CSE 473: Artificial Intelligence
Spring 2014

Hidden Markov Models & Particle Filtering

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Many slides adapted from Dan Weld, Pieter Abbeel, Dan Klein, Stuart Russell, Andrew Moore & Luke Zettlemoyer

Outline

- Probabilistic sequence models (and inference)
  - Probability and Uncertainty – Preview
  - Markov Chains
  - Hidden Markov Models
  - Exact Inference
  - Particle Filters
  - Applications

Example

- A robot move in a discrete grid
  - May fail to move in the desired direction with some probability
- Observation from noisy sensor at each time
  - Is a function of robot position
- Goal: Find the robot position (probability that a robot is at a specific position)
- Cannot always compute this probability exactly

➔ Approximation methods
Here: Approximate a distribution by sampling

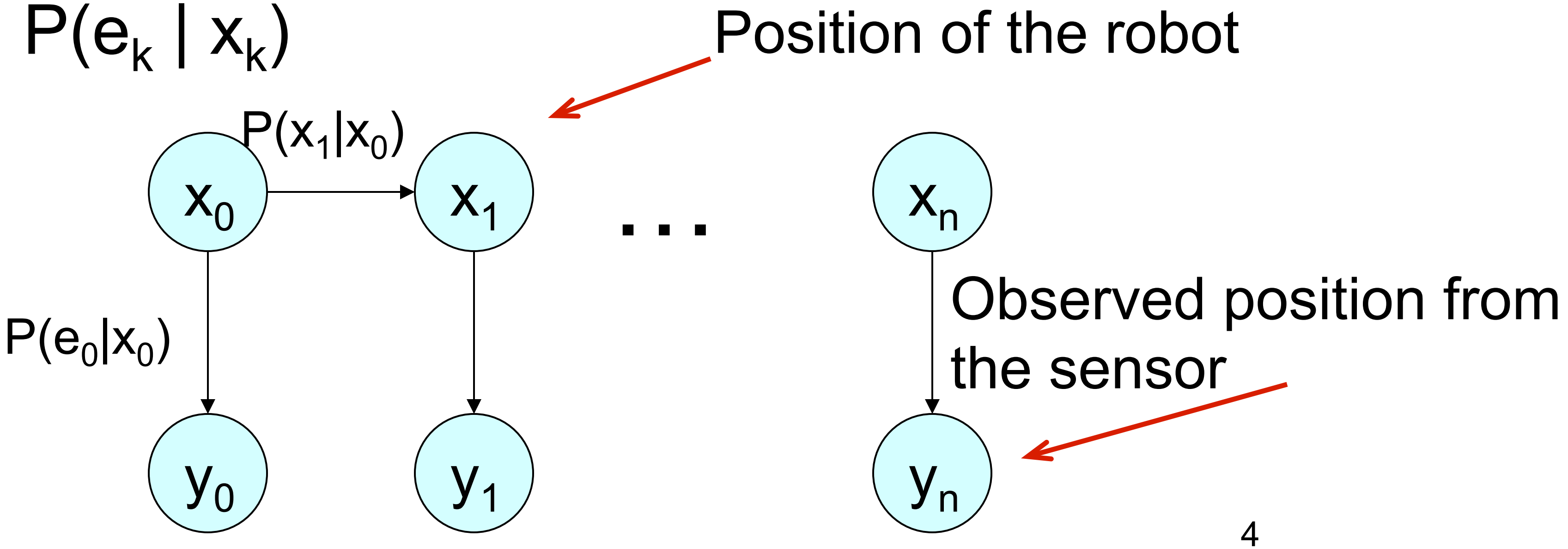
Hidden Markov Model

- State Space Model
  - Hidden states: Modeled as a Markov Process

$P(x_0)$, $P(x_k \mid x_{k-1})$

  - Observations: e_k

$P(e_k \mid x_k)$



Exact Solution: Forward Algorithm

- Filtering is the inference process of finding a distribution over X_T given e_1 through e_T : P(X_T | $e_{1:t}$)
- We first compute P(X_1 | e_1): $P(x_1|e_1) \propto P(x_1) \cdot P(e_1|x_1)$
- For each t from 2 to T, we have P(X_{t-1} | $e_{1:t-1}$)
- Elapse time: compute P(X_t | $e_{1:t-1}$)

$$P(x_t|e_{1:t-1}) = \sum_{x_{t-1}} P(x_{t-1}|e_{1:t-1}) \cdot P(x_t|x_{t-1})$$

- Observe: compute P(X_t | $e_{1:t-1}$, e_t) = P(X_t | $e_{1:t}$)

$$P(x_t|e_{1:t}) \propto P(x_t|e_{1:t-1}) \cdot P(e_t|x_t)$$

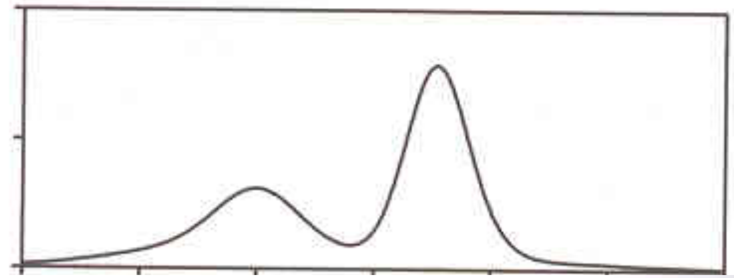
Approximate Inference:

- Sometimes |X| is too big for exact inference
  - |X| may be too big to even store B(X)
  - E.g. when X is continuous
  - $|X|^2$ may be too big to do updates

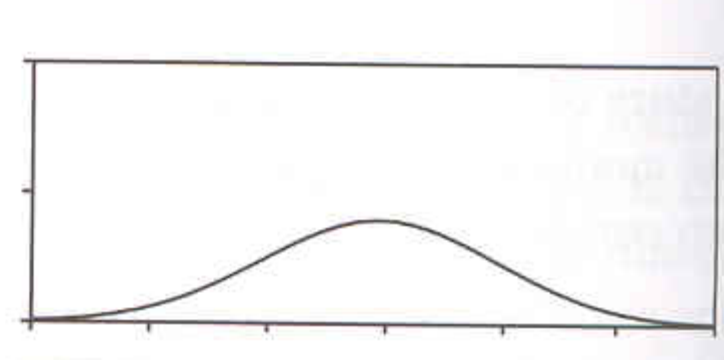
- Solution: approximate inference by sampling
- How robot localization works in practice

What is Sampling?

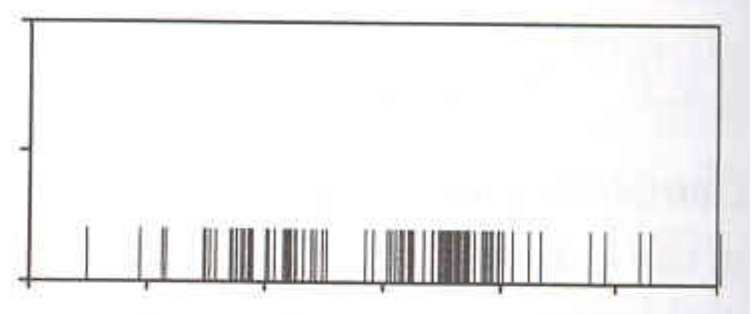
- Goal: Approximate the original distribution:



- Approximate with Gaussian distribution



- Draw samples from a distribution close enough to the original distribution
- Here: A general framework for a sampling method

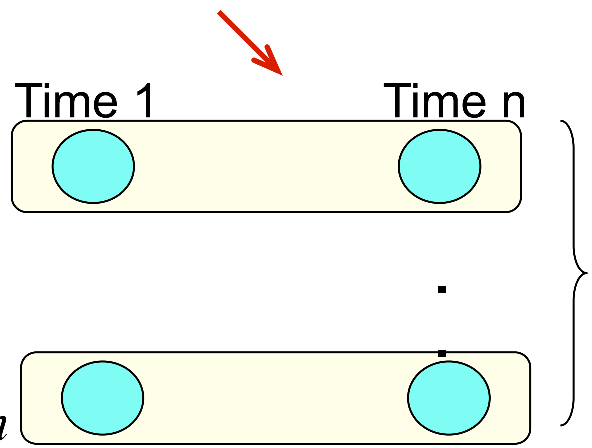


Approximate Solution: Perfect Sampling

Robot path till time n

Particle 1 $x^1_{0:n}$

Particle N $x^N_{0:n}$



Assume we can sample from the original distribution

$$p(x_{0:n} \mid y_{0:n})$$

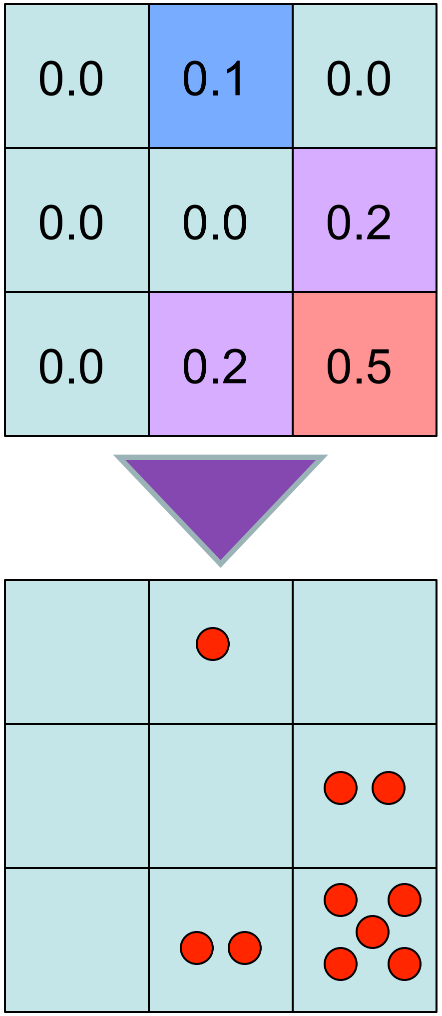
$$P(x_{0:n} \mid y_{0:n}) = \frac{1}{N} \left(\text{Number of samples that match with query} \right)$$

Converges to the exact value for large N

Approximate Inference: Particle Filtering

- Solution: approximate inference
  - Track samples of X, not all values
  - Samples are called ***particles***
  - Time per step is linear in the number of samples
  - But: number needed may be large
  - In memory: list of particles, not states

- How robot localization works in practice

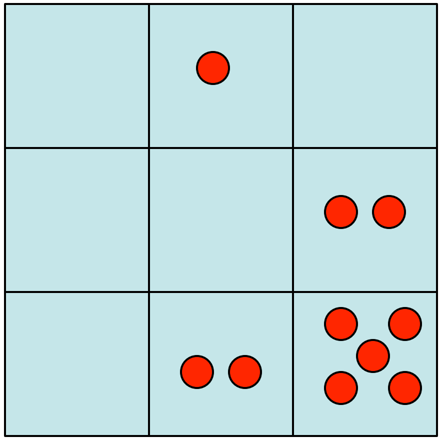


Representation: Particles

- Our representation of P(X) is now a list of N particles (samples)
  - ***Generally, N << |X|***
  - Storing map from X to counts would defeat the point

- P(x) approximated by number of particles with value x
  - So, many x will have P(x) = 0!
  - More particles, more accuracy

- For now, all particles have a weight of 1



Particles:
(3,3)
(2,3)
(3,3)
(3,2)
(3,3)
(3,2)
(2,1)
(3,3)
(3,3)
(2,1)

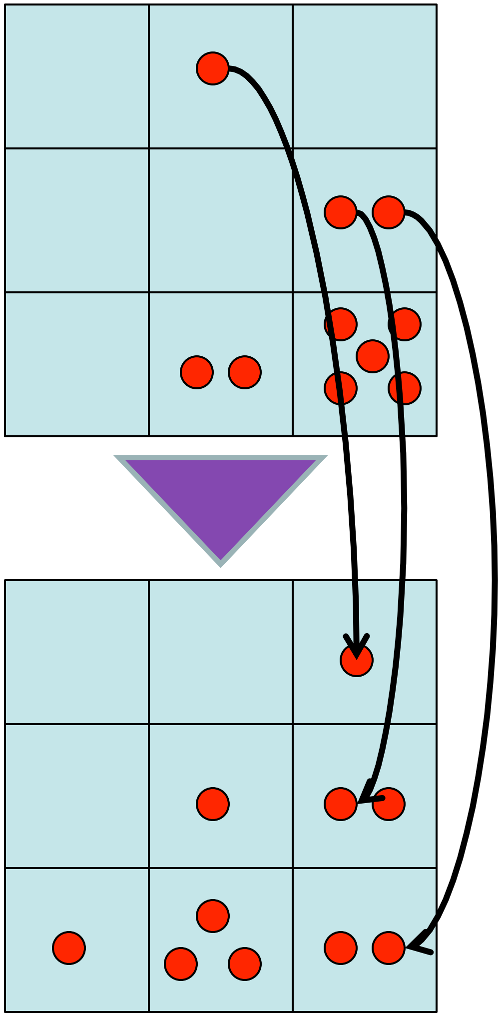
Particle Filtering: Elapse Time

- Each particle is moved by sampling its next position from the transition model

$$x' = \text{sample}(P(X'|x))$$

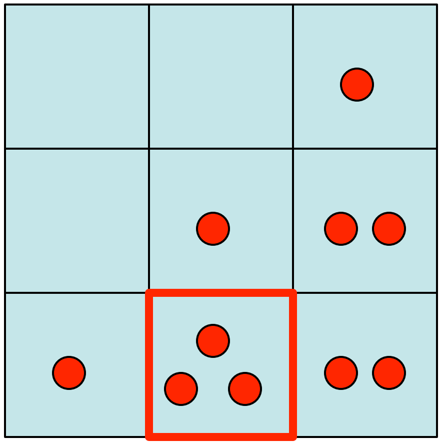
  - This is like prior sampling – samples' frequencies reflect the transition probs
  - Here, most samples move clockwise, but some move in another direction or stay in place

- This captures the passage of time
  - If we have enough samples, close to the exact values before and after (consistent)



Particle Filtering: Observe

- How handle noisy observations?
- Suppose sensor gives red reading?



Particle Filtering: Observe

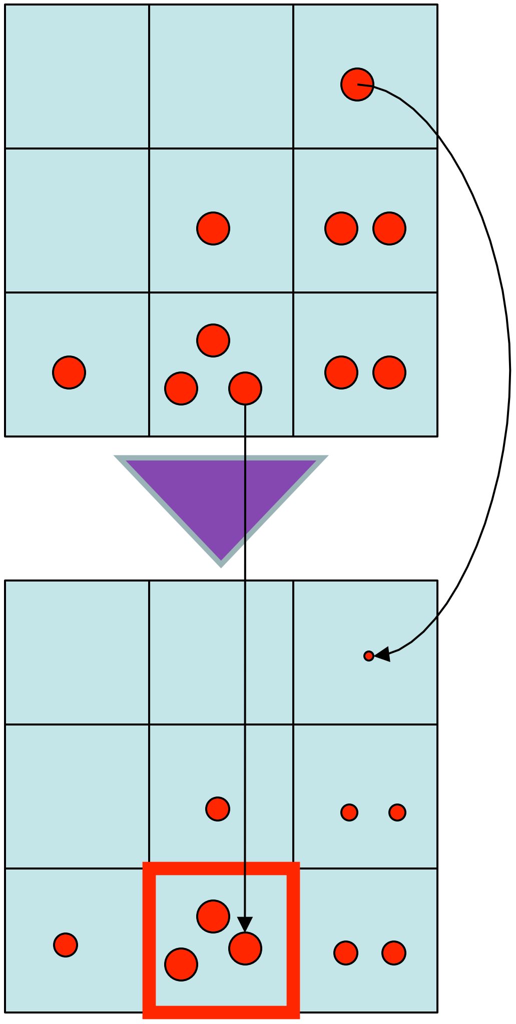
Slightly trickier:

- We don't sample the observation, we fix it
- Instead: ***downweight samples*** based on the evidence (form of likelihood weighting)

$$w(x) = P(e|x)$$

$$B(X) \propto P(e|X)B'(X)$$

- Note: as before, probabilities ***don't sum to one***, since most have been downweighted (in fact they sum to an approximation of P(e))

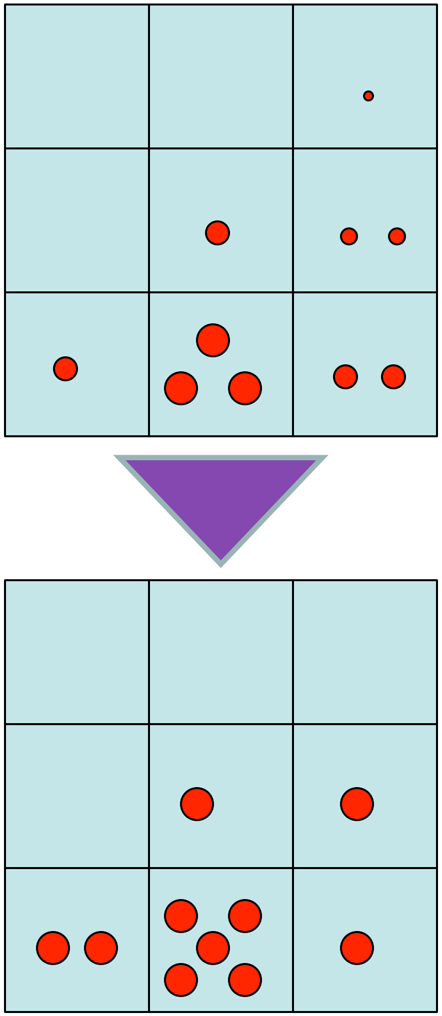


Particle Filtering: Resample

- Rather than tracking weighted samples, we resample
- N times, we choose from our weighted sample distribution (i.e. draw with replacement)
- This is equivalent to renormalizing the distribution
- Now the update is complete for this time step, continue with the next one

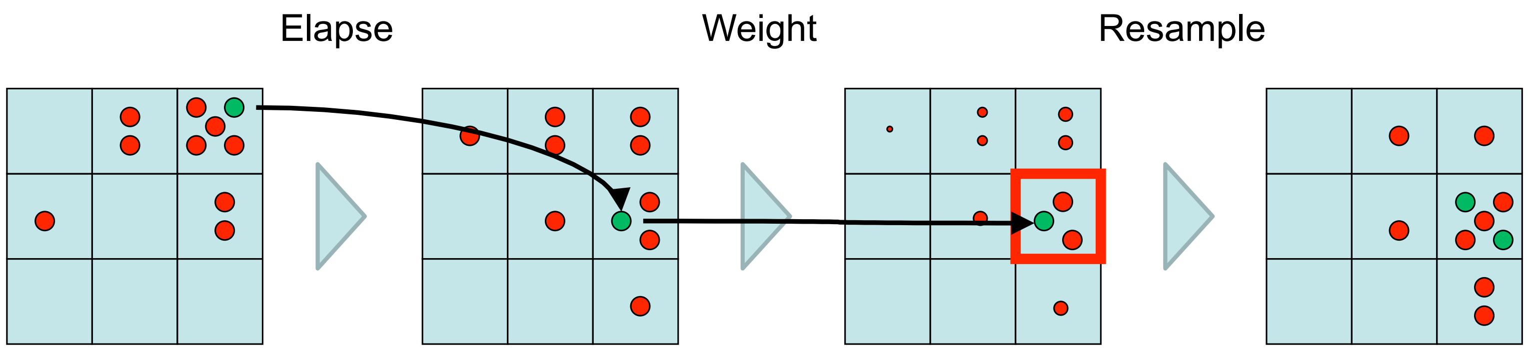
Old Particles:
(3,3) w=0.1
(2,1) w=0.9
(2,1) w=0.9
(3,1) w=0.4
(3,2) w=0.3
(2,2) w=0.4
(1,1) w=0.4
(3,1) w=0.4
(2,1) w=0.9
(3,2) w=0.3

New Particles:
(2,1) w=1
(2,1) w=1
(2,1) w=1
(3,2) w=1
(2,2) w=1
(2,1) w=1
(1,1) w=1
(3,1) w=1
(2,1) w=1
(1,1) w=1



Particle Filter (Recap)

- Particles: track samples of states rather than an explicit distribution



Particles:	Particles:	Particles:	(New) Particles:
(3,3)	(3,2)	(3,2) w=.9	(3,2)
(2,3)	(2,3)	(2,3) w=.2	(2,2)
(3,3)	(3,2)	(3,2) w=.9	(3,2)
(3,2)	(3,1)	(3,1) w=.4	(2,3)
(3,3)	(3,3)	(3,3) w=.4	(3,3)
(3,2)	(3,2)	(3,2) w=.9	(3,2)
(1,2)	(1,3)	(1,3) w=.1	(1,3)
(3,3)	(2,3)	(2,3) w=.2	(2,3)
(3,3)	(3,2)	(3,2) w=.9	(3,2)
(2,3)	(2,2)	(2,2) w=.4	(3,2)

Particle Filtering Summary

- Represent current belief P(X | evidence to date) as set of n samples (actual assignments X=x)
- For each new observation e:
  1. Sample transition, once for each current particle x

     $$x' = \text{sample}(P(X'|x))$$

  2. For each new sample x', compute importance weights for the new evidence e:

     $$w(x') = P(e|x')$$

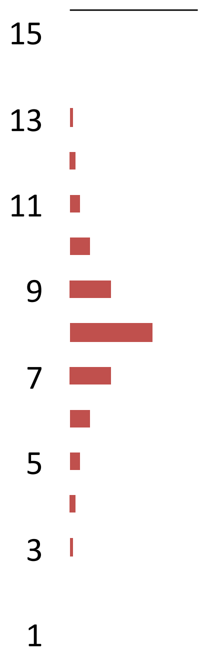
  3. Finally, normalize by resampling the importance weights to create N new particles

HMM Examples & Applications

P4: Ghostbusters

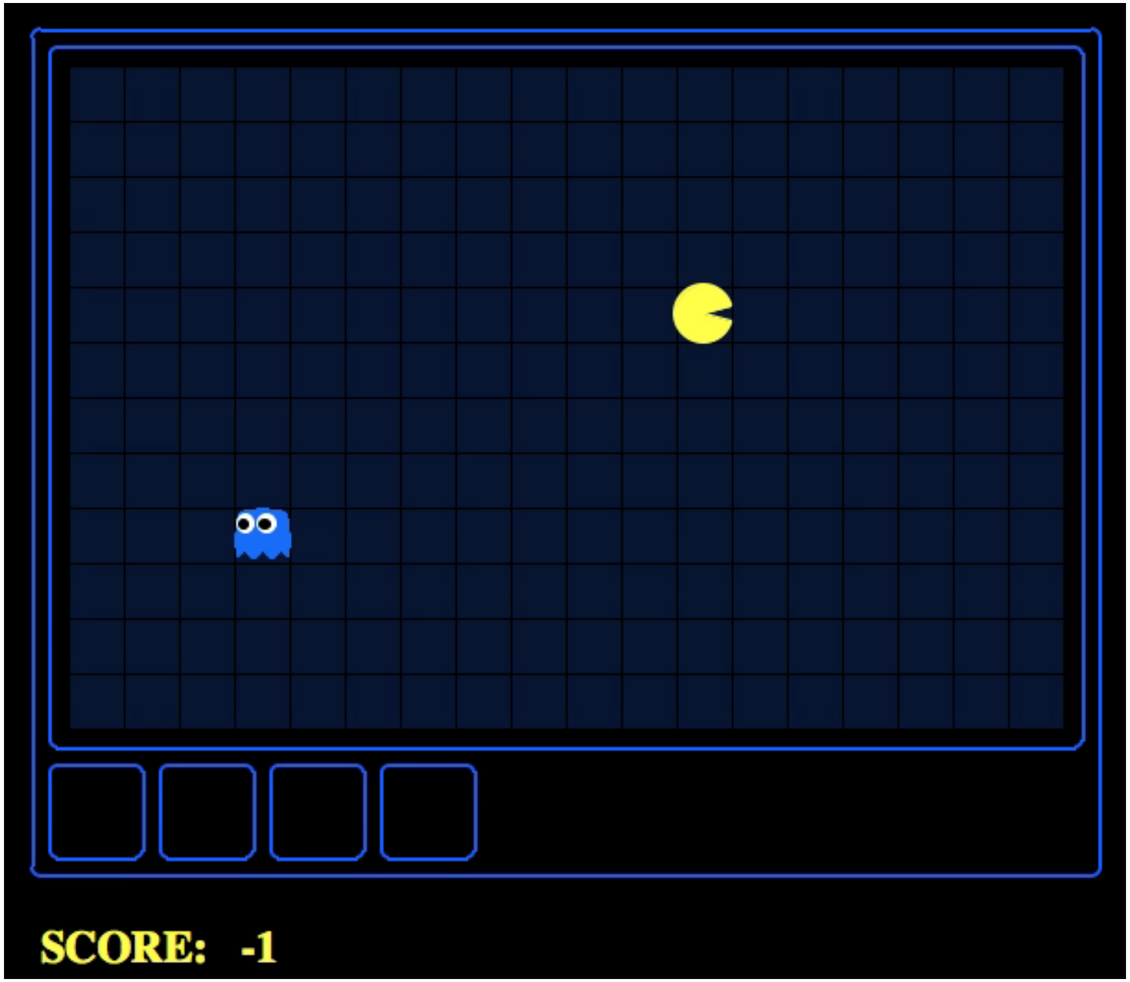
- Plot: Pacman's grandfather, Grandpac, learned to hunt ghosts for sport.
- He was blinded by his power, but could hear the ghosts' banging and clanging.
- Transition Model: All ghosts move randomly, but are sometimes biased
- Emission Model: Pacman knows a "noisy" distance to each ghost

Noisy distance prob
True distance = 8



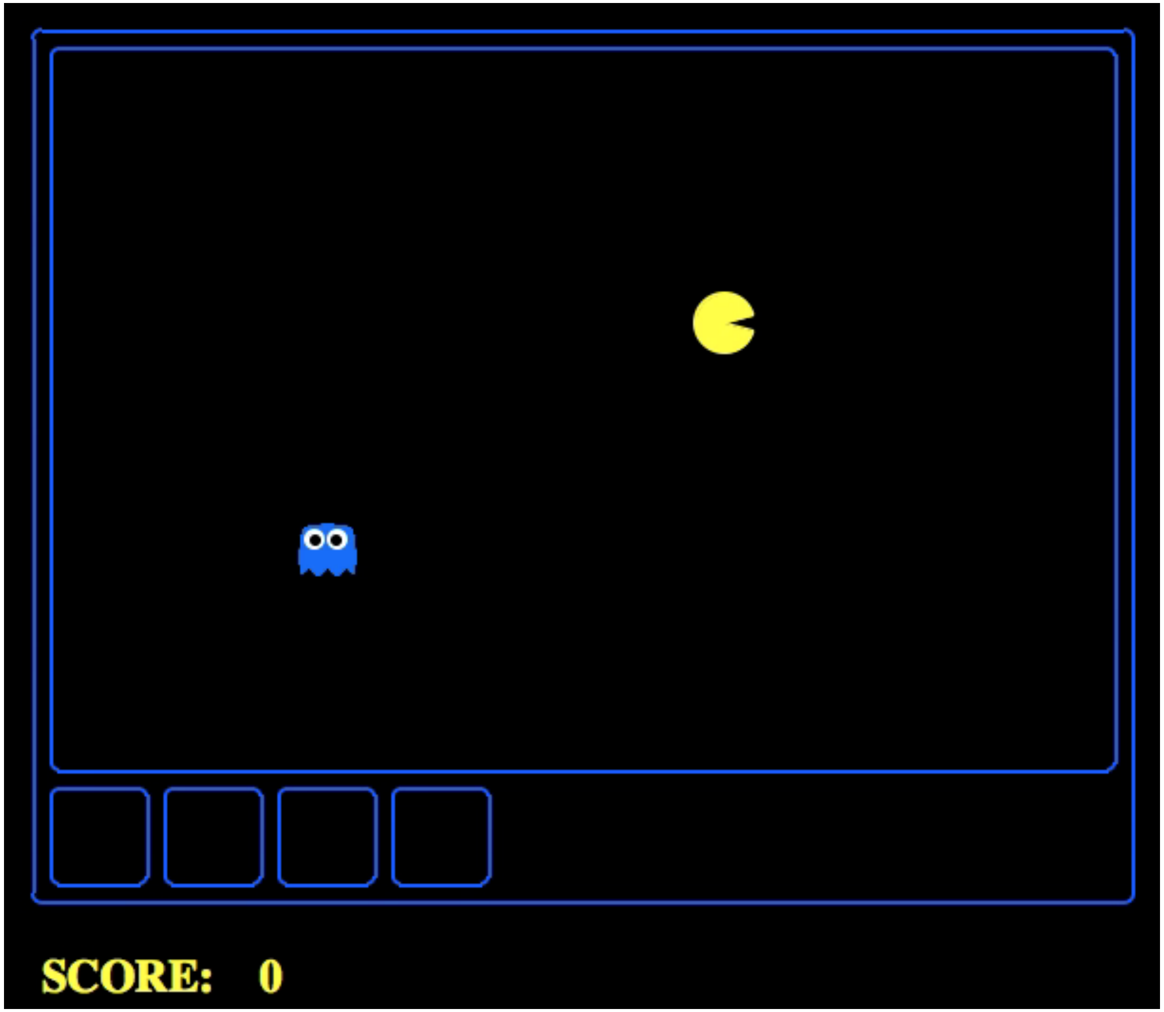
Which Algorithm?

Exact filter, uniform initial beliefs



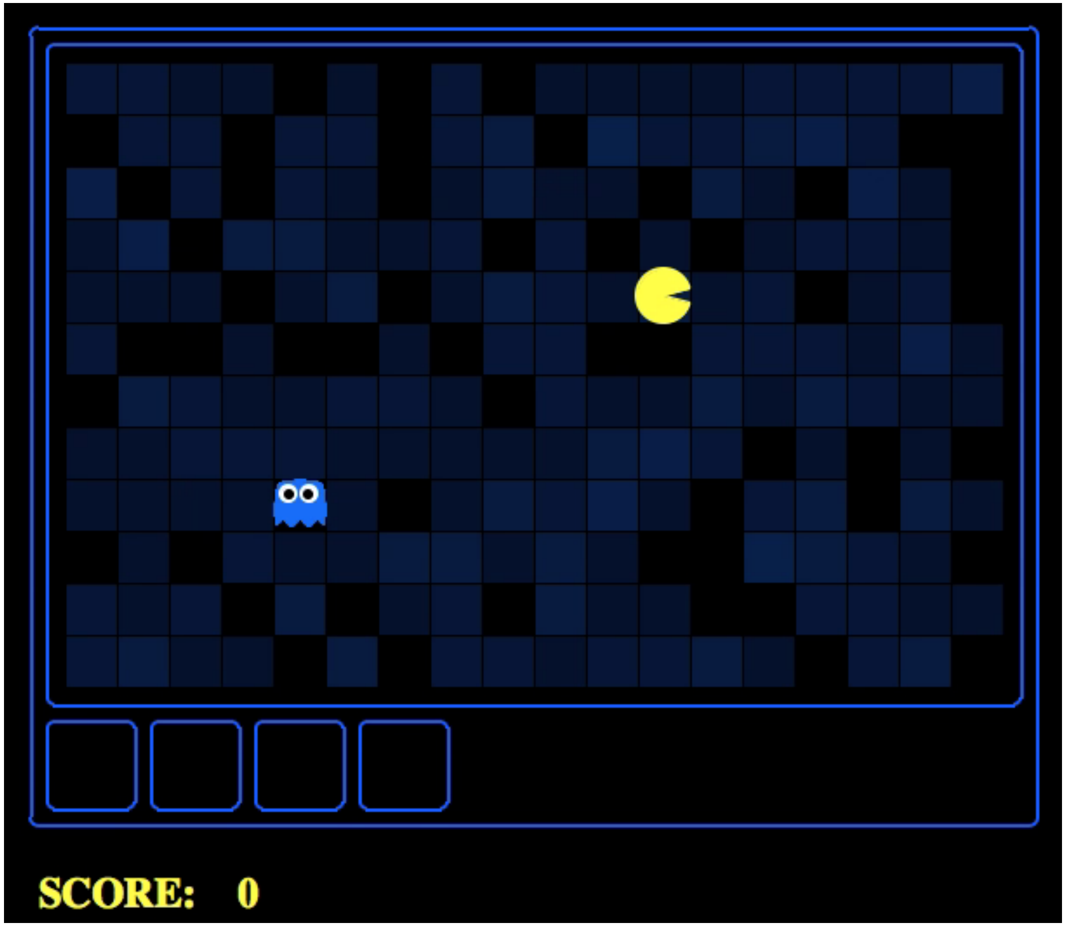
Which Algorithm?

Particle filter, uniform initial beliefs, 25 particles



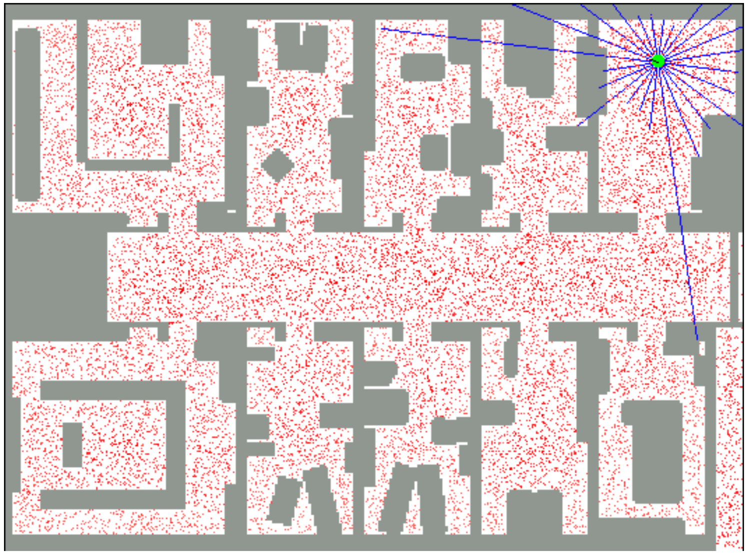
Which Algorithm?

Particle filter, uniform initial beliefs, 300 particles



Robot Localization

- In robot localization:
  - We know the map, but not the robot's position
  - Observations may be vectors of range finder readings
  - State space and readings are typically continuous (works basically like a very fine grid) and so we cannot store B(X)
  - Particle filtering is a main technique



Robot Localization

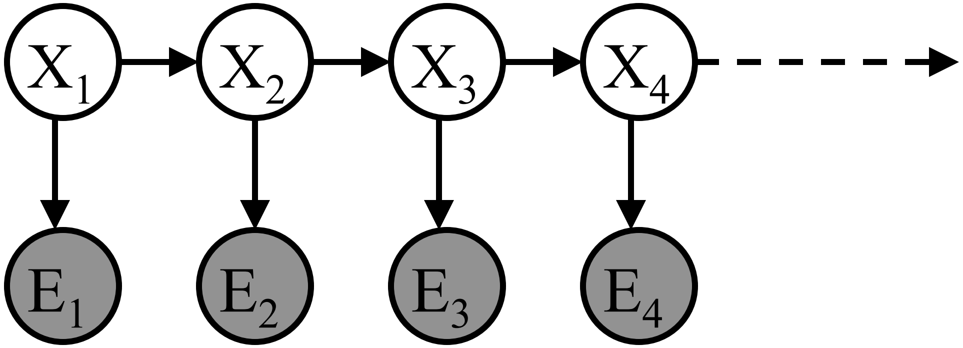
QuickTime™ and a
GIF decompressor
are needed to see this picture.

SLAM

- SLAM = Simultaneous Localization And Mapping
  - We do not know the map or our location
  - Our belief state is over maps and positions!
  - Main techniques: Kalman filtering (Gaussian HMMs) and particle methods

DP-SLAM, Ron Parr

Best Explanation Queries

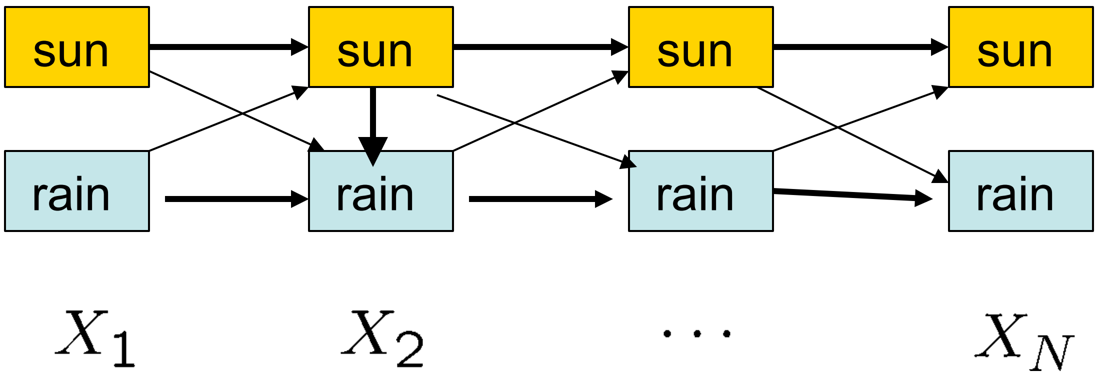


- Query: most likely seq:

$$\arg\max_{x_{1:t}} P(x_{1:t}|e_{1:t})$$

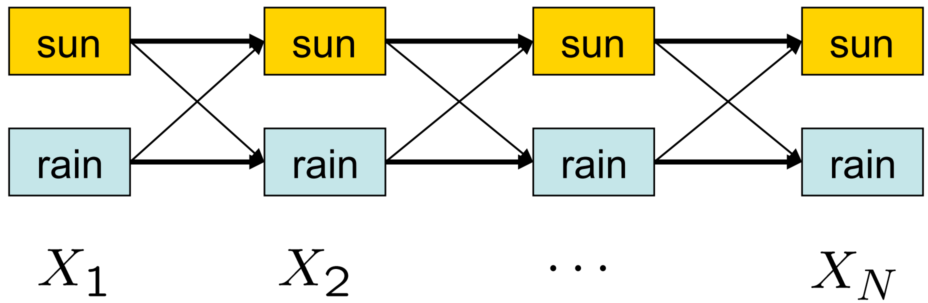
State Path Trellis

- State trellis: graph of states and transitions over time



- Each arc represents some transition $x_{t-1} \rightarrow x_t$
- Each arc has weight $P(x_t|x_{t-1})P(e_t|x_t)$
- Each path is a sequence of states
- The product of weights on a path is the seq's probability
- Can think of the Forward (and now Viterbi) algorithms as computing sums of all paths (best paths) in this graph

*Forward/Viterbi Algorithm



Forward Algorithm (Sum)

$$f_t[x_t] = P(x_t, e_{1:t})$$

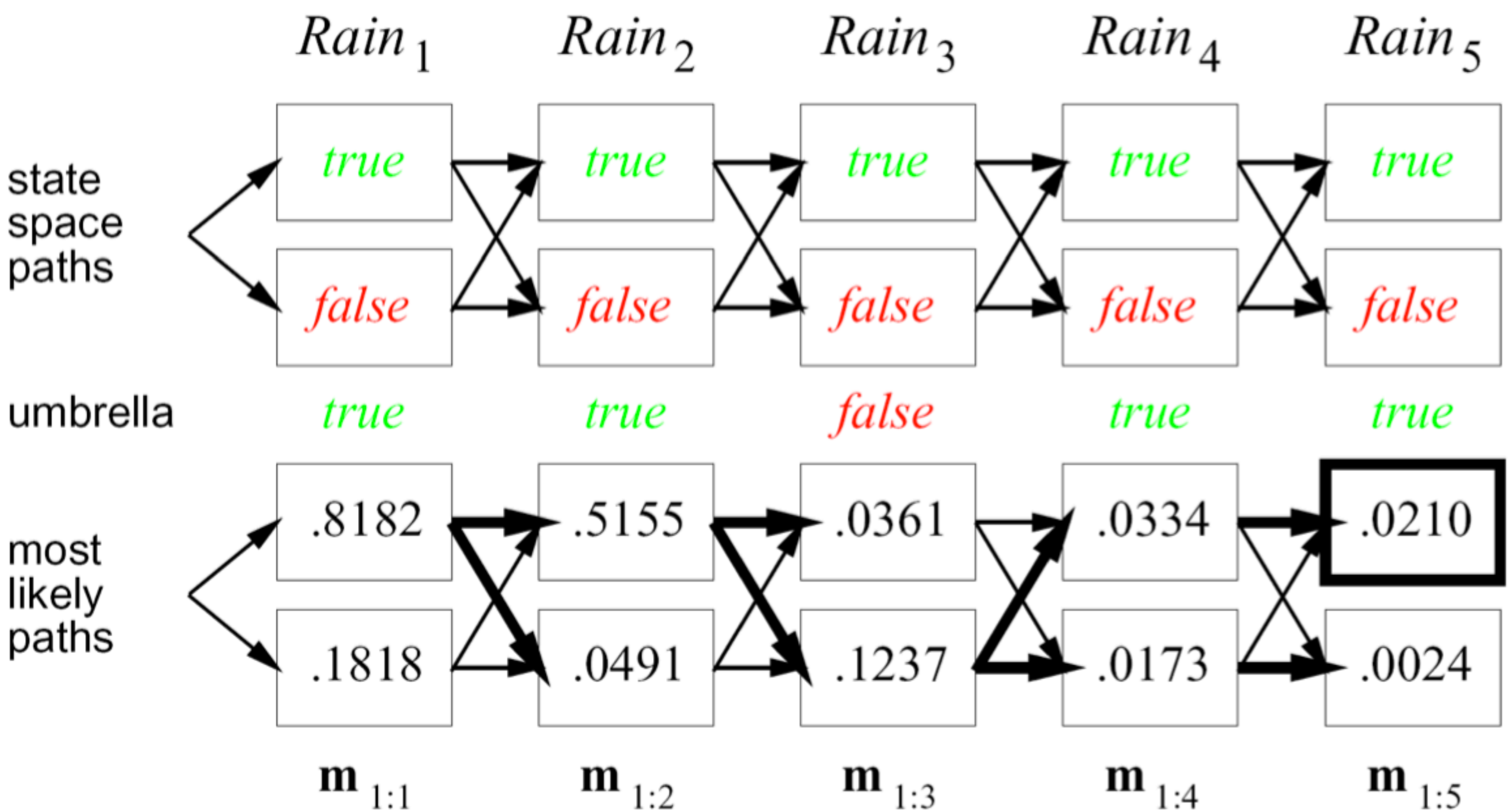
$$= P(e_t|x_t) \sum_{x_{t-1}} P(x_t|x_{t-1}) f_{t-1}[x_{t-1}]$$

Viterbi Algorithm (Max)

$$m_t[x_t] = \max_{x_{1:t-1}} P(x_{1:t-1}, x_t, e_{1:t})$$

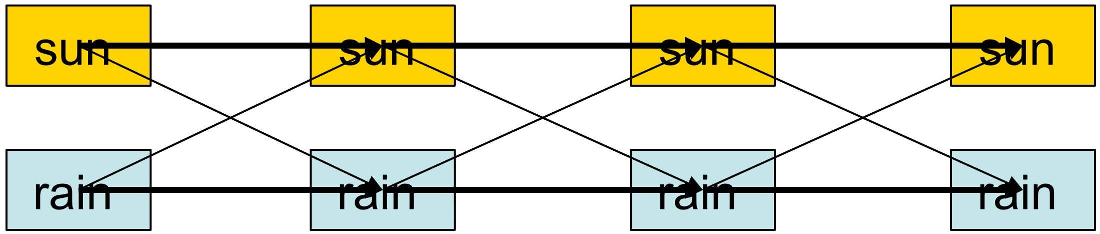
$$= P(e_t|x_t) \max_{x_{t-1}} P(x_t|x_{t-1}) m_{t-1}[x_{t-1}]$$

Example



Rain 1
Rain 2
Rain 3
Rain 4
Rain 5
state space paths
true
false
true
false
true
false
true
false
true
false
umbrella
true
true
false
true
true
most likely paths
.8182
.1818
.5155
.0491
.0361
.1237
.0334
.0173
.0210
.0024
m 1:1
m 1:2
m 1:3
m 1:4
m 1:5

* Viterbi Algorithm



$$x^*_{1:T} = \arg\max_{x_{1:T}} P(x_{1:T}|e_{1:T}) = \arg\max_{x_{1:T}} P(x_{1:T}, e_{1:T})$$

$$\begin{aligned}
m_t[x_t] &= \max_{x_{1:t-1}} P(x_{1:t-1}, x_t, e_{1:t}) \\
&= \max_{x_{1:t-1}} P(x_{1:t-1}, e_{1:t-1}) P(x_t|x_{t-1}) P(e_t|x_t) \\
&= P(e_t|x_t) \max_{x_{t-1}} P(x_t|x_{t-1}) \max_{x_{1:t-2}} P(x_{1:t-1}, e_{1:t-1}) \\
&= P(e_t|x_t) \max_{x_{t-1}} P(x_t|x_{t-1}) m_{t-1}[x_{t-1}]
\end{aligned}$$