Wavelet Transform

- Wavelet Transform
  - A family of transformations that filters the data into low resolution data plus detail data.

Wavelet Transform Coding

• Wavelet Transform
  - Low pass filter
  - High pass filter

Wavelet Transformed Barbara (Enhanced)

Wavelet Transformed Barbara (Actual)

most of the details are small so they are very dark.

Wavelet Transform Compression

Decoder

Wavelet coder transmits wavelet transformed image in bit plane order with the most significant bits first.

Bit Planes of Coefficients

Coefficients are normalized between -1 and 1
Why Wavelet Compression Works

- Wavelet coefficients are transmitted in bit-plane order.
  - In most significant bit planes most coefficients are 0 so they can be coded efficiently.
  - Only some of the bit planes are transmitted. This is where fidelity is lost when compression is gained.
- Natural progressive transmission

![Wavelet Compression Diagram]

Rate-Fidelity Curve

![Rate-Fidelity Curve Graph]

Wavelet Coding Methods

- EZW - Shapiro, 1993
  - Embedded Zerotree coding.
- SPIHT - Said and Pearlman, 1996
  - Set Partitioning in Hierarchical Trees coding. Also uses "zerotrees".
- ECECOW - Wu, 1997
  - Uses arithmetic coding with context.
- EBCOT – Taubman, 2000
  - Uses arithmetic coding with different context.
- JPEG 2000 – new standard based largely on EBCOT
- GTW – Hong, Ladner 2000
  - Uses group testing which is closely related to Golomb codes
- PACW - Ladner, Askew, Barney 2003
  - Like GTW but uses arithmetic coding

![Wavelet Coding Methods Diagram]

One-Dimensional Average Transform

(1)

![One-Dimensional Average Transform Diagram (1)]

(2)

![One-Dimensional Average Transform Diagram (2)]
One-Dimensional Average Transform

\[ L = B[0 \cdots n/2 - 1] \]
\[ H = B[n/2 \cdots n - 1] \]

\[ A[2i] = B[i] + B[n/2 + i], \quad 0 \leq i < \frac{n}{2} \]
\[ A[2i+1] = B[i] + B[n/2 + i], \quad 0 \leq i < \frac{n}{2} \]

Note that the low resolution version and the detail together have the same number of values as the original.

One-Dimensional Average Inverse Transform

Two Dimensional Transform (1)

Transform each row in LL
Transform each column in L and H
2 levels of transform gives 7 subbands.
k levels of transform gives \(3k + 1\) subbands.

Two Dimensional Transform (1)

Transform each row in L
Transform each column in H
3 detail subbands

Two Dimensional Average Transform

\[ L = B[0 \cdots n/2 - 1] \]
\[ H = B[n/2 \cdots n - 1] \]
Wavelet Transformed Image

2 levels of wavelet transform
1 low resolution subband
6 detail subbands

Wavelet Transform Details

- Conversion to reals.
  - Convert gray scale to floating point.
  - Convert color to YUV and then convert each to band to floating point. Compress separately.
- After several levels (3-8) of transform we have a matrix of floating point numbers called the wavelet transformed image (coefficients).

Wavelet Transforms

- Technically wavelet transforms are special kinds of linear transformations. Easiest to think of them as filters.
  - The filters depend only on a constant number of values. (bounded support)
  - Preserve energy (norm of the pixels = norm of the coefficients)
  - Inverse filters also have bounded support.
- Well-known wavelet transforms
  - Haar – like the average but orthogonal to preserve energy. Not used in practice.
  - Daubechies 9/7 – biorthogonal (inverse is not the transpose). Most commonly used in practice.

Haar Filters

\[
\text{low pass } \frac{1}{\sqrt{2}} \quad \frac{1}{\sqrt{2}} \\
\text{high pass } -0.5
\]

\[
\text{low pass } B[i] = \frac{1}{\sqrt{2}} A[2i] + \frac{1}{\sqrt{2}} A[2i + 1], \quad 0 \leq i < \frac{n}{2} \\
\text{high pass } B[n/2 + i] = -\frac{1}{\sqrt{2}} A[2i] + \frac{1}{\sqrt{2}} A[2i + 1], \quad 0 \leq i < \frac{n}{2}
\]

Want the sum of squares of the filter coefficients = 1

Daubechies 9/7 Filters

\[
\text{low pass filter } h_i \\
\text{high pass filter } g_i
\]

\[
\text{low pass } B[i] = \sum_{j=0}^{n/4} h_j A[2i + j], \quad 0 \leq i < \frac{n}{2} \\
\text{high pass } B[n/2 + i] = \sum_{j=0}^{n/2} g_j A[2i + j], \quad 0 \leq i < \frac{n}{2}
\]

Reflection used near boundaries

Linear Time Complexity of 2D Wavelet Transform

- Let n = number of pixels and let b be the number of coefficients in the filters.
- One level of transform takes time
  - O(bn)
- k levels of transform takes time proportional to
  - bn + bn/4 + ... + bn/d/4 -1 < (4/3)bn.
- The wavelet transform is linear time when the filters have constant size.
  - The point of wavelets is to use constant size filters unlike many other transforms.
Wavelet Transform

Encoder

- Wavelet transform
- Coefficients
- Wavelet coding
- Bit stream

Decoder

- Wavelet decoding
- Approx coefficients
- Inverse wavelet transform
- Distorted image

Wavelet coder transmits wavelet transformed image in bit plane order with the most significant bits first.

Bit-Plane Coding

- Normalize the coefficients to be between –1 and 1
- Transmit one bit-plane at a time
- For each bit-plane
  - Significance pass: Find the newly significant coefficients, transmit their signs.
  - Refinement pass: transmit the bits of the known significant coefficients.

Divide into Bit-Planes

Significance Coefficients

Significance & Refinement Passes

- Code a bit-plane in two passes
  - Significance pass
    - codes previously insignificant coefficients
    - also codes sign bit
  - Refinement pass
    - refines values for previously significant coefficients
- Main idea:
  - Significance-pass bits likely to be 0;
  - Refinement-pass bit are not

Coefficient List

<table>
<thead>
<tr>
<th>#</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0001010010</td>
</tr>
<tr>
<td>2</td>
<td>0011101110</td>
</tr>
<tr>
<td>3</td>
<td>0001001001</td>
</tr>
<tr>
<td>4</td>
<td>0010110010</td>
</tr>
<tr>
<td>5</td>
<td>0001111010</td>
</tr>
<tr>
<td>6</td>
<td>0010110011</td>
</tr>
<tr>
<td>7</td>
<td>0001001001</td>
</tr>
<tr>
<td>8</td>
<td>0010111111</td>
</tr>
<tr>
<td>9</td>
<td>0010110111</td>
</tr>
<tr>
<td>10</td>
<td>0001001001</td>
</tr>
</tbody>
</table>
bit planes 1 – 2
dpp .0033
PSNR 16.8
Compressed size

bit planes 1 – 3
dpp .0072
PSNR 18.8
Compressed size

bit planes 1 – 4
dpp .015
ratio 533:1
PSNR 20.5
Compressed size

bit planes 1 – 5
dpp .035
ratio 229:1
PSNR 22.2
Compressed size

bit planes 1 – 6
dpp .118
ratio 68:1
PSNR 24.8
Compressed size
bit planes
1 – 7
bpp .303
ratio 26 : 1
PSNR 28.7

bit planes
1 – 8
bpp .619
ratio 13 : 1
PSNR 32.9

bit planes
1 – 9
bpp 1.116
ratio 7 : 1
PSNR 37.5

PACW
- A simple image coder based on
  - Bit-plane coding
    - Significance pass
    - Refinement pass
  - Arithmetic coding
  - Careful selection of contexts based on statistical studies
- Implemented by undergraduates Amanda Askew and Dane Barney in Summer 2003.

Arithmetic Coding in PACW
- Performed on each individual bit plane.
  - Alphabet is Σ={0,1}
  - Signs are coded as needed
- Uses integer implementation with 32-bit integers. (Initialize L = 0, R = 2^{32} - 1)
  - Uses scaling and adaptation.
  - Uses contexts based on statistical studies.
Encoding the Bit-Planes

- Code most significant bit-planes first
- Significance pass for a bit-plane
  - First code those coefficients that were insignificant in the previous bit-plane.
  - Code these in a priority order.
  - If a coefficient becomes significant then code its sign.
- Refinement pass for a bit-plane
  - Code the refinement bit for each coefficient that is significant in a previous bit-plane

Decoding

- Emulate the encoder to find the bit planes.
  - The decoder know which bit-plane is being decoded
  - Whether it is the significant or refinement pass
  - Which coefficient is being decoded.
- Interpolate to estimate the coefficients.

Contexts (per bit plane)

- Significance pass contexts:
  - Contexts based on
    - Subband level
    - Number of significant neighbors
    - Sign context
- Refinement contexts
  - 1st refinement bit is always 1 so no context needed
  - 2nd refinement bit has a context
  - All other refinement bits have a context
- Context Principles
  - Bits in a given context have a probability distribution
  - Bits in different contexts have different probability distributions

Subband Level

- Image is divided into subbands until LL band (subband level 0) is less than 16x16
- Barbara image has 7 subband levels

Statistics for Subband Levels

<table>
<thead>
<tr>
<th>Subband Level</th>
<th># significant</th>
<th># insignificant</th>
<th>% significant</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>144</td>
<td>364</td>
<td>28.3%</td>
</tr>
<tr>
<td>1</td>
<td>272</td>
<td>1048</td>
<td>20.6%</td>
</tr>
<tr>
<td>2</td>
<td>848</td>
<td>4592</td>
<td>15.6%</td>
</tr>
<tr>
<td>3</td>
<td>3134</td>
<td>23568</td>
<td>11.7%</td>
</tr>
<tr>
<td>4</td>
<td>12268</td>
<td>113886</td>
<td>9.7%</td>
</tr>
<tr>
<td>5</td>
<td>48282</td>
<td>504633</td>
<td>8.7%</td>
</tr>
<tr>
<td>6</td>
<td>190003</td>
<td>2226904</td>
<td>7.8%</td>
</tr>
</tbody>
</table>
### Number of Significant Neighbors

**Barbara (8bpp)**

<table>
<thead>
<tr>
<th>Significant neighbors</th>
<th># significant</th>
<th># insignificant</th>
<th>% significant</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>4849</td>
<td>2252468</td>
<td>.2%</td>
</tr>
<tr>
<td>1</td>
<td>13319</td>
<td>210695</td>
<td>5.9%</td>
</tr>
<tr>
<td>2</td>
<td>22276</td>
<td>104252</td>
<td>17.6%</td>
</tr>
<tr>
<td>3</td>
<td>30206</td>
<td>78899</td>
<td>27.7%</td>
</tr>
<tr>
<td>4</td>
<td>33244</td>
<td>55841</td>
<td>37.3%</td>
</tr>
<tr>
<td>5</td>
<td>27354</td>
<td>39189</td>
<td>41.1%</td>
</tr>
<tr>
<td>6</td>
<td>36482</td>
<td>44225</td>
<td>45.2%</td>
</tr>
<tr>
<td>7</td>
<td>87566</td>
<td>91760</td>
<td>48.8%</td>
</tr>
</tbody>
</table>

### Refinement Bit Context Statistics

**Barbara (8bpp)**

<table>
<thead>
<tr>
<th></th>
<th>0’s</th>
<th>1’s</th>
<th>% 0’s</th>
</tr>
</thead>
<tbody>
<tr>
<td>2nd Refinement Bits</td>
<td>146,293</td>
<td>100,521</td>
<td>59.3%</td>
</tr>
<tr>
<td>Other Refinement Bits</td>
<td>475,941</td>
<td>433,962</td>
<td>53.3%</td>
</tr>
<tr>
<td>Sign Bits</td>
<td>128,145</td>
<td>120,100</td>
<td>49.6%</td>
</tr>
</tbody>
</table>

- Barbara at 2bpp: 2nd Refinement bit % 0’s = 65.8%

### Context Details

- Significance pass contexts per bit-plane:
  - Max neighbors’ num subband levels contexts
  - For Barbara: contexts for sig neighbor counts of 0 - 3 and subband levels of 0.6 = 4'7 = 28 contexts
  - Index of a context:
    - Max neighbors’ subband level + num sig neighbors
    - Example sum sig neighbors = 2, subband level = 3, index = 4'3 + 2’ = 14
  - Sign context
    - 1 contexts
  - 2 Refinement contexts
    - 1st refinement bit is always 1 not transmitted
    - 2nd refinement bit has a context
  - Number of contexts per bit plane for Barbara = 28 + 1 + 2 = 31

### Priority Queue

- Used in significance pass to decide which coefficient to code next
  - Goal code coefficients most likely to become significant
  - All non-empty contexts are kept in a max heap
  - Priority is determined by:
    - # sig coefficients coded / total coefficients coded

### Reconstruction of Coefficients

- Coefficients are decoded to a certain number of bit planes
  - .10110XXXX What should X’s be?
  - .10110000 < .101110000 < .101110111...
  - .10111000000 is half-way
- Handled the same as SPIHT and GTW
  - if coefficient is still insignificant, do no interpolation
  - if newly significant, add on .38 to scale
  - if significant, add on .5 to scale

\[ |A| = k \]

- .A000...
- .38
- .5
- .A01100
- .A100...
- .A111...

### Original Barbara Image
Barbara at .5 bpp (PSNR = 31.68)

Barbara at .25 bpp (PSNR = 27.75)

Barbara at .1 bpp (PSNR = 24.53)

Results
Compression of Barbara

Results
Compression of Lena

Results
Compression of RoughWall
PACW Notes

• PACW competitive with JPEG 2000, SPIHT-AC, and GTW.
• Developed in Java from scratch by two undergraduates, Dane Barney and Amanda Askew, in 2 months.
• Dane’s final version is slightly different than the one describe here. See his senior thesis.