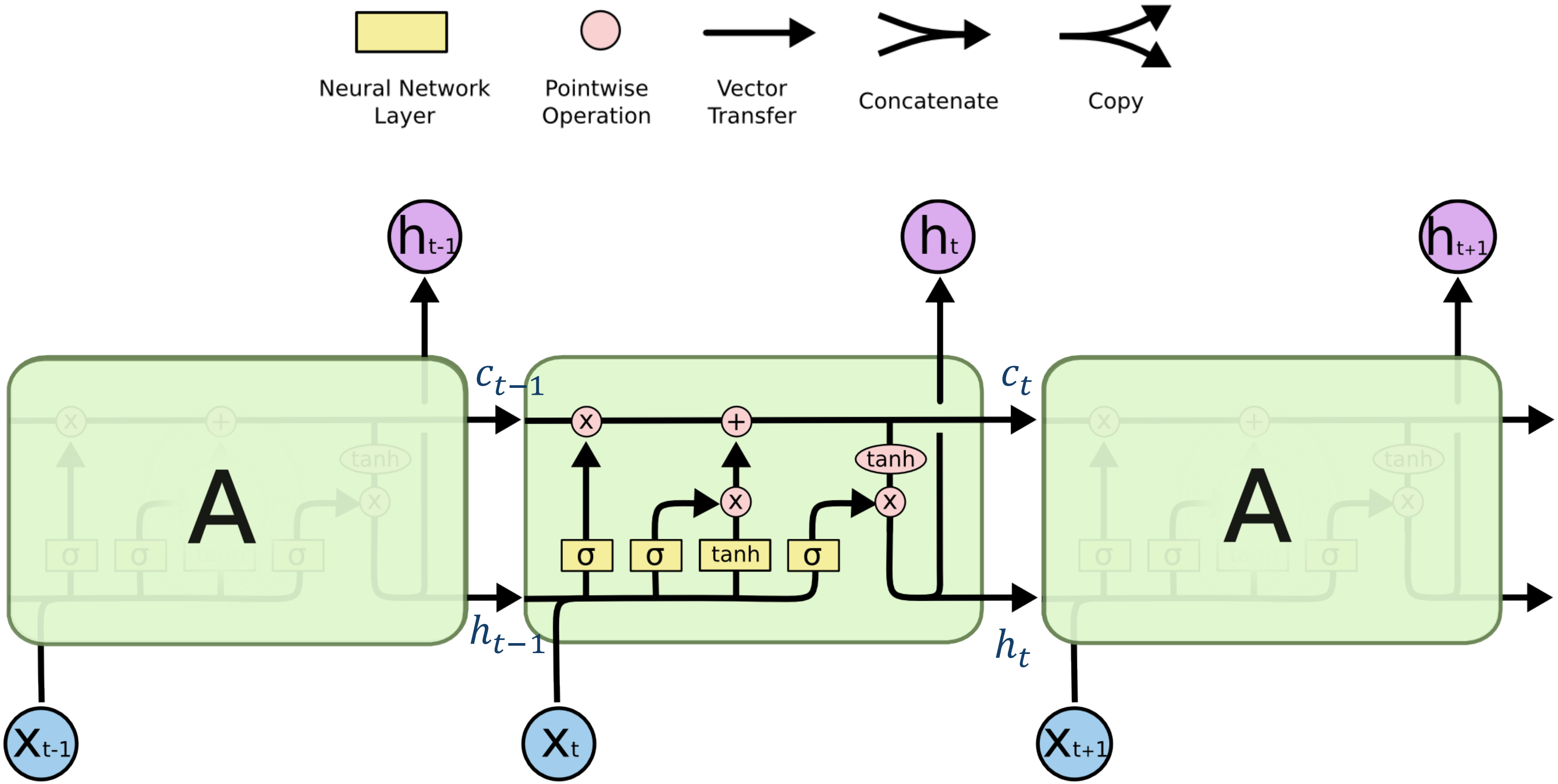
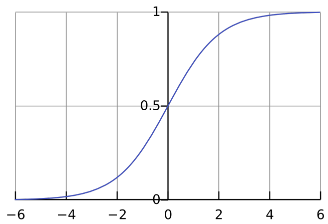



Figure by Christopher Olah (colah.github.io)

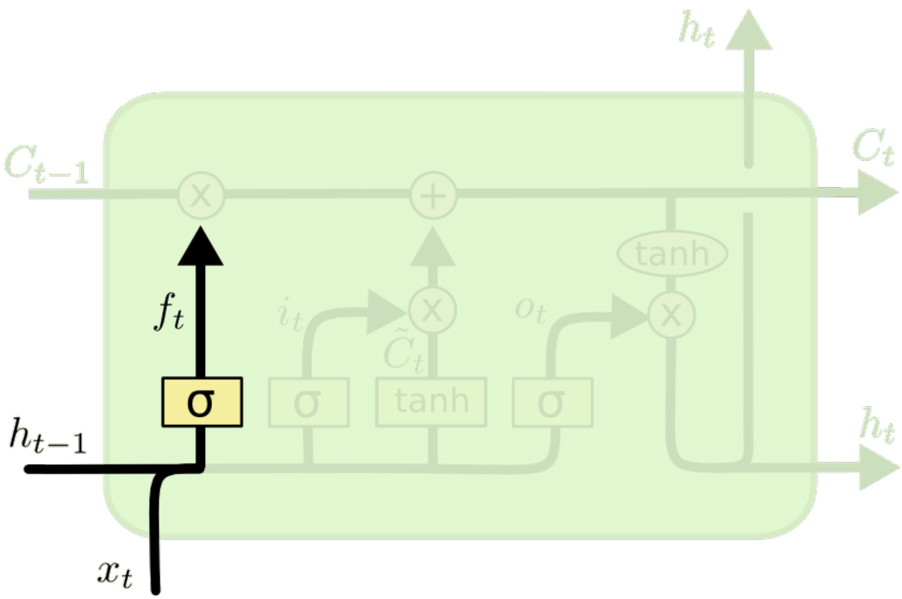
# LSTMS (LONG SHORT-TERM MEMORY NETWORKS

sigmoid:
[0,1]




Forget gate: forget the past or not

$$f_t = \sigma(U^{(f)}x_t + W^{(f)}h_{t-1} + b^{(f)})$$

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# LSTMS (LONG SHORT-TERM MEMORY NETWORKS

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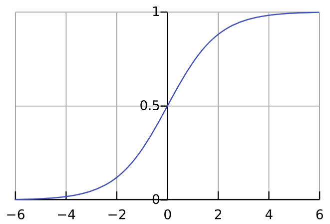
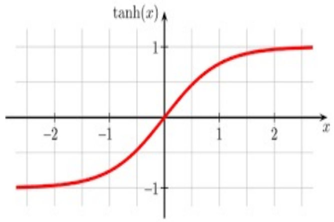


tanh:
[-1,1]



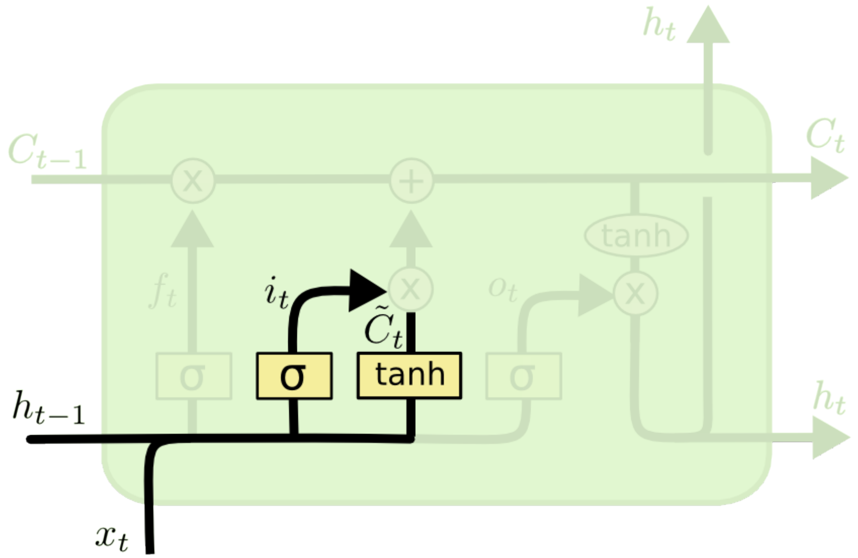



Figure by Christopher Olah (colah.github.io)

Forget gate: forget the past or not

$$f_t = \sigma(U^{(f)}x_t + W^{(f)}h_{t-1} + b^{(f)})$$

Input gate: use the input or not

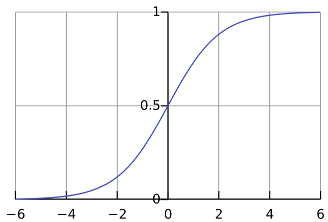
$$i_t = \sigma(U^{(i)}x_t + W^{(i)}h_{t-1} + b^{(i)})$$

New cell content (temp):

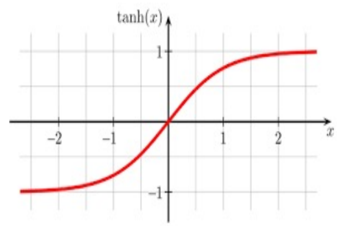
$$\tilde{c}_t = \tanh(U^{(c)}x_t + W^{(c)}h_{t-1} + b^{(c)})$$

# LSTMS (LONG SHORT-TERM MEMORY NETWORKS

sigmoid:
[0,1]




tanh:
[-1,1]



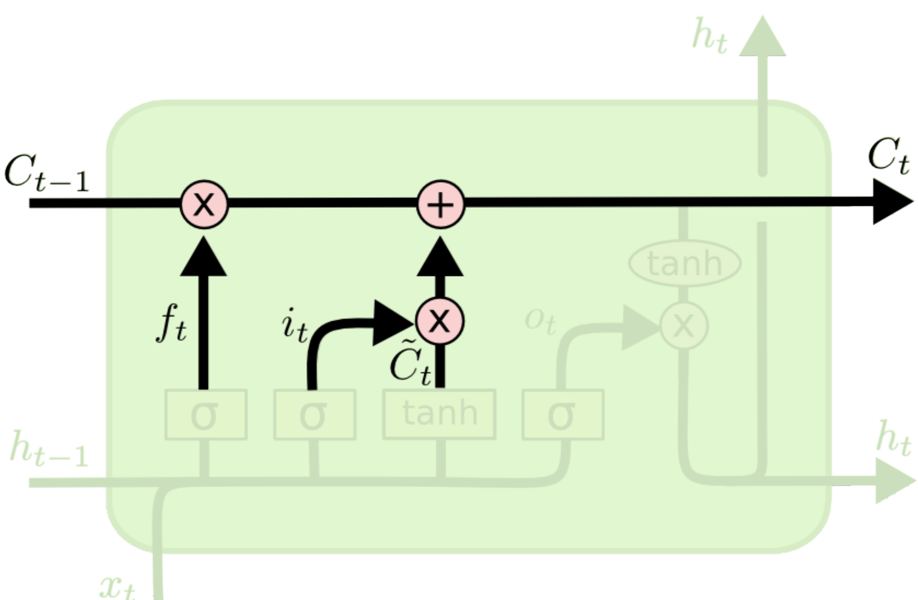




Figure by Christopher Olah (colah.github.io)

Forget gate: forget the past or not

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Input gate: use the input or not

$$i_t = \sigma(U^{(i)}x_t + W^{(i)}h_{t-1} + b^{(i)})$$

New cell content (temp):

$$\tilde{c}_t = \tanh(U^{(c)}x_t + W^{(c)}h_{t-1} + b^{(c)})$$

New cell content:

- mix old cell with the new temp cell

$$c_t = f_t \circ c_{t-1} + i_t \circ \tilde{c}_t$$

# LSTMS (LONG SHORT-TERM MEMORY NETWORKS

Output gate: output from the new cell or not

$$o_t = \sigma(U^{(o)}x_t + W^{(o)}h_{t-1} + b^{(o)})$$

Hidden state:

$$h_t = o_t \circ \tanh(c_t)$$

Forget gate: forget the past or not

$$f_t = \sigma(U^{(f)}x_t + W^{(f)}h_{t-1} + b^{(f)})$$

Input gate: use the input or not

$$i_t = \sigma(U^{(i)}x_t + W^{(i)}h_{t-1} + b^{(i)})$$

New cell content (temp):

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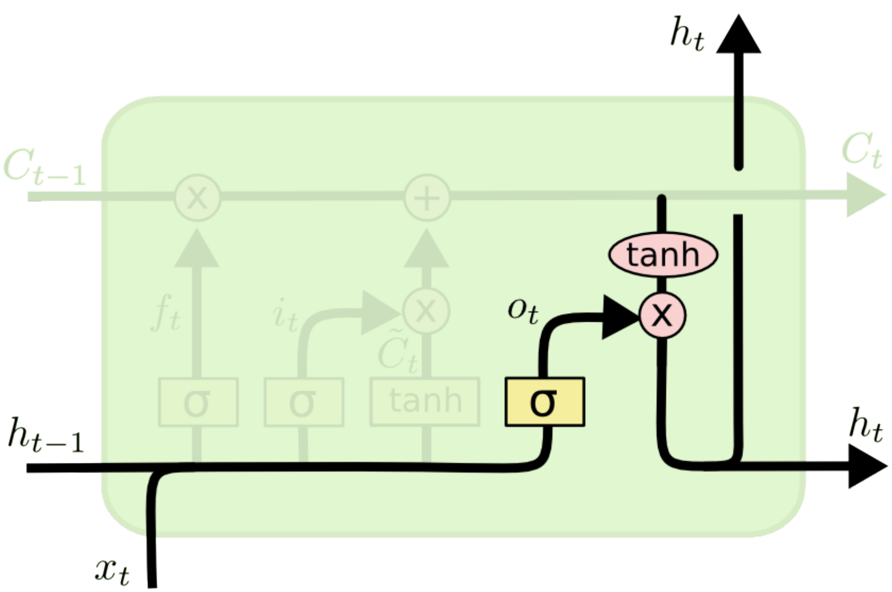



Figure by Christopher Olah (colah.github.io)

# LSTMS (LONG SHORT-TERM MEMORY NETWORKS

Forget gate: forget the past or not

Input gate: use the input or not

Output gate: output from the new cell or not

$$f_t = \sigma(U^{(f)}x_t + W^{(f)}h_{t-1} + b^{(f)})$$
$$i_t = \sigma(U^{(i)}x_t + W^{(i)}h_{t-1} + b^{(i)})$$
$$o_t = \sigma(U^{(o)}x_t + W^{(o)}h_{t-1} + b^{(o)})$$

---

New cell content (temp):

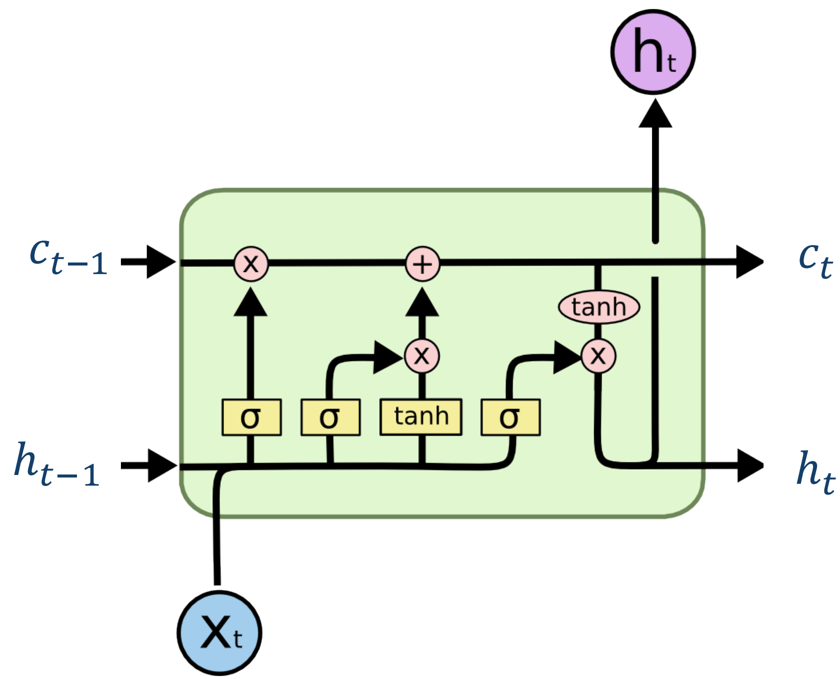
New cell content:

- mix old cell with the new temp cell

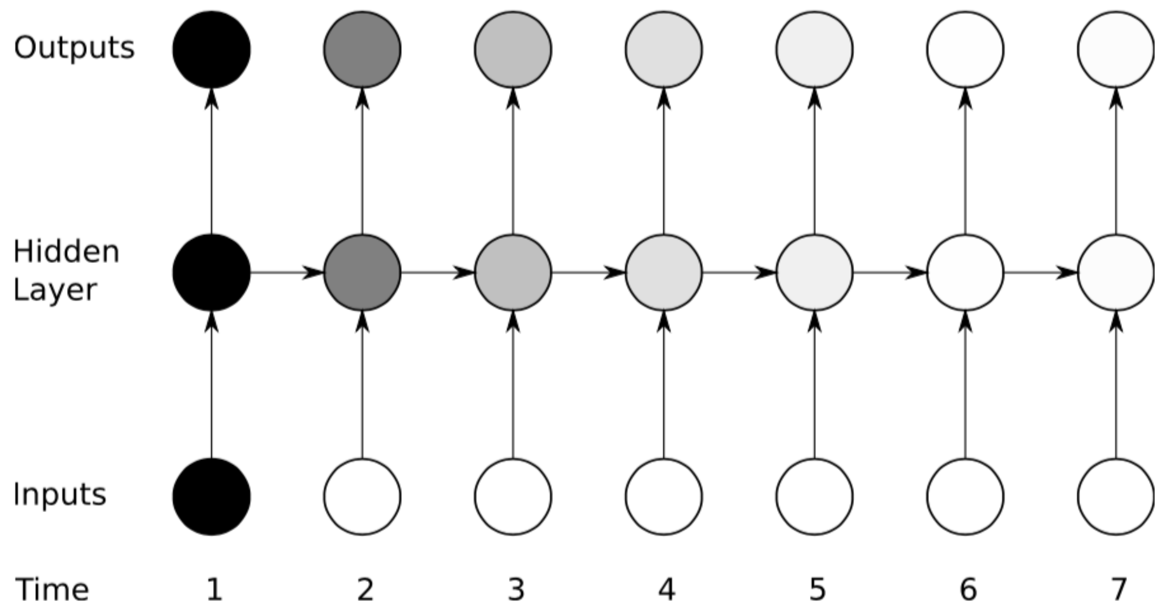
$$\tilde{c}_t = \tanh(U^{(c)}x_t + W^{(c)}h_{t-1} + b^{(c)})$$
$$c_t = f_t \circ c_{t-1} + i_t \circ \tilde{c}_t$$

Hidden state:

$$h_t = o_t \circ \tanh(c_t)$$



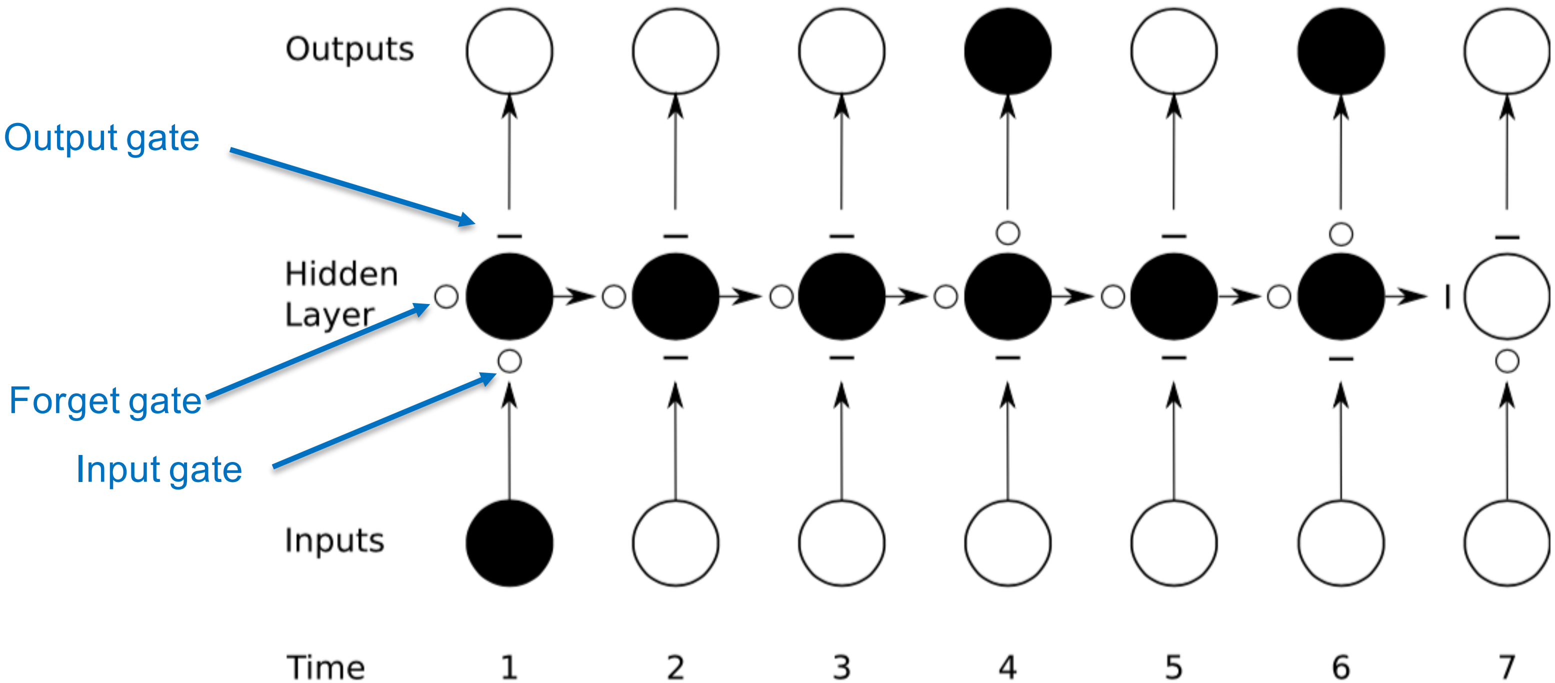
# vanishing gradient problem for RNNs.




- The shading of the nodes in the unfolded network indicates their sensitivity to the inputs at time one (the darker the shade, the greater the sensitivity).
- The sensitivity decays over time as new inputs overwrite the activations of the hidden layer, and the network 'forgets' the first inputs.

Example from Graves 2012

# Preservation of gradient information by LSTM




- For simplicity, all gates are either entirely open ('O') or closed ('—').
- The memory cell 'remembers' the first input as long as the forget gate is open and the input gate is closed.
- The sensitivity of the output layer can be switched on and off by the output gate without affecting the cell.

Example from Graves 2012

# Recurrent Neural Networks (RNNs)

- Generic RNNs: $h_t = f(x_t, h_{t-1})$
- Vanilla RNNs: $h_t = \tanh(Ux_t + Wh_{t-1} + b)$
- GRUs (**G**ated **R**ecurrent **U**nits):

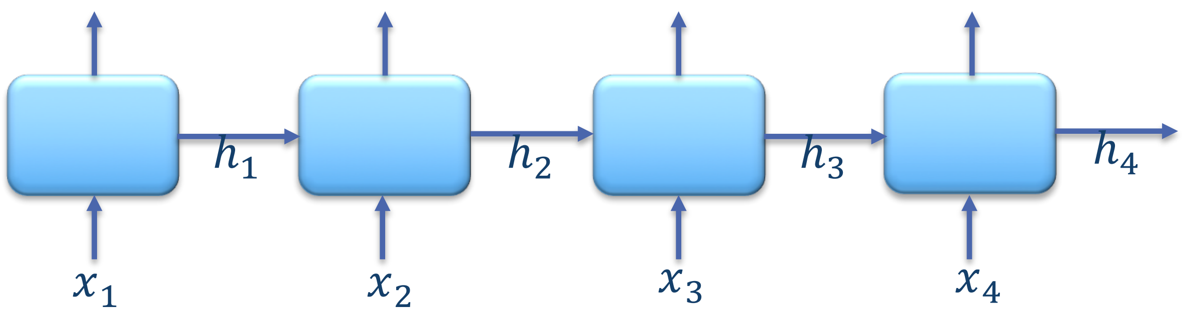
$$z_t = \sigma(U^{(z)}x_t + W^{(z)}h_{t-1} + b^{(z)})$$

$$r_t = \sigma(U^{(r)}x_t + W^{(r)}h_{t-1} + b^{(r)})$$

$$\tilde{h_t} = \tanh(U^{(h)}x_t + W^{(h)}(r_t \circ h_{t-1}) + b^{(h)})$$

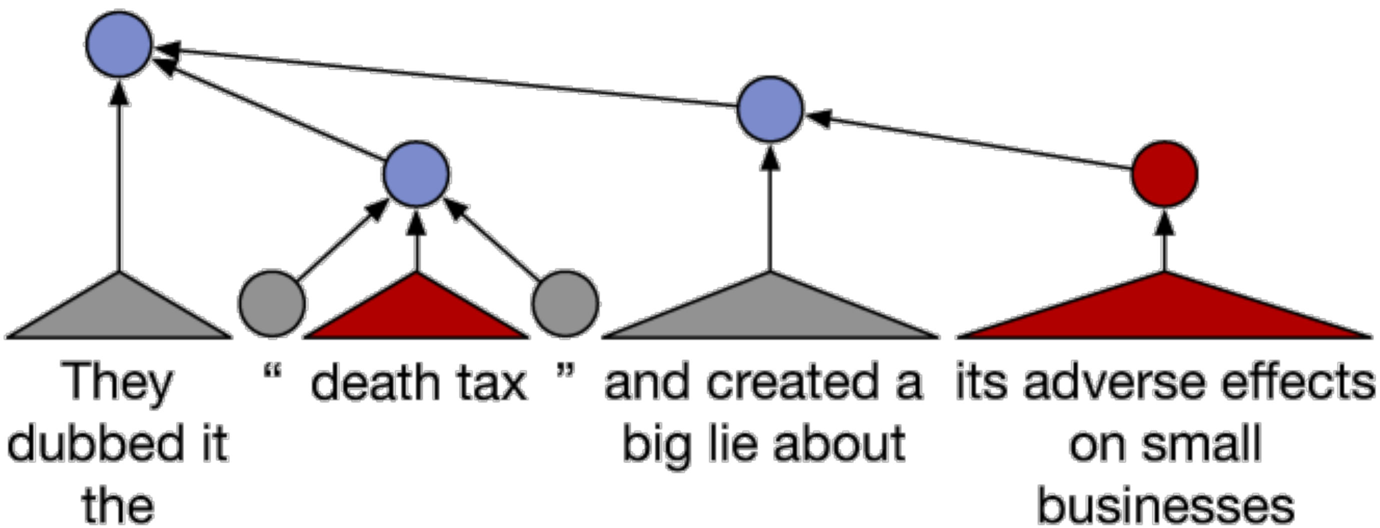
$$h_t = (1 - z_t) \circ h_{t-1} + z_t \circ \tilde{h_t}$$

Less parameters than LSTMs. Easier to train for comparable performance!



# Recursive Neural Networks

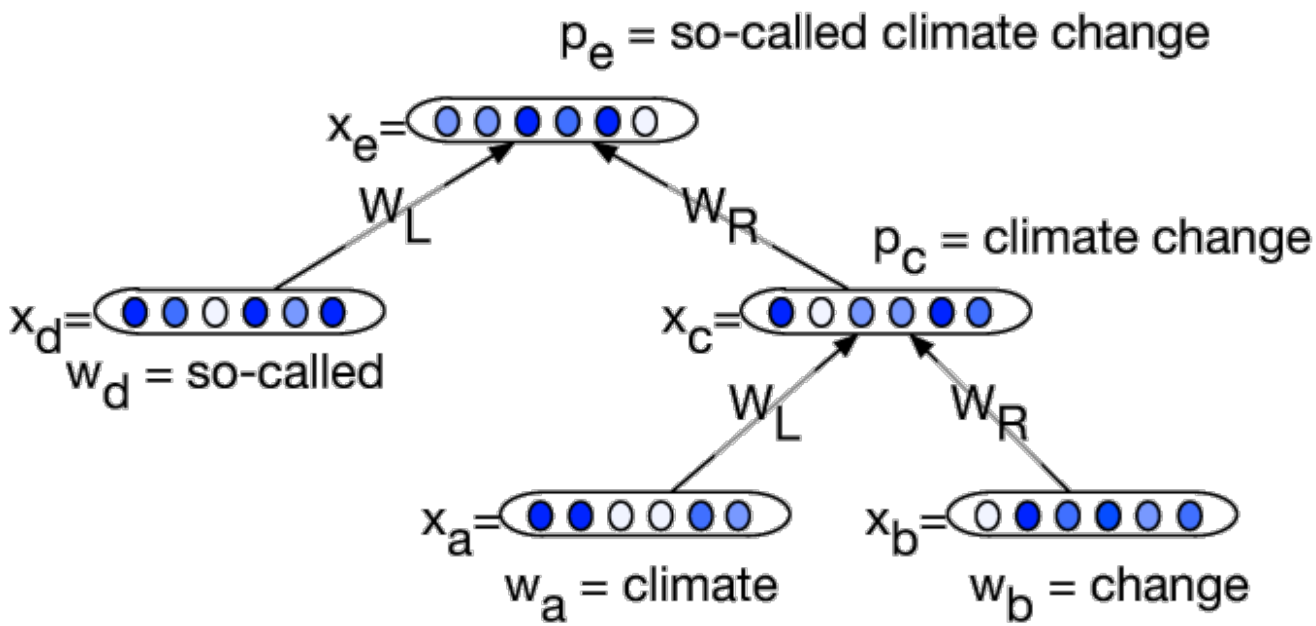
- Sometimes, inference over a tree structure makes more sense than sequential structure
- An example of compositionality in ideological bias detection (red → conservative, blue → liberal, gray → neutral) in which modifier phrases and punctuation cause polarity switches at higher levels of the parse tree




Example from Iyyer et al., 2014

# Recursive Neural Networks

- NNs connected as a tree
- Tree structure is fixed a priori
- Parameters are shared, similarly as RNNs




Example from Iyyer et al., 2014

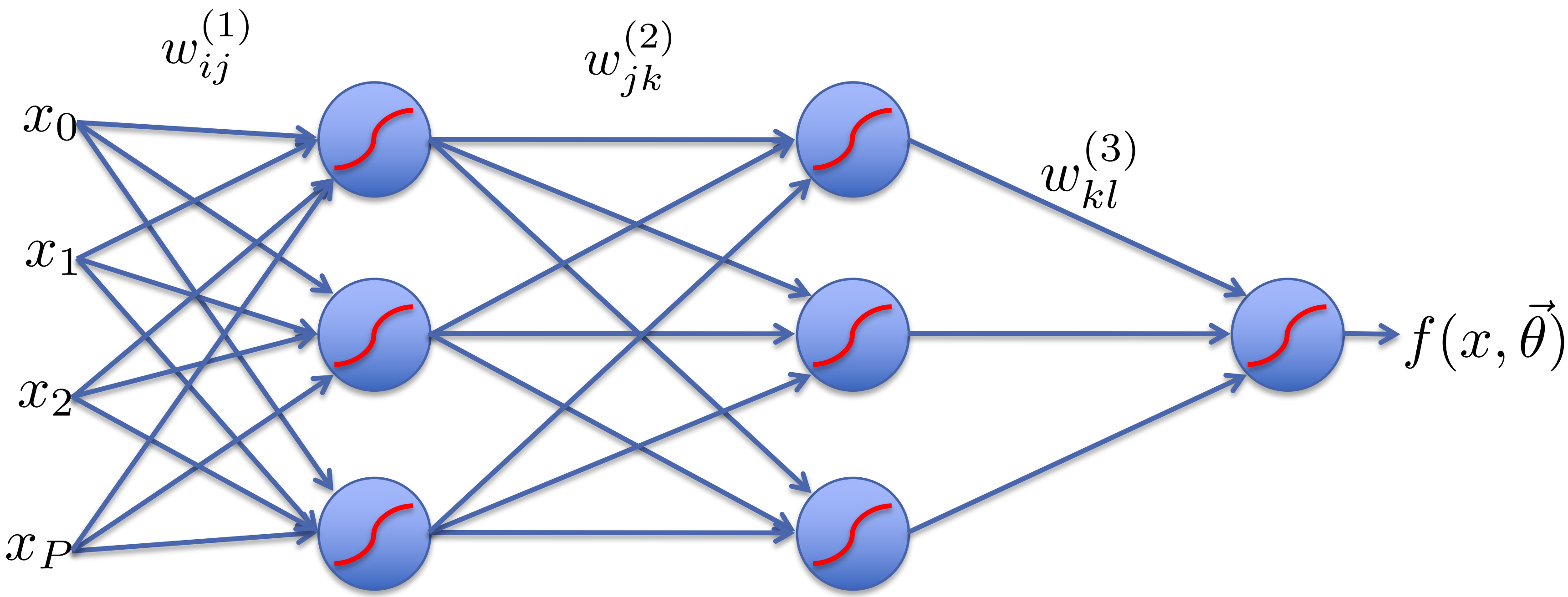
# LEARNING: BACKPROPAGATION

Next 10 slides on back propagation are adapted from Andrew Rosenberg

# Error Backpropagation

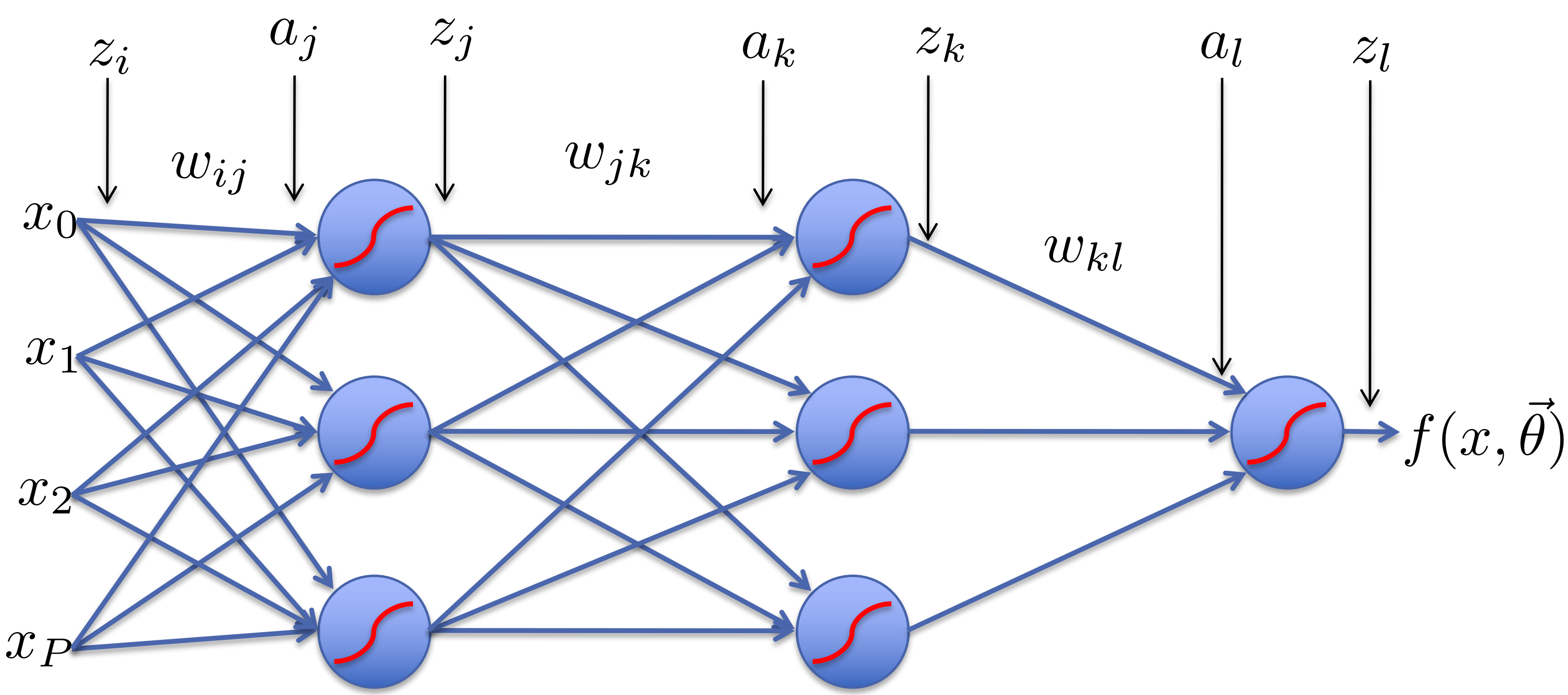
- Model parameters: $\vec{\theta} = \{w_{ij}^{(1)}, w_{jk}^{(2)}, w_{kl}^{(3)}\}$

  for brevity: $\vec{\theta} = \{w_{ij}, w_{jk}, w_{kl}\}$



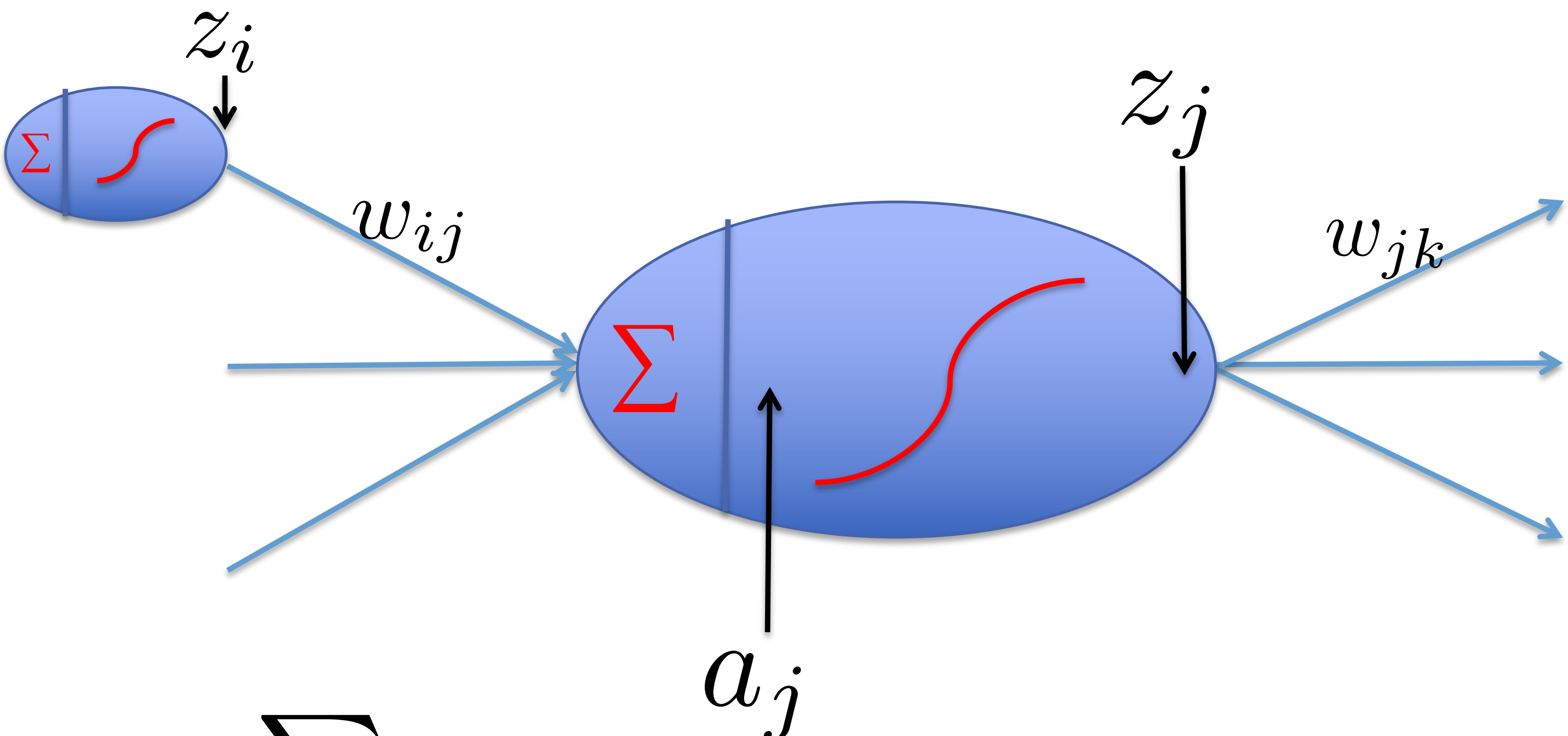
# Error Backpropagation

- Model parameters: $\vec{\theta} = \{w_{ij}, w_{jk}, w_{kl}\}$
- *Let* $a$ and $z$ *be the input and output of each node*



# Error Backpropagation

$$z_j = g(a_j)$$

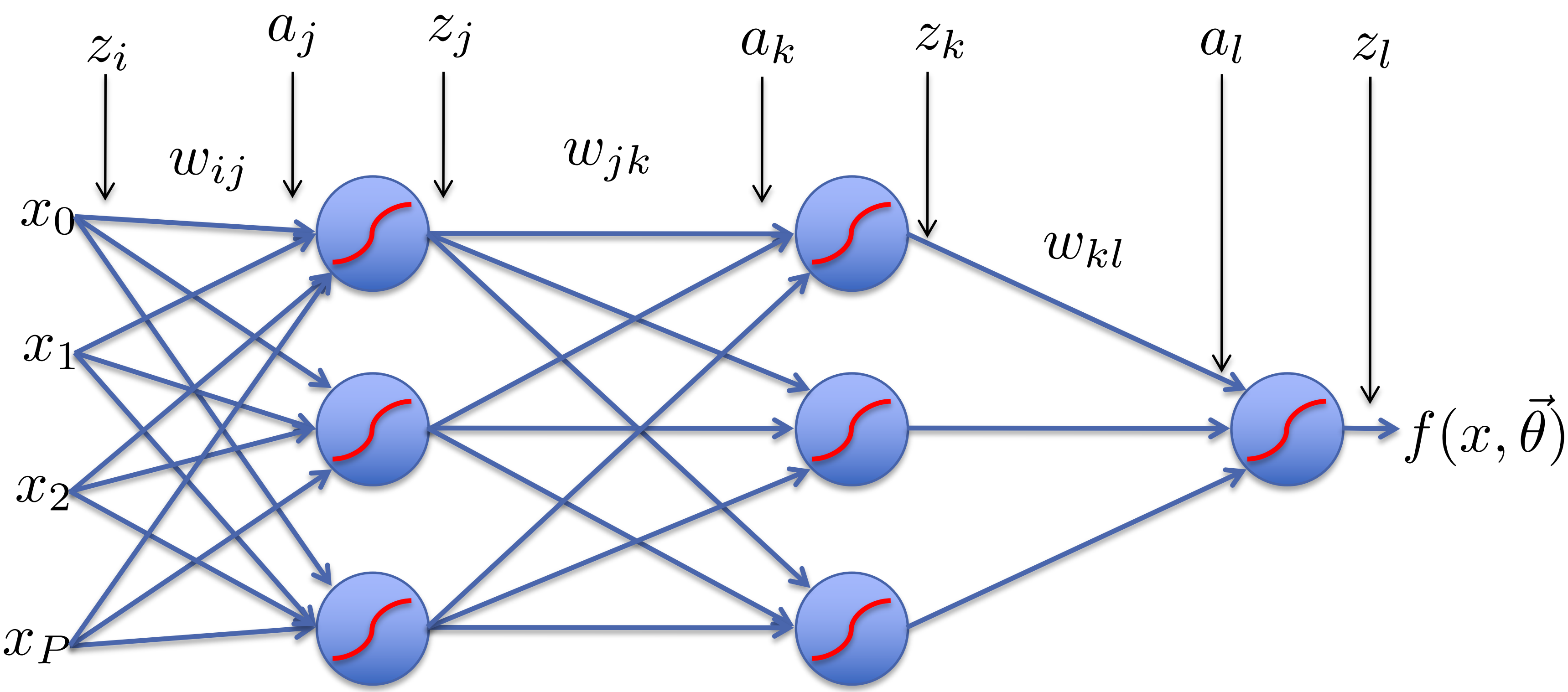



$$a_j = \sum_i w_{ij} z_i$$

- *Let* $a$ and $z$ *be the input and output of each node*

$$a_j = \sum_i w_{ij} z_i \qquad a_k = \boxed{\qquad\qquad} \qquad a_l = \boxed{\qquad\qquad}$$

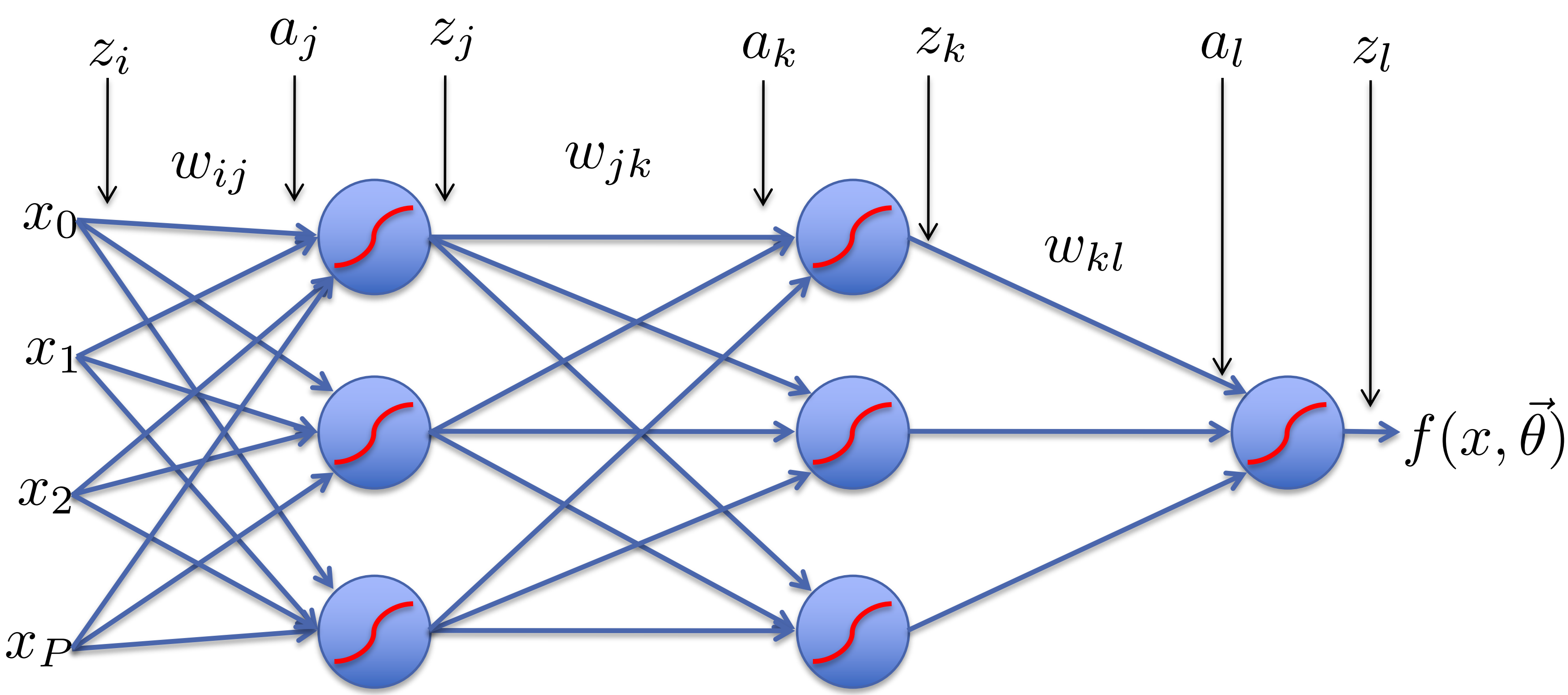
$$z_j = g(a_j) \qquad z_k = \boxed{\qquad} \qquad z_l = \boxed{\qquad}$$



- *Let* $a$ and $z$ *be the input and output of each node*

$$a_j = \sum_i w_{ij} z_i \qquad a_k = \sum_j w_{jk} z_j \qquad a_l = \sum_k w_{kl} z_k$$

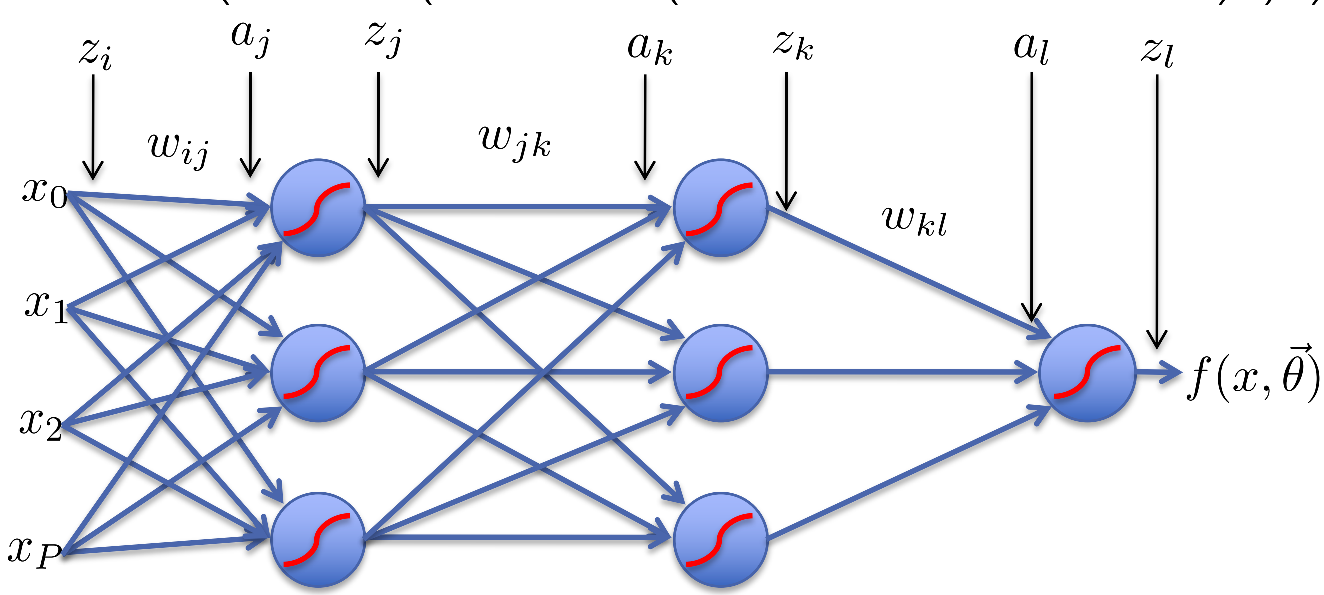
$$z_j = g(a_j) \qquad z_k = g(a_k) \qquad z_l = g(a_l)$$



# Training: minimize loss

$$
\begin{aligned}
R(\theta) &= \frac{1}{N}\sum_{n=0}^{N} L(y_n - f(x_n)) \\
&= \frac{1}{N}\sum_{n=0}^{N} \frac{1}{2}\left(y_n - f(x_n)\right)^2 \\
&= \frac{1}{N}\sum_{n=0}^{N} \frac{1}{2}\left(y_n - g\left( \qquad g\left( \qquad g\left( \qquad x_{n,i}\right)\right)\right)\right)^2
\end{aligned}
$$

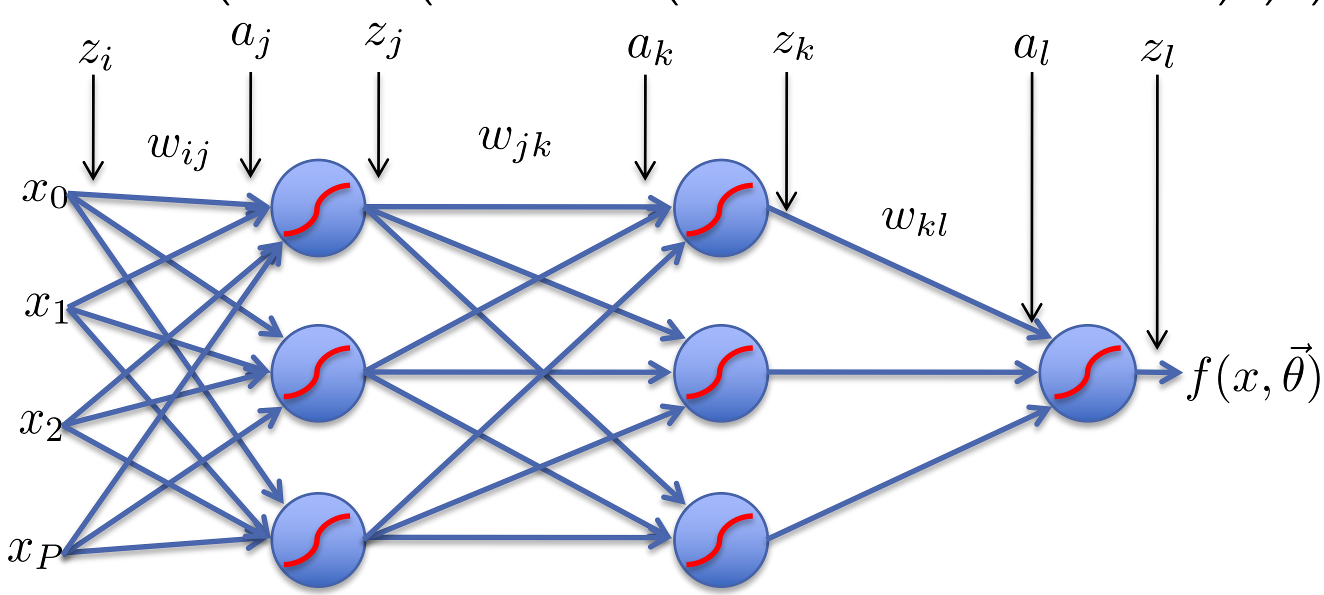
Empirical Risk Function



# Training: minimize loss

$$
\begin{aligned}
R(\theta) &= \frac{1}{N}\sum_{n=0}^{N} L(y_n - f(x_n)) \\
&= \frac{1}{N}\sum_{n=0}^{N} \frac{1}{2}\left(y_n - f(x_n)\right)^2 \\
&= \frac{1}{N}\sum_{n=0}^{N} \frac{1}{2}\left(y_n - g\left(\sum_k w_{kl}\, g\left(\sum_j w_{jk}\, g\left(\sum_i w_{ij} x_{n,i}\right)\right)\right)\right)^2
\end{aligned}
$$

$\boxed{\text{Empirical Risk Function}}$



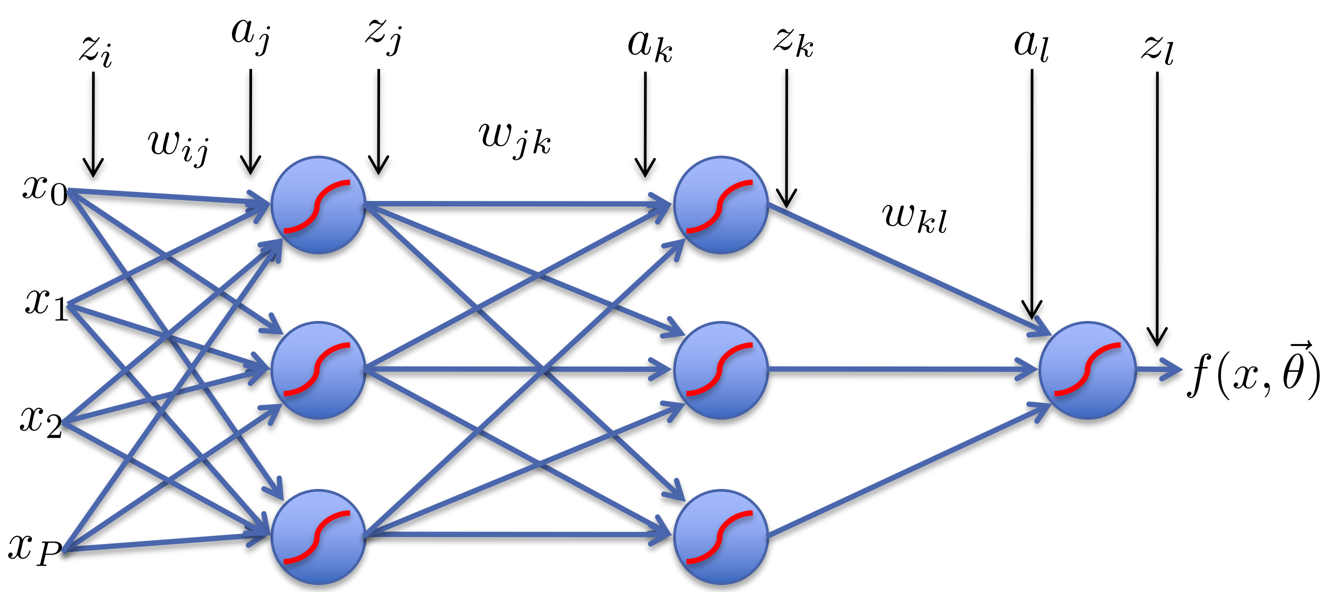
# Error Backpropagation

Optimize last layer weights $w_{kl}$

$$L_n = \frac{1}{2}\left(y_n - f(x_n)\right)^2$$

$$\frac{\partial R}{\partial w_{kl}} = \frac{1}{N}\sum_n \left[\frac{\partial L_n}{\partial a_{l,n}}\right]\left[\frac{\partial a_{l,n}}{\partial w_{kl}}\right]$$

Calculus chain rule



# Error Backpropagation

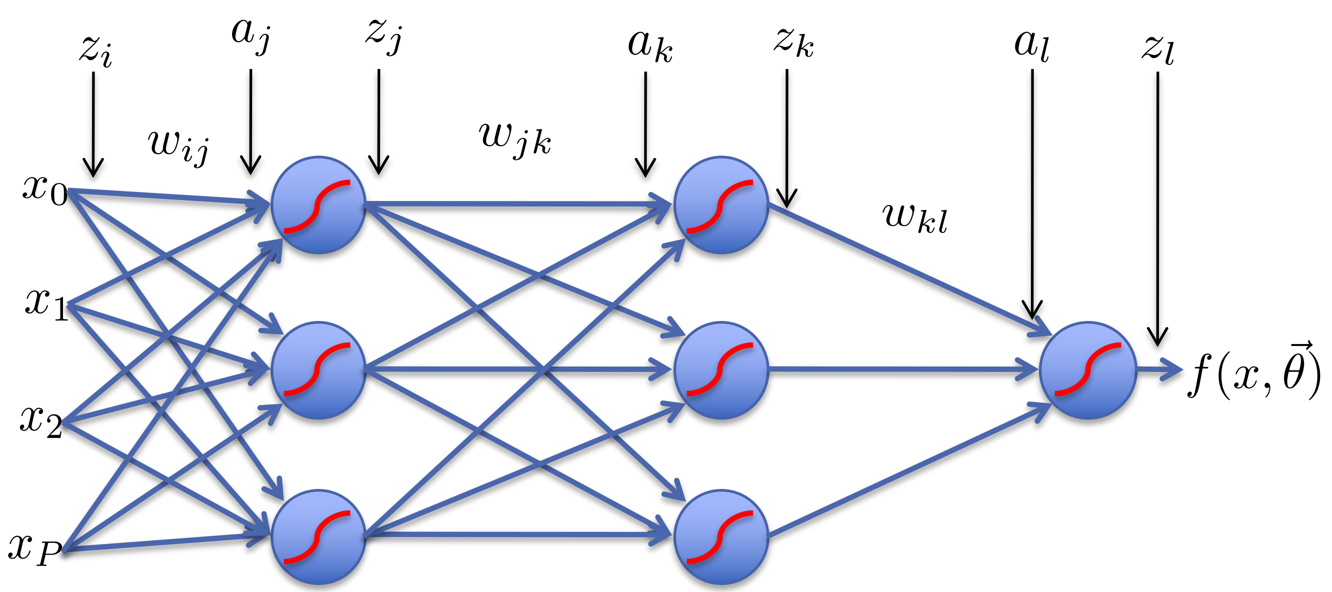
Optimize last layer weights $w_{kl}$

$$L_n = \frac{1}{2}\left(y_n - f(x_n)\right)^2$$

$$\frac{\partial R}{\partial w_{kl}} = \frac{1}{N}\sum_n \left[\frac{\partial L_n}{\partial a_{l,n}}\right]\left[\frac{\partial a_{l,n}}{\partial w_{kl}}\right]$$

Calculus chain rule

$$\frac{\partial R}{\partial w_{kl}} = \frac{1}{N}\sum_n \left[\frac{\partial \frac{1}{2}(y_n - g(a_{l,n}))^2}{\partial a_{l,n}}\right]\left[\frac{\partial a_{l,n}}{\partial w_{kl}}\right]$$



# Error Backpropagation

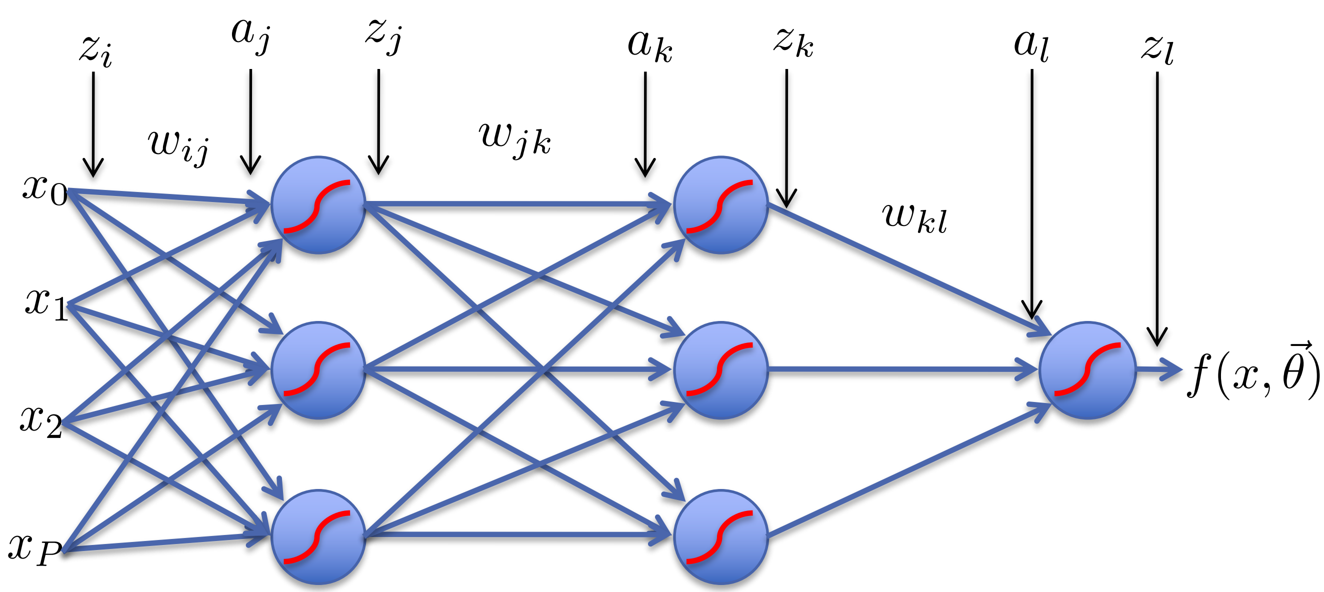
Optimize last layer weights $w_{kl}$

$$L_n = \frac{1}{2}\left(y_n - f(x_n)\right)^2$$

$$\frac{\partial R}{\partial w_{kl}} = \frac{1}{N}\sum_n \left[\frac{\partial L_n}{\partial a_{l,n}}\right]\left[\frac{\partial a_{l,n}}{\partial w_{kl}}\right]$$

Calculus chain rule

$$\frac{\partial R}{\partial w_{kl}} = \frac{1}{N}\sum_n \left[\frac{\partial \frac{1}{2}(y_n - g(a_{l,n}))^2}{\partial a_{l,n}}\right]\left[\frac{\partial z_{k,n} w_{kl}}{\partial w_{kl}}\right]$$



# Error Backpropagation

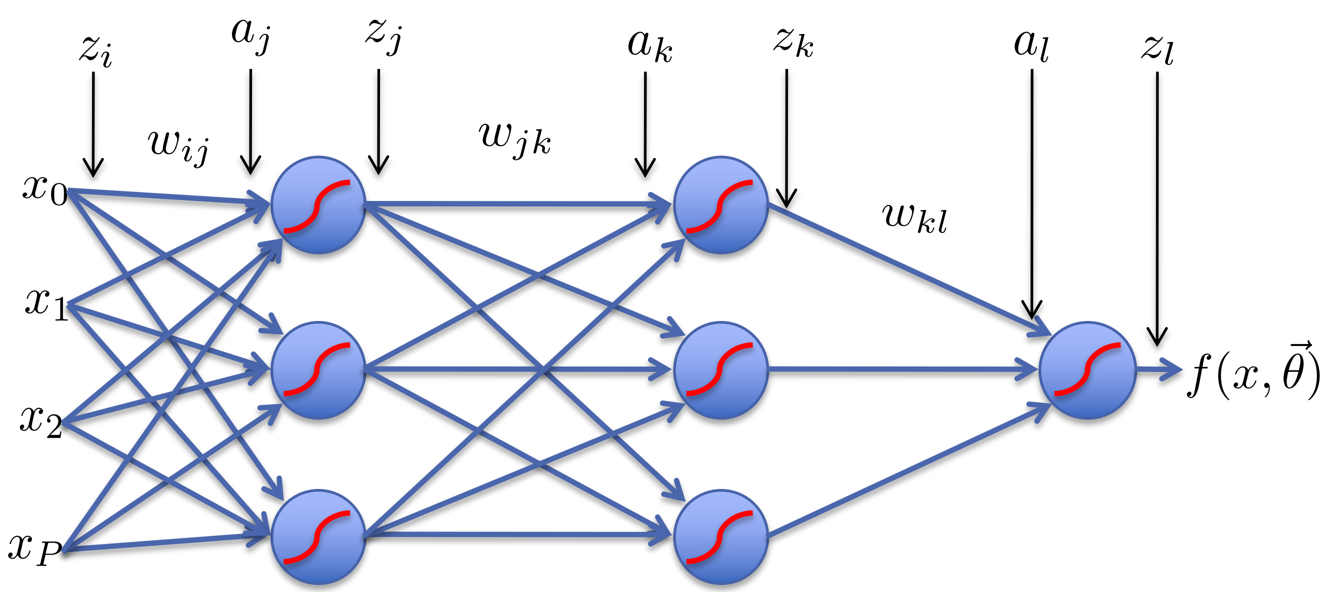
Optimize last layer weights $w_{kl}$

$$L_n = \frac{1}{2}\left(y_n - f(x_n)\right)^2$$

$$\frac{\partial R}{\partial w_{kl}} = \frac{1}{N}\sum_n \left[\frac{\partial L_n}{\partial a_{l,n}}\right]\left[\frac{\partial a_{l,n}}{\partial w_{kl}}\right]$$

Calculus chain rule

$$\frac{\partial R}{\partial w_{kl}} = \frac{1}{N}\sum_n \left[\frac{\partial \frac{1}{2}(y_n - g(a_{l,n}))^2}{\partial a_{l,n}}\right]\left[\frac{\partial z_{k,n} w_{kl}}{\partial w_{kl}}\right] = \frac{1}{N}\sum_n \left[-(y_n - z_{l,n})g'(a_{l,n})\right] z_{k,n}$$



# Error Backpropagation

Optimize last layer weights $w_{kl}$

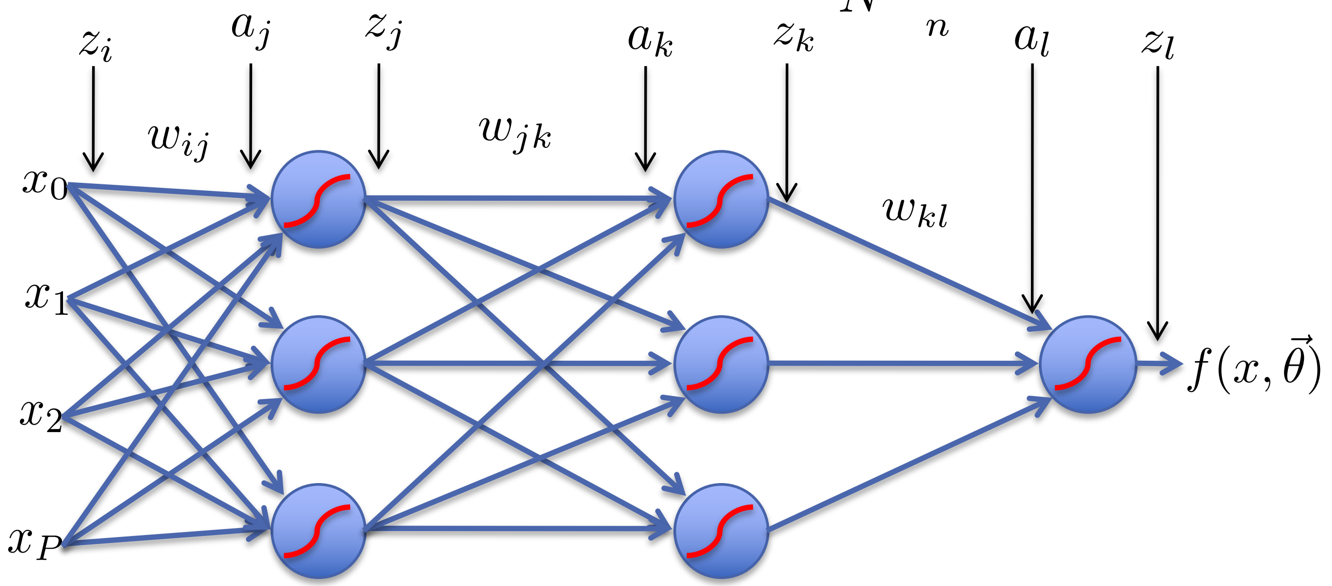
$$L_n = \frac{1}{2}\left(y_n - f(x_n)\right)^2$$

Calculus chain rule

$$\frac{\partial R}{\partial w_{kl}} = \frac{1}{N}\sum_n \left[\frac{\partial L_n}{\partial a_{l,n}}\right]\left[\frac{\partial a_{l,n}}{\partial w_{kl}}\right]$$

$$\frac{\partial R}{\partial w_{kl}} = \frac{1}{N}\sum_n \left[\frac{\partial \frac{1}{2}(y_n - g(a_{l,n}))^2}{\partial a_{l,n}}\right]\left[\frac{\partial z_{k,n} w_{kl}}{\partial w_{kl}}\right] = \frac{1}{N}\sum_n \boxed{\left[-(y_n - z_{l,n})g'(a_{l,n})\right]} z_{k,n}$$

$$= \frac{1}{N}\sum_n \delta_{l,n} \quad z_{k,n}$$



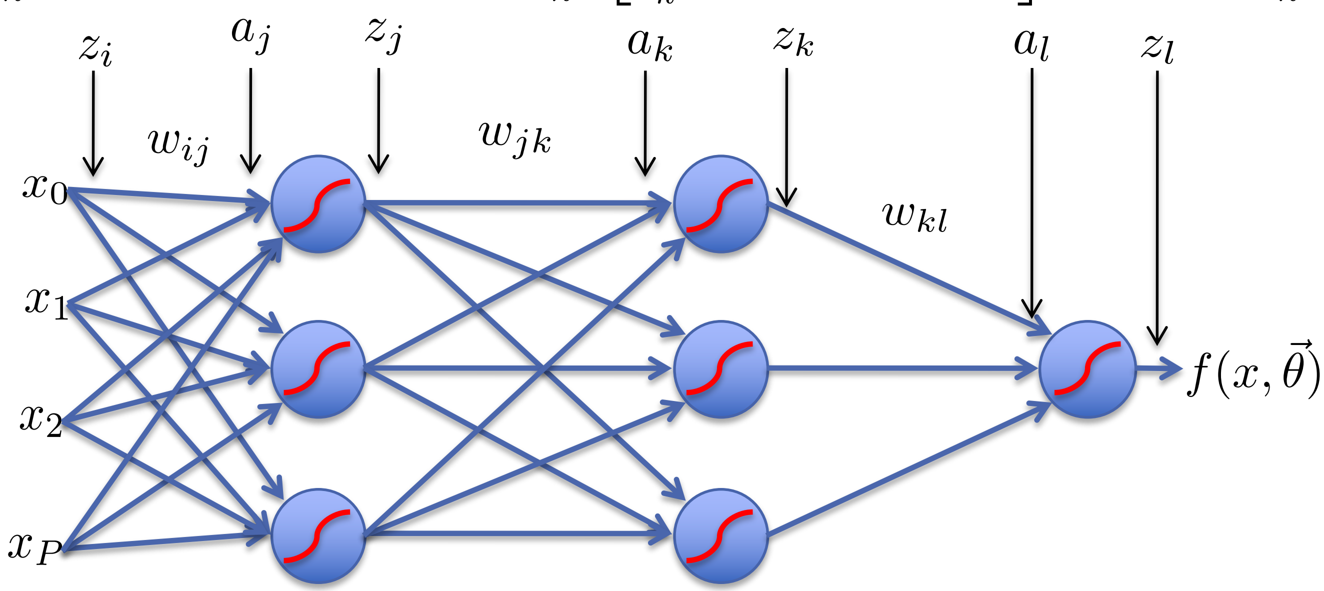
# Error Backpropagation

Repeat for all previous layers

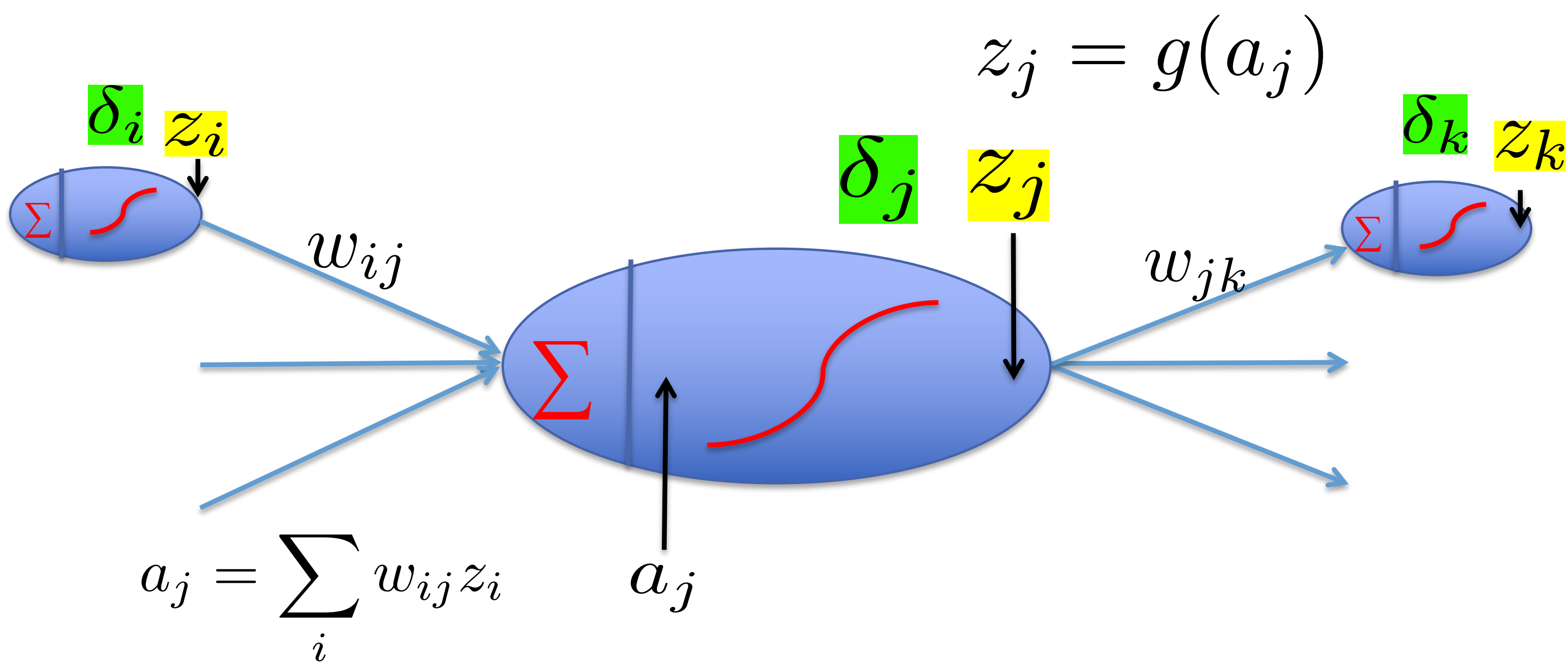
$$\frac{\partial R}{\partial w_{kl}} = \frac{1}{N}\sum_n \left[\frac{\partial L_n}{\partial a_{l,n}}\right]\left[\frac{\partial a_{l,n}}{\partial w_{kl}}\right] = \frac{1}{N}\sum_n \left[-(y_n - z_{l,n})g'(a_{l,n})\right] z_{k,n} = \frac{1}{N}\sum_n \delta_{l,n} z_{k,n}$$

$$\frac{\partial R}{\partial w_{jk}} = \frac{1}{N}\sum_n \left[\frac{\partial L_n}{\partial a_{k,n}}\right]\left[\frac{\partial a_{k,n}}{\partial w_{jk}}\right] = \frac{1}{N}\sum_n \left[\sum_l \delta_{l,n} w_{kl} g'(a_{k,n})\right] z_{j,n} = \frac{1}{N}\sum_n \delta_{k,n} z_{j,n}$$

$$\frac{\partial R}{\partial w_{ij}} = \frac{1}{N}\sum_n \left[\frac{\partial L_n}{\partial a_{j,n}}\right]\left[\frac{\partial a_{j,n}}{\partial w_{ij}}\right] = \frac{1}{N}\sum_n \left[\sum_k \delta_{k,n} w_{jk} g'(a_{j,n})\right] z_{i,n} = \frac{1}{N}\sum_n \delta_{j,n} z_{i,n}$$



# Backprop Recursion

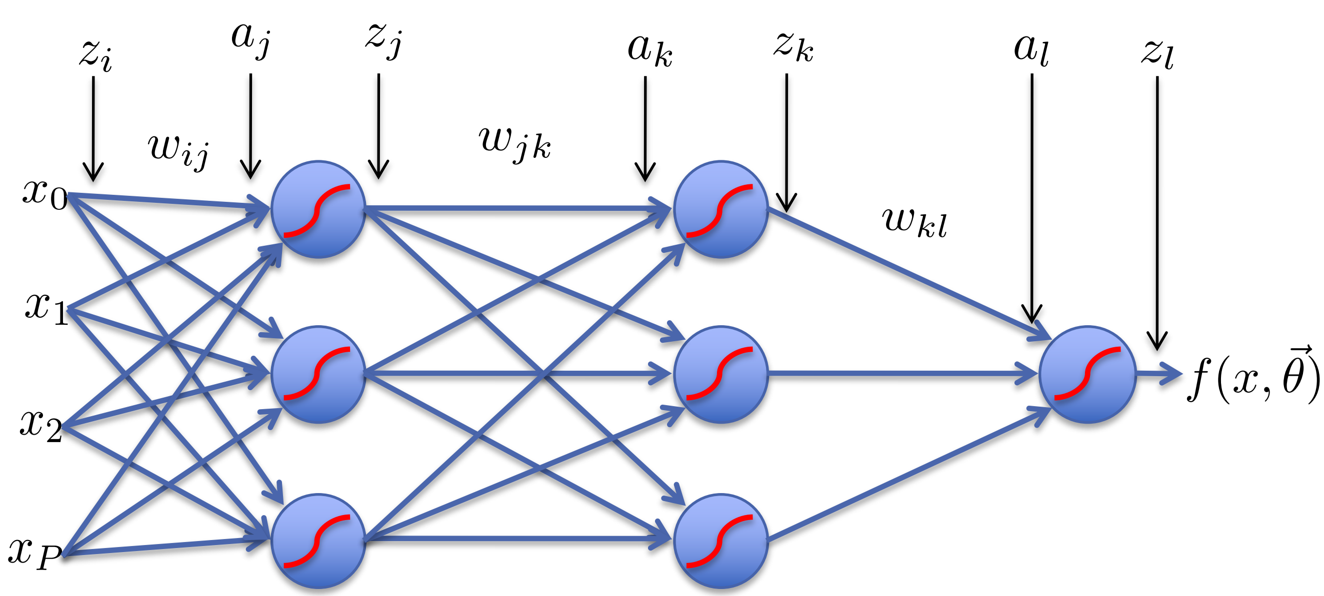



$$\frac{\partial R}{\partial w_{jk}} = \frac{1}{N}\sum_n \left[\frac{\partial L_n}{\partial a_{k,n}}\right]\left[\frac{\partial a_{k,n}}{\partial w_{jk}}\right] = \frac{1}{N}\sum_n \left[\sum_l \delta_{l,n} w_{kl} g'(a_{k,n})\right] z_{j,n} = \frac{1}{N}\sum_n \delta_{k,n} z_{j,n}$$

$$\frac{\partial R}{\partial w_{ij}} = \frac{1}{N}\sum_n \left[\frac{\partial L_n}{\partial a_{j,n}}\right]\boxed{\left[\frac{\partial a_{j,n}}{\partial w_{ij}}\right]} = \frac{1}{N}\sum_n \boxed{\left[\sum_k \delta_{k,n} w_{jk} g'(a_{j,n})\right]} z_{i,n} = \frac{1}{N}\boxed{\sum_n \delta_{j,n}}\boxed{z_{i,n}}$$

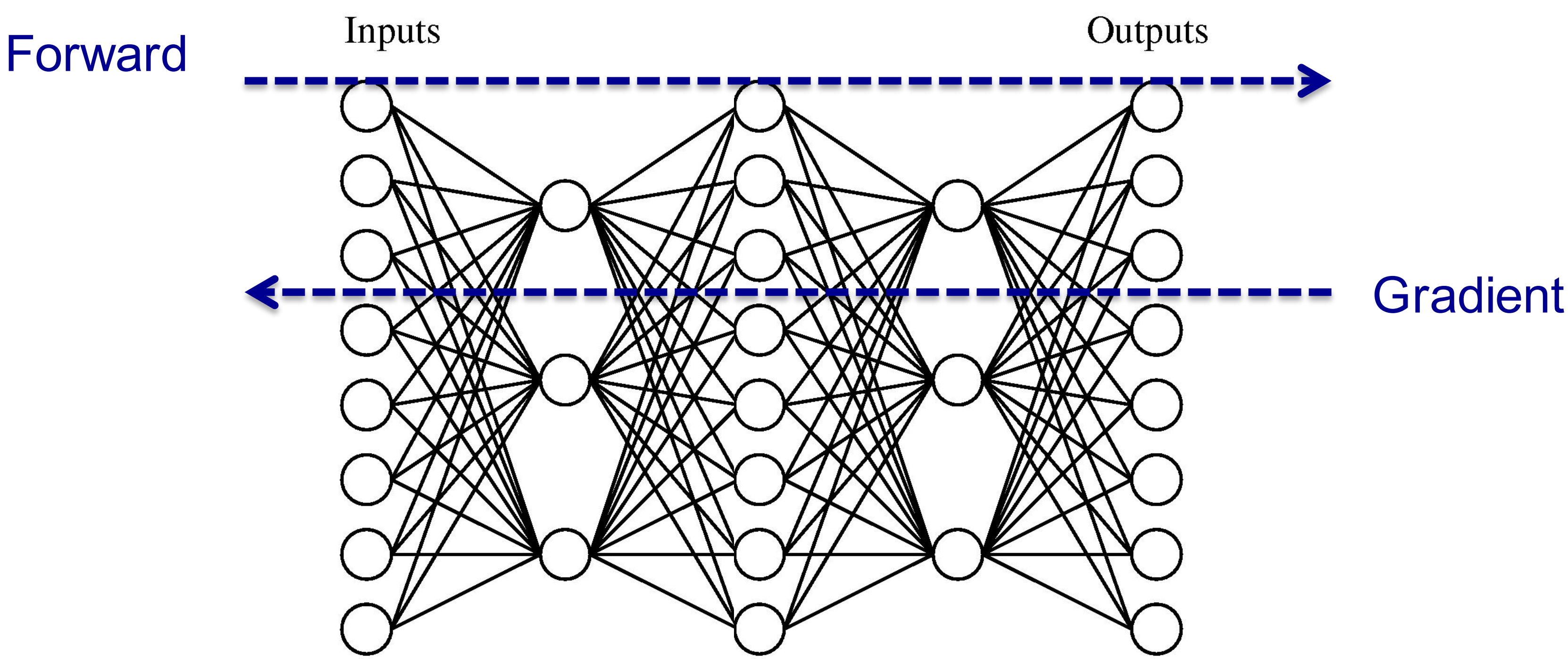
# Learning: Gradient Descent

$$
\begin{aligned}
w_{ij}^{t+1} &= w_{ij}^{t} - \eta \frac{\partial R}{w_{ij}} \\
w_{jk}^{t+1} &= w_{jk}^{t} - \eta \frac{\partial R}{w_{kl}} \\
w_{kl}^{t+1} &= w_{kl}^{t} - \eta \frac{\partial R}{w_{kl}}
\end{aligned}
$$



# Backpropagation

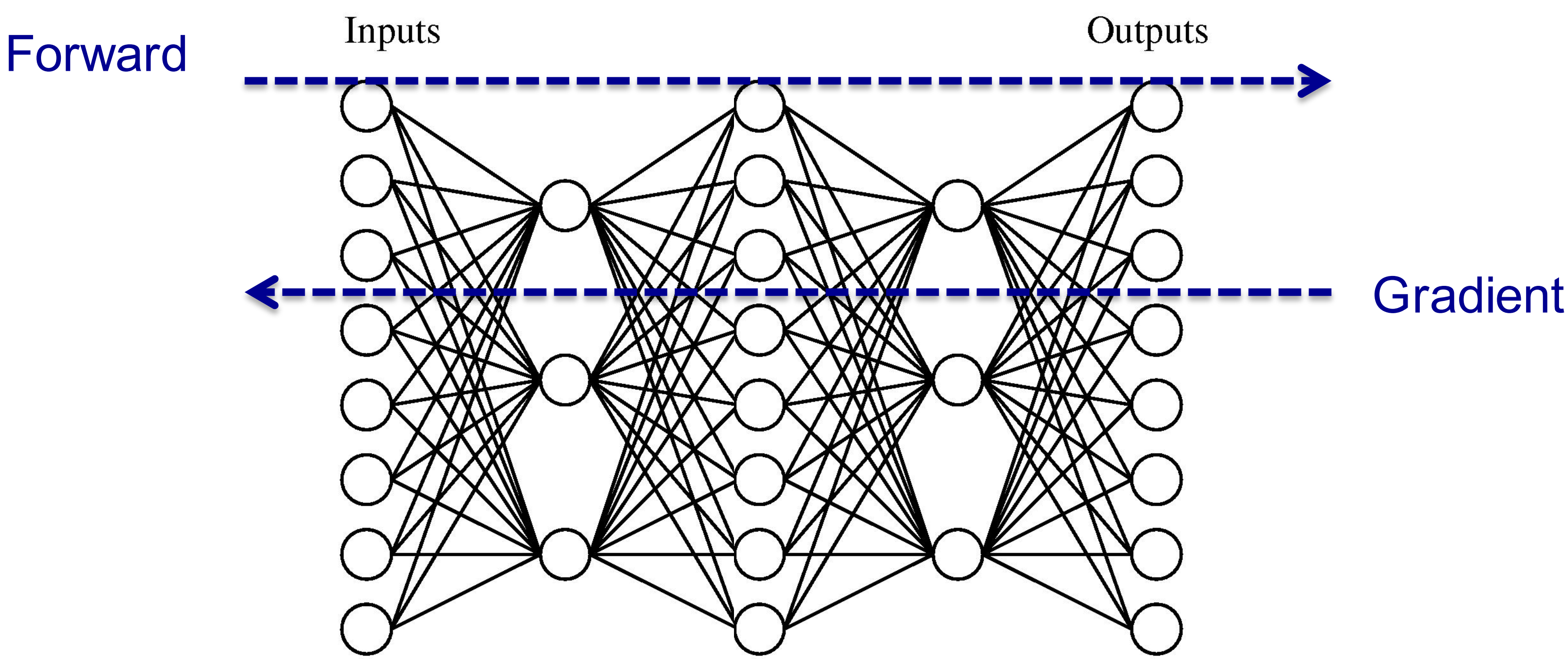
- Starts with a forward sweep to compute all the intermediate function values $z_i$
- Through backprop, computes the partial derivatives recursively $\delta_j$ $\frac{\partial R}{\partial w_{ij}}$
- A form of dynamic programming
  - Instead of considering exponentially many paths between a weight w_ij and the final loss (risk), store and reuse intermediate results.
- A type of automatic differentiation. (there are other variants e.g., recursive differentiation only through forward propagation.



# Backpropagation

Primary Interface Language:

- TensorFlow (https://www.tensorflow.org/) — Python
- Torch (http://torch.ch/) — Lua
- Theano (http://deeplearning.net/software/theano/) — Python
- CNTK (https://github.com/Microsoft/CNTK) — C++
- cnn (https://github.com/clab/cnn) — C++
- Caffe (http://caffe.berkeleyvision.org/) — C++



# Cross Entropy Loss (aka log loss, logistic loss)

- Cross Entropy

$$H(p, q) = -\sum_y p(y) \log q(y)$$

Predicted prob

True prob

- Related quantities
  - Entropy

$$H(p) = \sum_y p(y) \log p(y)$$

  - KL divergence (the distance between two distributions p and q)

$$D_{KL}(p||q) = \sum_y p(y) \log \frac{p(y)}{q(y)}$$

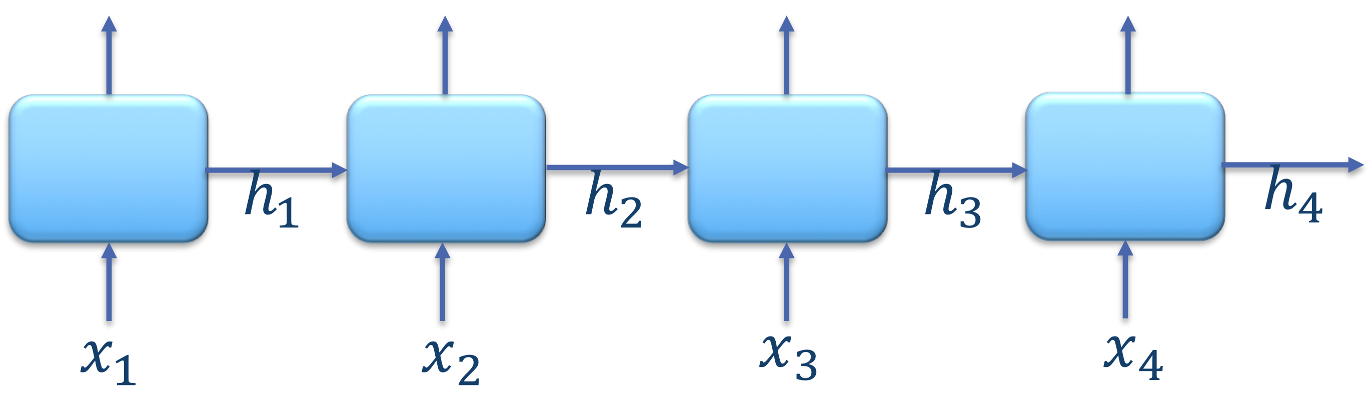
$$H(p, q) = E_p[-\log q] = H(p) + D_{KL}(p||q)$$

- Use Cross Entropy for models that should have more probabilistic flavor (e.g., language models)
- Use Mean Squared Error loss for models that focus on correct/incorrect predictions

$$\text{MSE} = \frac{1}{2}(y - f(x))^2$$

# RNN Learning: **B**ack**p**rop **T**hrough **T**ime (BPTT)

- Similar to backprop with non-recurrent NNs
- But unlike feedforward (non-recurrent) NNs, each unit in the computation graph repeats the exact same parameters…
- Backprop gradients of the parameters of each unit as if they are different parameters
- When updating the parameters using the gradients, use the average gradients throughout the entire chain of units.



# Convergence of backprop

- Without non-linearity or hidden layers, learning is convex optimization
  - Gradient descent reaches **global minima**
- Multilayer neural nets (with nonlinearity) are **not convex**
  - Gradient descent gets stuck in local minima
  - Selecting number of hidden units and layers = fuzzy process
  - NNs have made a HUGE comeback in the last few years
    - Neural nets are back with a new name
      - Deep belief networks
      - Huge error reduction when trained with lots of data on GPUs

# Overfitting in NNs

- Are NNs likely to overfit?
  - Yes, they can represent arbitrary functions!!!
- Avoiding overfitting?
  - More training data
  - Fewer hidden nodes / better topology
  - Random perturbation to the graph topology (“Dropout”)
  - Regularization
  - Early stopping