CSE 473: Artificial Intelligence
Spring 2015

Adversarial Search

Based on slides adapted Luke Zettlemoyer, Dan Klein, Pieter Abbeel, Dan Weld, Stuart Russell or Andrew Moore

Adversarial Search

Game Playing State-of-the-Art

- Checkers: Chinook ended 40-year-reign of human world champion Marion Tinsley in 1994. Used an endgame database defining perfect play for all positions involving 8 or fewer pieces on the board, a total of 443,748,401,247 positions. Checkers is now solved!
- Chess: Deep Blue defeated human world champion Gary Kasparov in a six-game match in 1997. Deep Blue examined 200 million positions per second, used very sophisticated evaluation and undisclosed methods for extending some lines of search up to 40 ply. Current programs are even better, if less historic.
- Othello: Human champions refuse to compete against computers, which are too good.
- Go: Human champions are beginning to be challenged by machines, though the best humans still beat the best machines. In go, \( b > 300 \), so most programs use pattern knowledge bases to suggest plausible moves, along with aggressive pruning.
- Pacman: unknown

Adversarial Search

Game Playing

- Many different kinds of games!
- Choices:
  - Deterministic or stochastic?
  - One, two, or more players?
  - Perfect information (can you see the state)?
- Want algorithms for calculating a strategy (policy) which recommends a move in each state

Deterministic Games

- Many possible formalizations, one is:
  - States: \( S \) (start at \( s_0 \))
  - Players: \( P = \{1 \ldots N\} \) (usually take turns)
  - Actions: \( A \) (may depend on player / state)
  - Transition Function: \( S \times A \rightarrow S \)
  - Terminal Test: \( S \rightarrow \{t,f\} \)
  - Terminal Utilities: \( S \times P \rightarrow R \)
- Solution for a player is a policy: \( S \rightarrow A \)

Zero-Sum Games

- Zero-Sum Games
  - Agents have opposite utilities (values on outcomes)
  - Lets us think of a single value that one maximizes and the other minimizes
  - Adversarial, pure competition
- General Games
  - Agents have independent utilities (values on outcomes)
  - Cooperation, indifference, competition, & more are possible
Single-Agent Trees

Value of a State

Non-Terminal States:
- The best achievable outcome (utility)
- Value of a state: $V(s) = \max_{\pi(\text{successor}(s))} V'(\pi')$
- Terminal States: $V(s) = \text{final result}$

Adversarial Game Trees

Minimax Values

States Under Agent’s Control:
- $V(s) = \max_{\pi(\text{successor}(s))} V'(\pi')$
- Terminal States:
- $V(s) = \text{final result}$

Tic-tac-toe Game Tree

Adversarial Search (Minimax)

- Deterministic, zero-sum games:
- Tic-tac-toe, chess, checkers
- One player maximizes result
- The other minimizes result

- Minimax search:
- A state-space search tree
- Players alternate turns
- Compute each node’s minimax value: the best achievable utility against a rational (optimal) adversary

Slide from Dan Klein & Pieter Abbeel - ai.berkeley.edu
Minimax Implementation

```python
def max_value(state):
    v = +\infty
    for each successor of state:
        v = min(v, max_value(successor))
    return v

def min_value(state):
    v = -\infty
    for each successor of state:
        v = max(v, min_value(successor))
    return v
```

Concrete Minimax Example

```
max

min
```

Minimax Properties

- Optimal?
  - Yes, against perfect player. Otherwise?
- Time complexity
  - \(O(b^m)\)
- Space complexity?
  - \(O(bm)\)
- For chess, \(b \approx 35, m \approx 100\)
  - Exact solution is completely infeasible
  - But, do we need to explore the whole tree?

α-β Pruning

- General configuration
  - \(\alpha\) is the best value that MAX can get at any choice point along the current path
  - If \(n\) becomes worse than \(\alpha\), MAX will avoid it, so can stop considering \(n\)'s other children
  - Define \(\beta\) similarly for MIN
```

Progress of search...
Alpha-Beta Pruning
Alpha-Beta Pruning Properties

- This pruning has **no effect** on final result at the root
- Values of intermediate nodes might be wrong!
  - but, they are bounds
- Good child ordering improves effectiveness of pruning
- With “perfect ordering”:
  - Time complexity drops to $O(b^{m/2})$
  - Doubles solvable depth!
  - Full search of, e.g. chess, is still hopeless...

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**Alpha-Beta Implementation**

```python
def min_value(state, α, β):
    iniBalize
    v = +∞
    for each successor of state:
        v = min(v, value(successor, α, β))
        if v ≤ α return v
    α = max(α, v)
    return v
```

```
def max_value(state, α, β):
    iniBalize
    v = -∞
    for each successor of state:
        v = max(v, value(successor, α, β))
        if v ≥ β return v
    β = min(β, v)
    return v
```

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[Slide from Dan Klein & Pieter Abbeel - ai.berkeley.edu]