CSE 473: Artificial Intelligence

Uncertainty and Expectimax Tree Search

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[These slides were adapted from Dan Klein and Pieter Abbeel for CS188 Intro to AI at UC Berkeley. All CS188 materials are available at http://ai.berkeley.edu]
Uncertain Outcomes
Worst-Case vs. Average Case

Idea: Uncertain outcomes controlled by chance, not an adversary!
Expectimax Search

- Why wouldn’t we know what the result of an action will be?
  - Explicit randomness: rolling dice
  - Unpredictable opponents: the ghosts respond randomly
  - Actions can fail: when moving a robot, wheels might slip

- Values should now reflect average-case (expectimax) outcomes, not worst-case (minimax) outcomes

- Expectimax search: compute the average score under optimal play
  - Max nodes as in minimax search
  - Chance nodes are like min nodes but the outcome is uncertain
  - Calculate their expected utilities
  - I.e. take weighted average (expectation) of children

- Later, we’ll learn how to formalize the underlying uncertain-result problems as Markov Decision Processes

[Demo: min vs exp (L7D1,2)]
Video of Demo Min vs. Exp (Min)
Video of Demo Min vs. Exp (Exp)
def value(state):
    if the state is a terminal state: return the state’s utility
    if the next agent is MAX: return max-value(state)
    if the next agent is EXP: return exp-value(state)

def max-value(state):
    initialize v = -\infty
    for each successor of state:
        v = max(v, value(successor))
    return v

def exp-value(state):
    initialize v = 0
    for each successor of state:
        p = probability(successor)
        v += p * value(successor)
    return v
def exp-value(state):
    initialize v = 0
    for each successor of state:
        p = probability(successor)
        v += p * value(successor)
    return v

v = (1/2) (8) + (1/3) (24) + (1/6) (-12) = 10
Expectimax Example
Expectimax Pruning?
Depth-Limited Expectimax

Estimate of true expectimax value (which would require a lot of work to compute)
Probabilities
A random variable represents an event whose outcome is unknown
A probability distribution is an assignment of weights to outcomes

Example: Traffic on freeway
- Random variable: $T =$ whether there’s traffic
- Outcomes: $T$ in {$\text{none}$, $\text{light}$, $\text{heavy}$}
- Distribution: $P(T=\text{none}) = 0.25$, $P(T=\text{light}) = 0.50$, $P(T=\text{heavy}) = 0.25$

Some laws of probability (more later):
- Probabilities are always non-negative
- Probabilities over all possible outcomes sum to one

As we get more evidence, probabilities may change:
- $P(T=\text{heavy}) = 0.25$, $P(T=\text{heavy} \mid \text{Hour=8am}) = 0.60$
- We’ll talk about methods for reasoning and updating probabilities later
The expected value of a function of a random variable is the average, weighted by the probability distribution over outcomes.

Example: How long to get to the airport?

<table>
<thead>
<tr>
<th>Time</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>20 min</td>
<td>0.25</td>
</tr>
<tr>
<td>30 min</td>
<td>0.50</td>
</tr>
<tr>
<td>60 min</td>
<td>0.25</td>
</tr>
</tbody>
</table>

\[ 20 \text{ min} \times 0.25 + 30 \text{ min} \times 0.50 + 60 \text{ min} \times 0.25 = 35 \text{ min} \]
What Probabilities to Use?

- In expectimax search, we have a probabilistic model of how the opponent (or environment) will behave in any state:
  - Model could be a simple uniform distribution (roll a die)
  - Model could be sophisticated and require a great deal of computation
  - We have a chance node for any outcome out of our control: opponent or environment
  - The model might say that adversarial actions are likely!

- For now, assume each chance node magically comes along with probabilities that specify the distribution over its outcomes

*Having a probabilistic belief about another agent’s action does not mean that the agent is flipping any coins!*
Let’s say you know that your opponent is actually running a depth 2 minimax, using the result 80% of the time, and moving randomly otherwise.

**Question:** What tree search should you use?

**Answer:** Expectimax!

- To figure out EACH chance node’s probabilities, you have to run a simulation of your opponent.
- This kind of thing gets very slow very quickly.
- Even worse if you have to simulate your opponent simulating you...
- ... except for minimax, which has the nice property that it all collapses into one game tree.
Modeling Assumptions
The Dangers of Optimism and Pessimism

Dangerous Optimism
Assuming chance when the world is adversarial

Dangerous Pessimism
Assuming the worst case when it’s not likely
Assumptions vs. Reality

Pacman used depth 4 search with an eval function that avoids trouble.
Ghost used depth 2 search with an eval function that seeks Pacman.

Results from playing 5 games:

<table>
<thead>
<tr>
<th></th>
<th>Adversarial Ghost</th>
<th>Random Ghost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Minimax Pacman</td>
<td>Won 5/5</td>
<td>Won 5/5</td>
</tr>
<tr>
<td></td>
<td>Avg. Score: 483</td>
<td>Avg. Score: 493</td>
</tr>
<tr>
<td>Expectimax Pacman</td>
<td>Won 1/5</td>
<td>Won 5/5</td>
</tr>
<tr>
<td></td>
<td>Avg. Score: -303</td>
<td>Avg. Score: 503</td>
</tr>
</tbody>
</table>

[Demos: world assumptions (L7D3,4,5,6)]
Video of Demo World Assumptions
Random Ghost – Expectimax Pacman
Video of Demo World Assumptions
Adversarial Ghost – Minimax Pacman
Video of Demo World Assumptions
Adversarial Ghost – Expectimax Pacman
Video of Demo World Assumptions
Random Ghost – Minimax Pacman
Other Game Types
Mixed Layer Types

- E.g. Backgammon
- Expectiminimax
  - Environment is an extra “random agent” player that moves after each min/max agent
  - Each node computes the appropriate combination of its children
Example: Backgammon

- Dice rolls increase $b$: 21 possible rolls with 2 dice
  - Backgammon $\approx$ 20 legal moves
  - Depth 2 = $20 \times (21 \times 20)^3 = 1.2 \times 10^9$

- As depth increases, probability of reaching a given search node shrinks
  - So usefulness of search is diminished
  - So limiting depth is less damaging
  - But pruning is trickier...

- Historic AI: TDGammon uses depth-2 search + very good evaluation function + reinforcement learning: world-champion level play

- 1st AI world champion in any game!
Multi-Agent Utilities

- What if the game is not zero-sum, or has multiple players?

- Generalization of minimax:
  - Terminals have utility tuples
  - Node values are also utility tuples
  - Each player maximizes its own component
  - Can give rise to cooperation and competition dynamically...

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1,6,6  7,1,2  6,1,2  7,2,1  5,1,7  1,5,2  7,7,1  5,2,5
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