

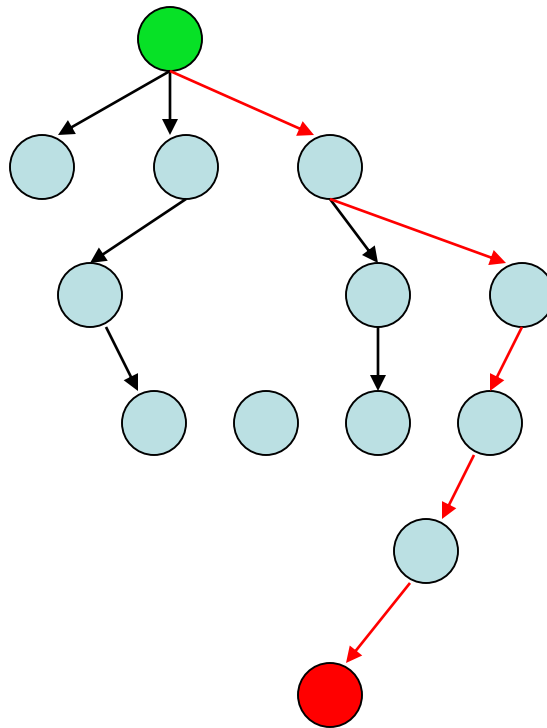
# HW 1: Warmup

## Missionaries and Cannibals

- Solve the Missionary-Cannibal Problem (with 3 missionaries and 3 cannibals) with a **RECURSIVE DEPTH-FIRST SEARCH** as follows:
  - You **MUST** use a **recursive** depth first search
  - No ancestor repeated states in a path
  - Keep counts of illegal states (cannibals eat missionaries), repeated states, total states searched
  - Use Python
  - Comment on each method and important code sections
  - Print all paths from start to goal
  - Print the final 3 counts.
- Due Jan 12 11:59pm. Late date Jan 14 11:59pm
- Your work must be **YOUR OWN**.

# Informed (Heuristic) Search

Idea: be **smart**  
about what paths  
to try.



# Blind Search vs. Informed Search

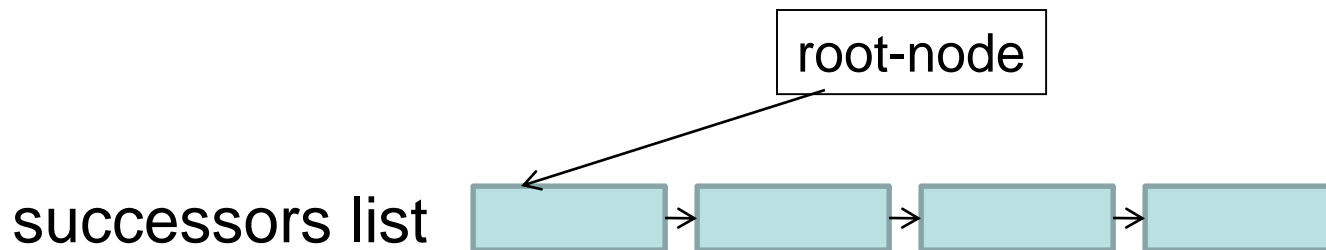
- What's the difference?

- How do we formally specify this?

A node is selected for expansion based on an evaluation function that estimates cost to goal.

# General Tree Search Paradigm

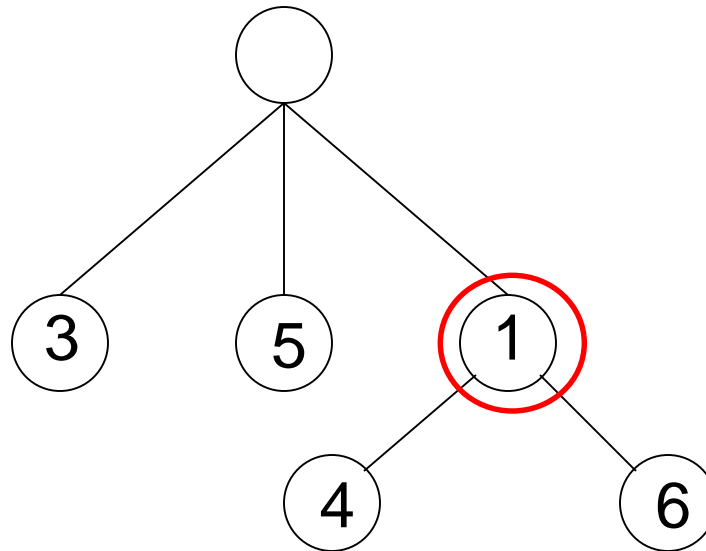
```
function tree-search(root-node)
  fringe ← successors(root-node)
  while ( notempty(fringe) )
    {node ← remove-first(fringe)
     state ← state(node)
     if goal-test(state) return solution(node)
     fringe ← insert-all(successors(node),fringe) }
  return failure
end tree-search
```



How do we order the successor list?

# Best-First Search

- Use an **evaluation function  $f(n)$**  for node  $n$ .
- Always choose the node from fringe that has the **lowest**  $f$  value.



# Heuristics

- What is a heuristic?
- What are some examples of heuristics we use?
- We'll call the heuristic function  $h(n)$ .

# Greedy Best-First Search

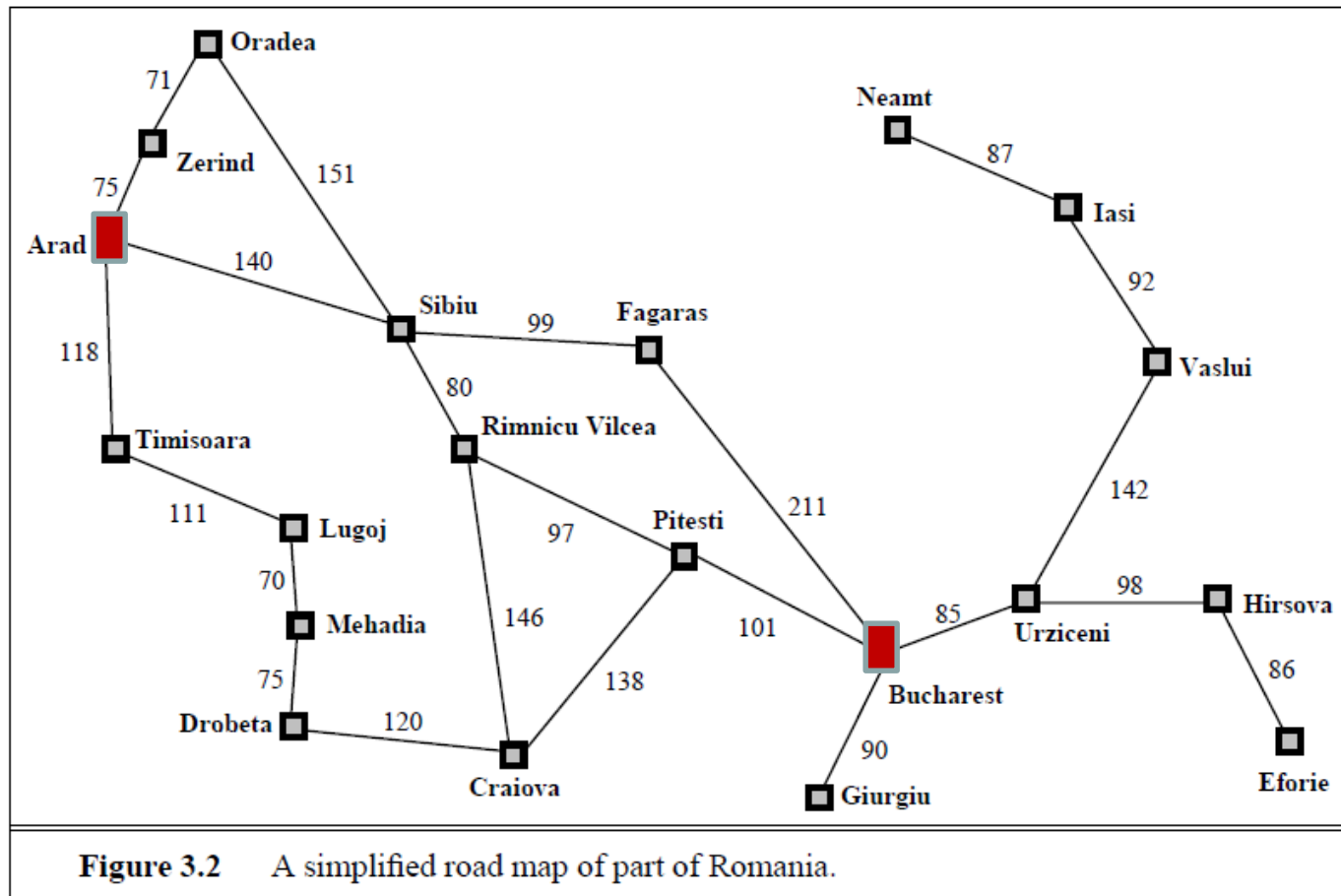
- $f(n) = h(n)$
- What does that mean?
- What is it ignoring?

# Romanian Route Finding

- **Problem**
  - Initial State: Arad
  - Goal State: Bucharest
  - $c(s,a,s')$  is the length of the road from  $s$  to  $s'$
- **Heuristic function:**  $h(s)$  = the straight line distance from  $s$  to Bucharest



# Original Road Map of Romania



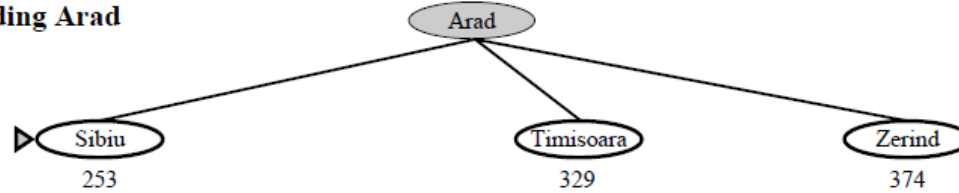
What's the real shortest path from Arad to Bucharest?  
What's the distance on that path?

# Greedy Search in Romania

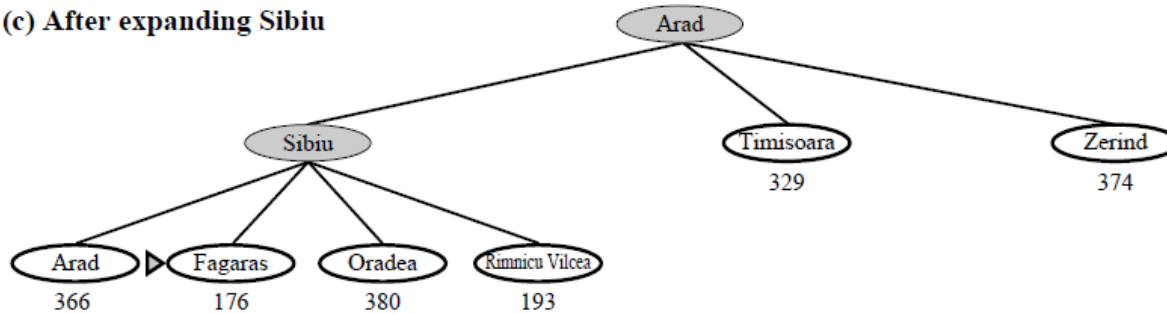
(a) The initial state



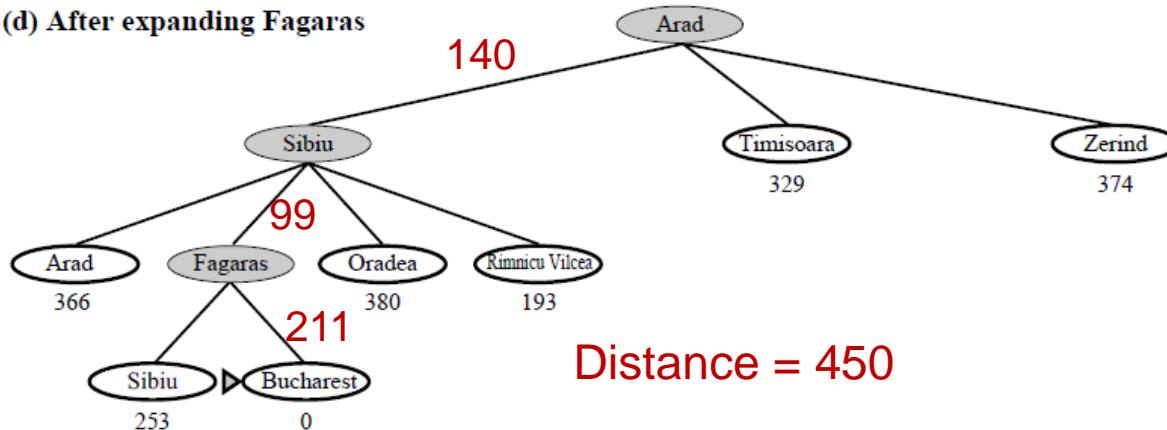
(b) After expanding Arad



(c) After expanding Sibiu



(d) After expanding Fagaras



# Greedy Best-First Search

- Is greedy search optimal?

- Is it complete?

No, can get into infinite loops in tree search.

Graph search is complete for finite spaces.

- What is its worst-case complexity for a tree search with branching factor  $b$  and maximum depth  $m$ ?

– time  $O(b^m)$

– space  $O(b^m)$

# Greedy Best-First Search

- When would we use greedy best-first search or greedy approaches in general?

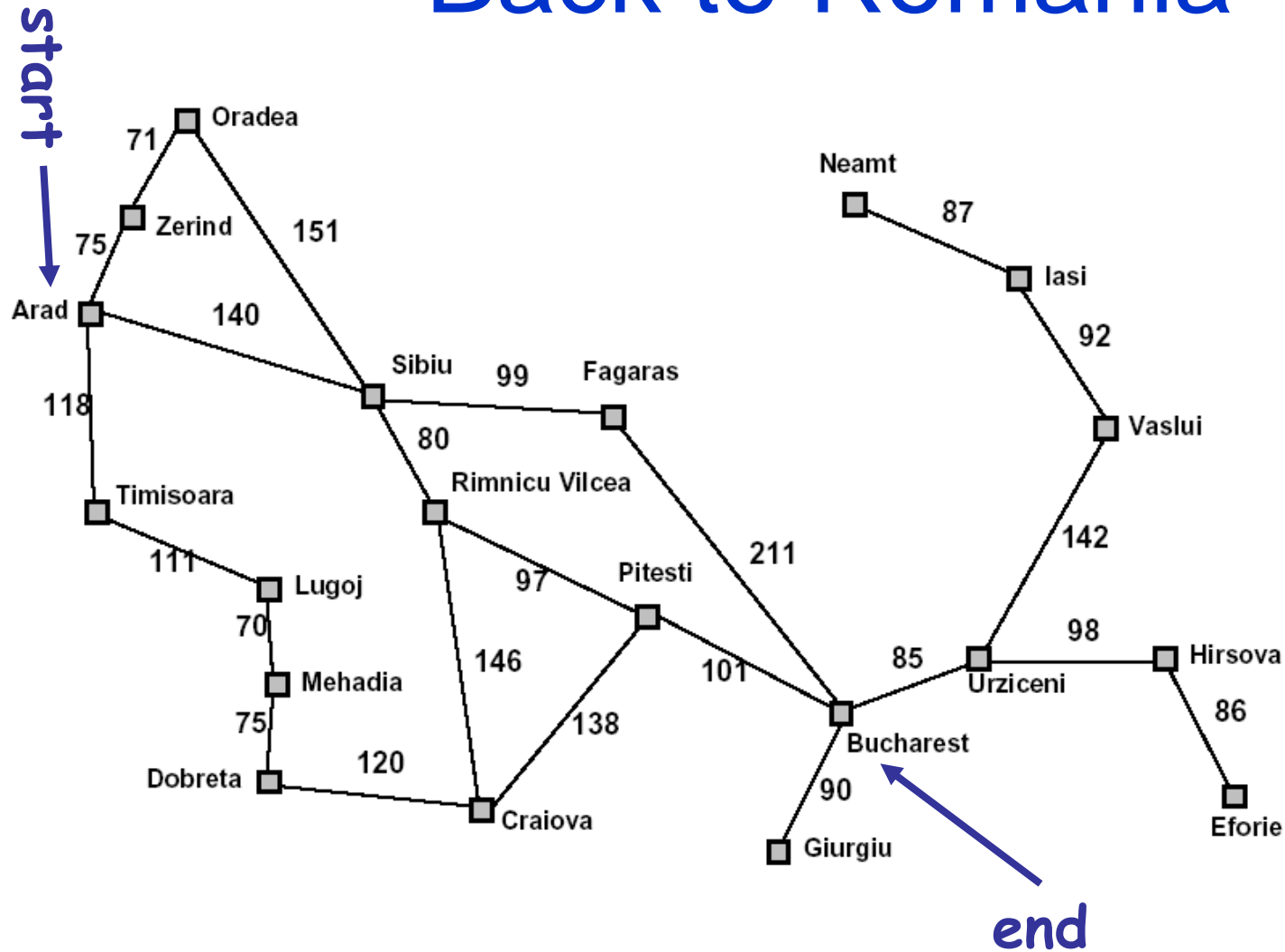
# A\* Search

- Hart, Nilsson & Rafael 1968
  - Best-first search with  $f(n) = g(n) + h(n)$   
where  $g(n)$  = sum of edge costs from start to  $n$   
and  $h(n)$  = estimate of lowest cost path  $n \rightarrow$  goal
  - If  $h(n)$  is **admissible** then search will find optimal solution.

↑ { Never overestimates the true cost of any solution which can be reached from a node.

Space bound since the queue must be maintained.

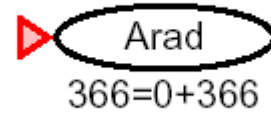
# Back to Romania



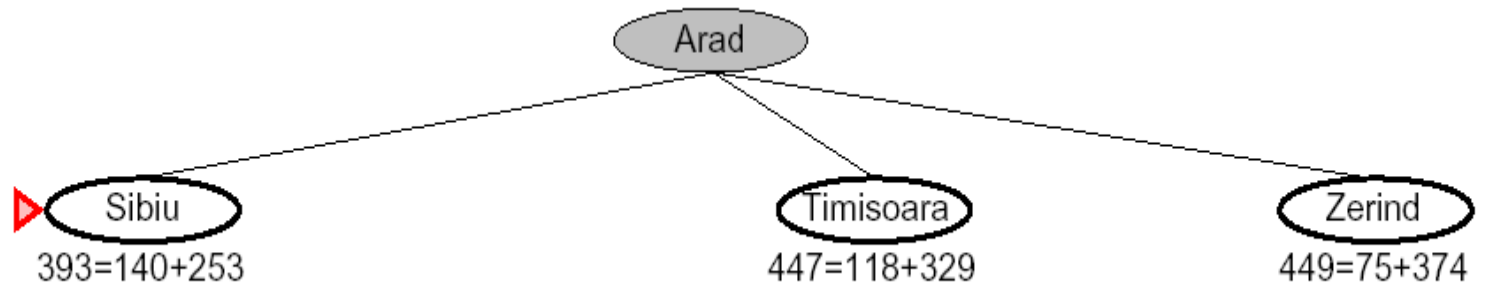
Straight-line distance to Bucharest

Arad	366
Bucharest	0
Craiova	160
Dobreta	242
Eforie	161
Fagaras	178
Giurgiu	77
Hirsova	151
Iasi	226
Lugoj	244
Mehadia	241
Neamt	234
Oradea	380
Pitesti	98
Rimnicu Vilcea	193
Sibiu	253
Timisoara	329
Urziceni	80
Vaslui	199
Zerind	374

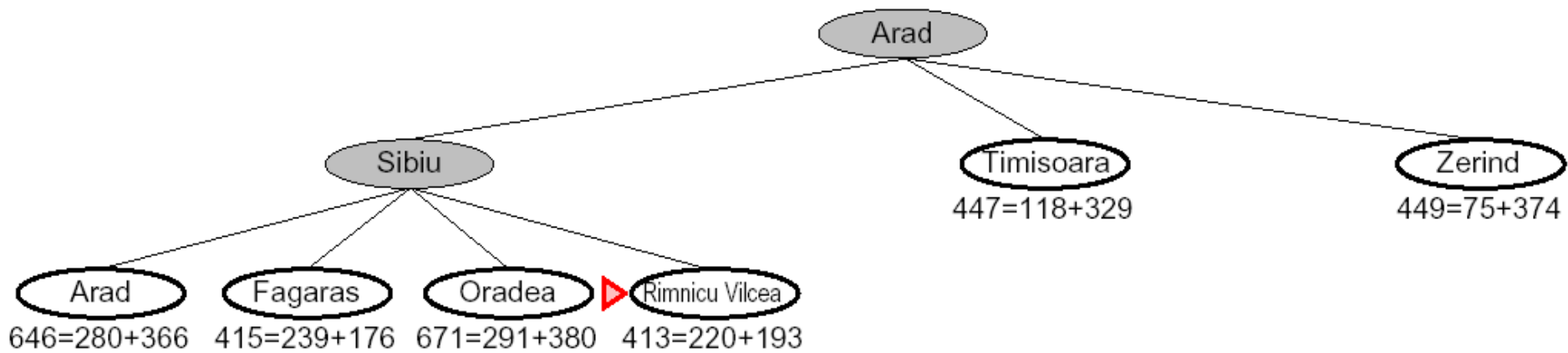
# A\* for Romanian Shortest Path

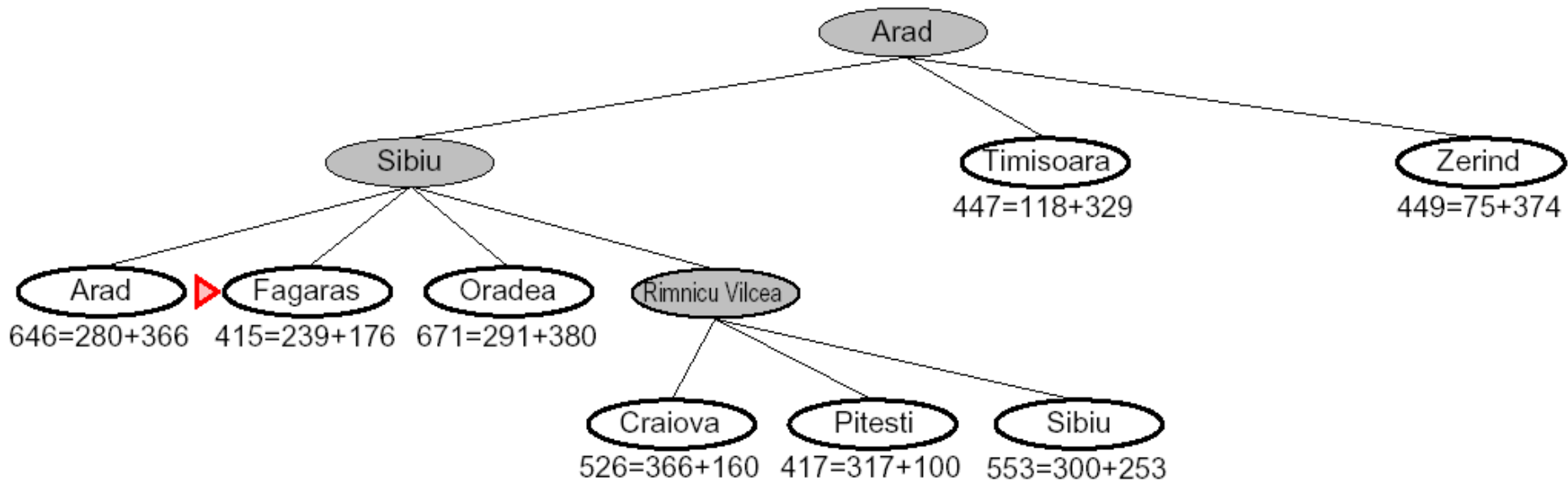


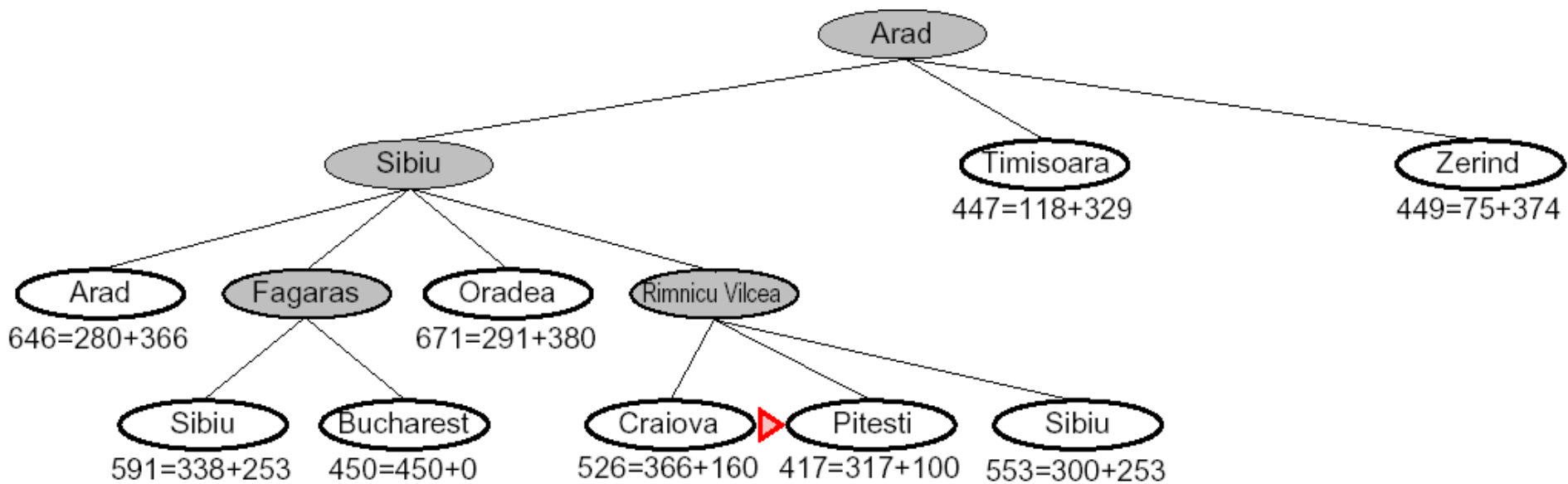
$$f(n) = g(n) + h(n)$$

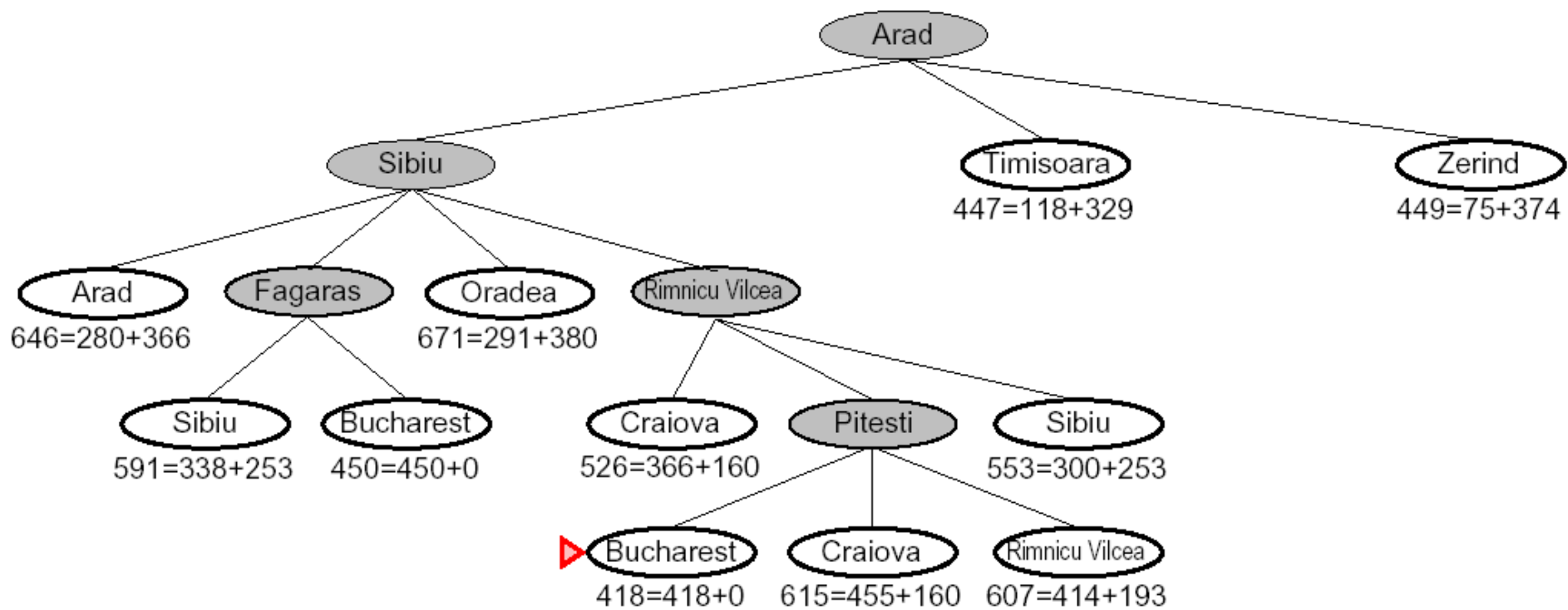












# 8 Puzzle Example

- $f(n) = g(n) + h(n)$
- What is the usual  $g(n)$ ?
- two well-known  $h(n)$ 's
  - $h_1$  = the number of misplaced tiles
  - $h_2$  = the sum of the distances of the tiles from their goal positions, using city block distance, which is the sum of the horizontal and vertical distances (Manhattan Distance)

# 8 Puzzle Using Number of Misplaced Tiles

1	2	3
8		4
7	6	5

goal

2	8	3
1	6	4
7		5

$g=0$

$h=4$

$f=4$

2	8	3
1		4
7	6	5

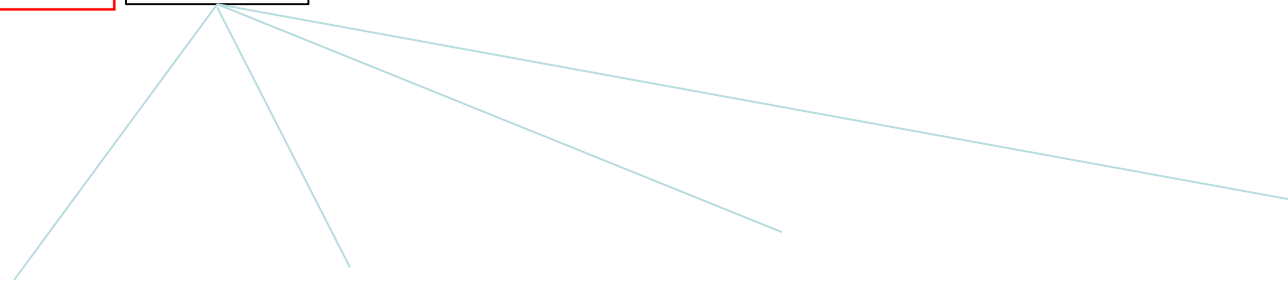
2	8	3
1	6	4
	7	5

2	8	3
1	6	4
7		5

1	2	3
8		4
7	6	5

2	8	3
1		4
7	6	5

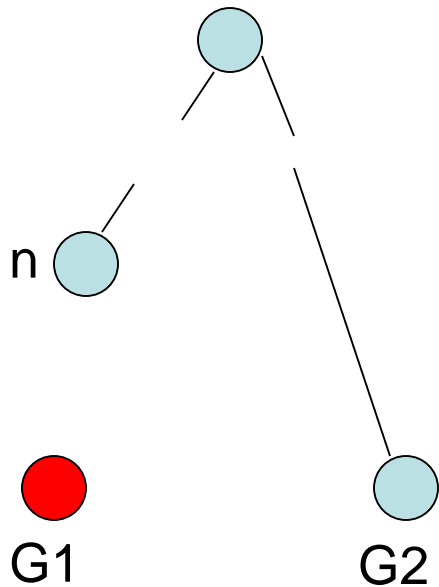
Exercise:  
What are its children and their  
f, g, h?



# Optimality of A\* with Admissibility

(h never overestimates the cost to the goal)

Suppose a suboptimal goal G2 has been generated and is in the queue. Let n be an unexpanded node on the shortest path to an optimal goal G1.



$$\begin{aligned} f(n) &= g(n) + h(n) \\ &\leq g(G1) \\ &< g(G2) \\ &= f(G2) \end{aligned}$$

Why?

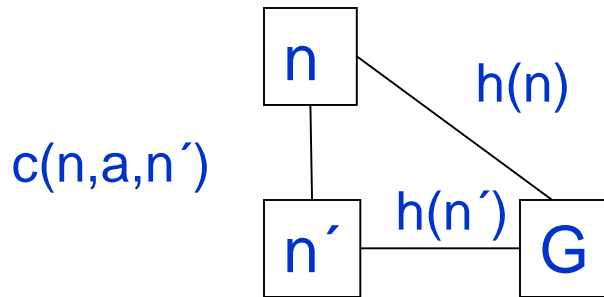
G2 is suboptimal  
 $f(G2) = g(G2)$

So  $f(n) < f(G2)$  and A\* will never select G2 for expansion.



# Optimality of A\* with Consistency (stronger condition)

- $h(n)$  is consistent if
  - for every node  $n$
  - for every successor  $n'$  due to legal action  $a$
  - $h(n) \leq c(n, a, n') + h(n')$



- Every consistent heuristic is also admissible.

# Algorithms for A\*

- Since Nilsson defined A\* search, many different authors have suggested algorithms.
- Using Tree-Search, the optimality argument holds, but you search too many states.
- Using Graph-Search, it can break down, because an optimal path to a **repeated state** can be discarded if it is not the first one found.
- One way to solve the problem is that whenever you come to a repeated node, discard the **longer** path to it.

# The Rich/Knight Implementation

- a **node** consists of
  - state
  - g, h, f values
  - list of successors
  - pointer to parent
- **OPEN** is the list of nodes that have been generated and had h applied, but not expanded and can be implemented as a priority queue.
- **CLOSED** is the list of nodes that have already been expanded.

# Rich/Knight

1) /\* Initialization \*/

OPEN <- start node

Initialize the start node

g:

h:

f:

CLOSED <- empty list

# Rich/Knight

2) repeat until goal (or time limit or space limit)

- if OPEN is empty, fail
- BESTNODE  $\leftarrow$  node on OPEN with lowest  $f$
- if BESTNODE is a goal, exit and succeed
- remove BESTNODE from OPEN and add it to CLOSED
- generate successors of BESTNODE

# Rich/Knight

for each successor  $s$  do

1. set its parent field

2. compute  $g(s)$

3. if there is a node **OLD** on OPEN with the same state info as  $s$

{ add **OLD** to successors(BESTNODE)

if  $g(s) < g(\text{OLD})$ , update **OLD** and

throw out  $s$  }

# Rich/Knight/Tanimoto

4. if ( $s$  is not on OPEN and there is a node **OLD** on CLOSED with the same state info as  $s$ )
  - { add **OLD** to successors(BESTNODE)
  - if  $g(s) < g(\text{OLD})$ , update **OLD**,
  - remove it from CLOSED
  - and put it on OPEN, throw out  $s$

# Rich/Knight

5. If  $s$  was not on OPEN or CLOSED

{ add  $s$  to OPEN

add  $s$  to successors(BESTNODE)

calculate  $g(s)$ ,  $h(s)$ ,  $f(s)$  }

end of repeat loop