Homework 5
Due on Jun 2, 2017

1. Bayes Net: Independence

Consider the Bayes Net shown below (Please use the right one for clarity. The left one is just an equivalent but cuter version). The following questions are worth 1 point each with a negative point for incorrect answers (don’t guess randomly). By independent we mean whether they are independent for any setting of the CPTs.

(a) Are A and B independent given C?
(b) Are A and H independent?
(c) Are A and H independent given E?
(d) Are E and F independent given H?
(e) Are E and F independent given C?
(f) Are E and F independent given C and D?
(g) Are A and F independent given C and H?
(h) Are A and F independent given C and D?
(i) Are A and F independent given C and G?
(j) Are A and F independent given C?
(k) Are C and G independent given H?

2. **Bayes Net: Inference**

Below you see the structure of a Bayesian network.

(a) What are the probability distributions that have to be specified in order to completely define the network?

(b) How can you compute \( P(D \mid A = a, E = e) \)? Describe the individual steps of your reasoning. You can assume that all variables are discrete, please use the following notation if you want to sum over the values of a variable, for example, \( X: \sum_x P(X = x) \). You should define new factors such as \( f_2(X, y) = \sum_z f_1(X, y, Z = z)P(X \mid Z = z) \). When you eliminate variables, please do so in alphabetical order.
3. Probabilities

Consider the joint probability distribution below.

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>P(A,B,C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>false</td>
<td>false</td>
<td>false</td>
<td>0.2</td>
</tr>
<tr>
<td>false</td>
<td>false</td>
<td>true</td>
<td>0.05</td>
</tr>
<tr>
<td>false</td>
<td>true</td>
<td>false</td>
<td>0.2</td>
</tr>
<tr>
<td>false</td>
<td>true</td>
<td>true</td>
<td>0.05</td>
</tr>
<tr>
<td>true</td>
<td>false</td>
<td>false</td>
<td>0.1</td>
</tr>
<tr>
<td>true</td>
<td>false</td>
<td>true</td>
<td>0.15</td>
</tr>
<tr>
<td>true</td>
<td>true</td>
<td>false</td>
<td>0.1</td>
</tr>
<tr>
<td>true</td>
<td>true</td>
<td>true</td>
<td>0.15</td>
</tr>
</tbody>
</table>

(a) What is $P(A = true)$? Provide the individual terms involved in this probability.

(b) What is $P(A = false \mid B = true)$? Provide the individual terms involved in this probability.

(c) Are A and B independent, that is, $A \perp B$? Justify your answer.
4. (Optional, not graded) Hidden Markov Models

Consider a Hidden Markov Model where the hidden state $X_t$ can be one of three values \{A, B, C\}. The transition probabilities are provided in the following table, where the row corresponds to $X_{t-1}$ and the column to $X_t$.

\[
\begin{array}{ccc}
   & A & B & C \\
A & 0.7 & 0.3 & 0 \\
B & 0.1 & 0.7 & 0.2 \\
C & 0 & 0.4 & 0.6 \\
\end{array}
\]

For example, $P(X_t = A \mid X_{t-1} = B) = 0.1$.

The noisy sensor model for evidence $E_t$ corresponding to $X_t$ gives the true hidden state with probability 0.8, and one of the other two states each with probability 0.1. For example, $P(E_t = B \mid X_t = A) = 0.1$.

(a) Assume our belief about the hidden state $X_t$ is

\[
\begin{array}{c}
   X_t \\
A & 0.5 \\
B & 0.5 \\
C & 0 \\
\end{array}
\]

Compute the belief about the hidden state $X_{t+1}$ before considering noisy evidence (no need to normalize):

\[
\begin{array}{c}
   X_{t+1} \\
A \\
B \\
C \\
\end{array}
\]

(b) Given your answer from the previous question, now assume we have the noisy sensor reading $E_{t+1} = C$. Compute our posterior belief taking this evidence into account (no need to normalize):

\[
\begin{array}{c}
   X_{t+1} \\
A \\
B \\
C \\
\end{array}
\]

continued on next page
(c) Assume now we are using a particle filter with 3 particles to approximate our belief instead of using exact inference. Imagine we have just applied transition model sampling (elapse-time) from state $X_t$ to $X_{t+1}$, and now have the set of particles \{A, A, B\}. What is our belief about $X_{t+1}$ before considering noisy evidence?

<table>
<thead>
<tr>
<th>$X_{t+1}$</th>
<th>$P(X_{t+1})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td></td>
</tr>
<tr>
<td>B</td>
<td></td>
</tr>
<tr>
<td>C</td>
<td></td>
</tr>
</tbody>
</table>

(d) Now assume we receive sensor evidence $E_{t+1} = B$. What is the weight for each particle, and what is our belief now about $X_{t+1}$ (before weighted resampling)?

<table>
<thead>
<tr>
<th>Particle</th>
<th>Weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td></td>
</tr>
<tr>
<td>$A$</td>
<td></td>
</tr>
<tr>
<td>$B$</td>
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</tbody>
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<td>C</td>
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</table>

(e) Will performing weighted resampling on these weighted particles to obtain our final three particle representation for $X_{t+1}$ cause our belief to change? Briefly explain why or why not.
5. **(Optional, not graded) Create Bayes Net**

Create a Bayes net with exactly four states \{A,B,C,D\}, that follows all of the independence constraints below.

(a) \( A \perp\!\!\!\!\perp B \)

(b) \( A \not\perp\!\!\!\!\perp D|B \)

(c) \( A \perp\!\!\!\!\perp D|C \)

(d) \( A \not\perp\!\!\!\!\perp C \)

(e) \( B \not\perp\!\!\!\!\perp C \)

(f) \( A \not\perp\!\!\!\!\perp B|D \)

(g) \( B \perp\!\!\!\!\perp D|A,C \)