## Genetic Algorithms

- Start with random population of states
- Representation serialized (ie. strings of characters or bits)
- States are ranked with "fitness function"
- Produce new generation
- Select random pair(s) using probability:
- probability ~ fitness
- Randomly choose "crossover point"
- Offspring mix halves
- Randomly mutate bits


## Crossover Mutation

1746 29844710
77651094281

## 16461094281

776029844210

## Genetic Algorithm

- Given: population $P$ and fitness-function $f$
- repeat
- newP $\leftarrow$ empty set
- for $i=1$ to size(P)
$x \leftarrow$ RandomSelection(P,f)
$y \leftarrow$ RandomSelection(P,f)
child $\leftarrow$ Reproduce $(x, y)$
if (small random probability) then child $\leftarrow$ Mutate(child) add child to newP
- $P \leqslant$ new $P$
- until some individual is fit enough or enough time has elapsed
- return the best individual in $P$ according to $f$


## Using Genetic Algorithms

- 2 important aspects to using them
- 1. How to encode your real-life problem
-2. Choice of fitness function
- Research Example
- I have N variables V1, V2, ... VN
- I want to produce a single number from them that best satisfies my fitness function $F$
- I tried linear combinations, but that didn't work
- A guy named Stan (Matwin) I met at a workshop in Italy told me to try Genetic Programming


## Genetic Programming

- Like genetic algorithm, but instead of finding the best character string, we want to find the best arithmetic expression tree
- The leaves will be the variables and the non-terminals will be arithmetic operators
- It uses the same ideas of crossover and mutation to produce the arithmetic expression tree that maximizes the fitness function.


## Example: Classification and

## Quantification of Facial Abnormalities

- Input is 3D meshes of faces
- Disease is 22q11.2 Deletion Syndrome.
- Multiple different facial abnormalities
- We'd like to assign severity scores to the different abnormalities, so need a single number to represent our analysis of a portion of the face.


# 22q11.2 Deletion Syndrome (22q11.2DS) 

- Caused by genetic deletion
- Cardiac anomalies, learning disabilities
- Multiple subtle physical manifestations
- Assessment is subjective



## Data Collection



3dMD multi-camera stereo system
Reconstructed 3D mesh

## Learning 3D Shape Quantification

- Analyze 22q11.2DS and 9 associated facial features
- Goal: quantify different shape variations in different facial abnormalities



## Azimuth and Elevation Angles



# Learning 3D Shape Quantification 2D Histogram Azimuth Elevation 

- Using azimuth and elevation angles of surface normal vectors of points in selected region



## Learning 3D Shape Quantification -

 Feature Selection- Determine most discriminative bins
- Use Adaboost learning
- Obtain positional information of important region on face



# Learning 3D Shape Quantification - 

 Feature Combination- Use Genetic Programming (GP) to evolve mathematical expression
- Start with random population
- Individuals are evaluated with fitness measure
- Best individuals reproduce to form new population


# Learning 3D Shape Quantification Genetic Programming 

- Individual:
- Tree structure
- Terminals e.g variables eg. $3,5, \mathrm{x}, \mathrm{y}, \ldots$
- Function set e.g +, -, *, ...
- Fitness measure e.g sum of square ...


$$
5^{*}(x+y)
$$

Learning 3D Shape Quantification Feature Combination

- 22q11.2DS dataset
- Assessed by craniofacial experts
- Groundtruth is union of expert scores
- Goal: classify individual according to given facial abnormality


## Learning 3D Shape Quantification Feature Combination

- Individual
- Terminal: selected histogram bins
- Function set: +,-,, ,,min,max,sqrt,log,2x,5x,10x
- Fitness measure: F1-measure


$$
\begin{aligned}
& F(\text { prec }, \text { rec })=\frac{2 \times(\text { prec } \times r e c)}{p r e c+r e c} \\
& \text { precision }=\mathrm{TP} /(\mathrm{TP}+\mathrm{FP})
\end{aligned}
$$

recall $=$ TP/all positives

$$
X 6+X 7+(\max (X 7, X 6)-\sin (X 8)+(X 6+X 6))
$$

# Learning 3D Shape Quantification - Experiment 1 

- Objective: investigate function sets
- Combo1 $=\left\{+,-,{ }^{*}, \min , \max \right\}$
- Combo2 $=\left\{+,-,{ }^{*}, \min , \max\right.$, sqrt,log2,log10\}
- Combo3 = \{+,-, ${ }^{*}$, min, max,
$2 x, 5 x, 10 x, 20 x, 50 x, 100 x\}$
- Combo4 = \{+,-, *, min, max,sqrt,log2,log10, $2 x, 5 x, 10 x, 20 x, 50 x, 100 x\}$


## Learning 3D Shape

## Quantification - Experiment 1

- Best F-measure out of 10 runs

| Facial anomaly | Combo1 | Combo2 | Combo3 | Combo4 |
| :--- | :---: | :---: | :---: | :---: |
| Midface Hypoplasia | 0.8393 | 0.8364 | 0.8527 | 0.80 |
| Tubular Nose | 0.8571 | 0.875 | 0.8667 | 0.8813 |
| Bulbous Nasal Tip | 0.8545 | 0.8099 | 0.8103 | 0.7544 |
| Prominent Nasal Root | 0.8667 | 0.8430 | 0.8571 | 0.8335 |
| Small Nasal Alae | 0.8846 | 0.8454 | 0.8454 | 0.8571 |
| Retrusive Chin | 0.7952 | 0.8000 | 0.7342 | 0.7586 |
| Open Mouth | 0.9444 | 0.9714 | 0.9189 | 0.9189 |
| Small Mouth | 0.6849 | 0.7568 | 0.6829 | 0.7750 |
| Downturned mouth | 0.8000 | 0.7797 | 0.8000 | 0.8000 |

## Tree structure for quantifying midface hypoplasia



Xi are the selected histogram bins from an azimuthelevation histogram of the surface normals of the face.

## Learning 3D Shape

## Quantification - Experiment 2

- Objective: compare local facial shape descriptors

| Facial abnormality | Region Histogram | Selected Bins | GP |
| :--- | :---: | :---: | :---: |
| Midface hypoplasia | 0.697 | 0.721 | 0.853 |
| Tubular nose | 0.701 | 0.776 | 0.881 |
| Bulbous nasal tip | 0.617 | 0.641 | 0.855 |
| Prominent nasal root | 0.704 | 0.748 | 0.867 |
| Small nasal alae | 0.733 | 0.801 | 0.885 |
| Retrusive chin | 0.658 | 0.713 | 0.800 |
| Open mouth | 0.875 | 0.889 | 0.971 |
| Small mouth | 0.694 | 0.725 | 0.775 |
| Downturned mouth | 0.506 | 0.613 | 0.800 |

## Learning 3D Shape

## Quantification - Experiment 3

- Objective: predict 22q11.2DS

| Method | F-measure |
| :--- | :--- |
| Quantification vector with SVM | 0.709 |
| Quantification vector with Adaboost | 0.721 |
| Quantification vector with GP | 0.821 |
| Global saliency map | 0.764 |
| Selected bins of global saliency map | 0.9 |
| Global 2D histogram | 0.79 |
| Selected bins of global 2D histogram | 0.9 |
| Selected bins of global saliency map with GP | 0.96 |
| Selected bins of global 2D histogram with GP | 0.92 |
| Expert's median | 0.68 |

## Local Search in Continuous Spaces

- Given a continuous state space

$$
\mathrm{S}=\left\{\left(\mathrm{x}_{1}, \mathrm{x}_{2}, \ldots, \mathrm{x}_{\mathrm{N}}\right) \mid \mathrm{x}_{\mathrm{i}} \varepsilon \mathrm{R}\right\}
$$

- Given a continuous objective function $f\left(x_{1}, x_{2}, \ldots, x_{N}\right)$
- The gradient of the objective function is a vector $\nabla \mathfrak{f}=\left(\partial \mathrm{f} / \partial \mathrm{x}_{1}, \partial \mathrm{f} / \partial \mathrm{x}_{2}, \ldots, \partial \mathrm{f} / \partial \mathrm{x}_{\mathrm{N}}\right)$
- The gradient gives the magnitude and direction of the steepest slope at a point.


## Local Search in Continuous Spaces

- To find a maximum, the basic idea is to set $\nabla f=0$
- Then updating of the current state becomes $x \leftarrow x+\alpha \nabla f(x)$
where $\alpha$ is a small constant.
- Theory behind this is taught in numerical methods classes.
- Your book suggests the Newton-Raphson method. Luckily there are packages.....


## Computer Vision Pose Estimation Example

I have a 3D model of an object and an image of that object.

I want to find the pose: the position and orientation of the camera.


## Computer Vision Pose Estimation Example

 Initial pose from points/ellipses and final pose after optimization.
(a) Initial pose

(b) Final pose

Figure 21: Example pose hypothesis and final pose after constrained optimization.

- The optimization was searching a 6D space: ( $\mathrm{x}, \mathrm{y}, \mathrm{z}, \theta \mathrm{x}, \theta \mathrm{y}, \theta \mathrm{z}$ )
- The fitness function was how well the projection of the 3D object lined up with the edges on the image.


## Fitness Function

- Modified Hausdorf Distance between the image of the projected model and the image of the detected edges

The directed distance $d_{6}$ [18] is used to quantitatively evaluate how well the projected model point set $(A)$ overlays the edge image point set $(B)$, and it is defined as

$$
\begin{equation*}
d_{6}(A, B)=\frac{1}{N_{A}} \sum_{a \in A} d(a, B) \tag{41}
\end{equation*}
$$

where $N_{A}$ is the number of points in set $A$.


## Searching with Nondeterministic Actions

- Vacuum World (actions = \{left, right, suck\})



## Searching with Nondeterministic Actions

In the nondeterministic case, the result of an action can vary.

## Erratic Vacuum World:

- When sucking a dirty square, it cleans it and sometimes cleans up dirt in an adjacent square.
- When sucking a clean square, it sometimes deposits dirt on the carpet.


## Generalization of State-Space Model

1. Generalize the transition function to return a set of possible outcomes.

$$
\text { oldf: S x A ->S newf: S x A -> } 2^{S}
$$

2. Generalize the solution to a contingency plan.
if state=s then action-set-1 else action-set-2
3. Generalize the search tree to an AND-OR tree.

## AND-OR Search Tree



## Searching with Partial Observations

- The agent does not always know its state!
- Instead, it maintains a belief state: a set of possible states it might be in.
- Example: a robot can be used to build a map of a hostile environment. It will have sensors that allow it to "see" the world.


## Belief State Space for Sensorless Agent

Knows it's on the left

Knows left side clean


Figure 4.14 The reachable portion of the belief-state space for the deterministic, sensorless vacuum world. Each shaded box corresponds to a single belief state. At any given point, the agent is in a particular belief state but does not know which physical state it is in. The initial belief state (complete ignorance) is the top center box. Actions are represented by labeled links. Self-loons are omitted for claritv.

Knows it's on the right.

## Online Search Problems

- Active agent
- executes actions
- acquires percepts from sensors
- deterministic and fully observable
- has to perform an action to know the outcome
- Examples
- Web search
- Autonomous vehicle

