CSE 473

Lecture 9

Logic and Reasoning
Knowledge-based agents
Wumpus world
Logic in general
Propositional logic
  • Inference, validity, equivalence and satisfiability
  • Reasoning
    - Resolution
    - Forward/backward chaining
Knowledge-Based Logical Agents

Chess program doesn’t know that no piece can be on 2 different squares at the same time

Knowledge-based logical agents combine general knowledge about the world with current percepts to infer hidden aspects of their state

- Crucial in partially observable environments
Knowledge Base and Inference

**Knowledge Base**: set of sentences represented in a knowledge representation language

- stores assertions about the world

**Inference**: when you ASK the KB a question, answer should follow from what has been TELLed to the KB previously

![Diagram](image)
Abilities of a KB agent

Agent must be able to:

- Represent states and actions
- Incorporate new percepts
- Update internal representation of the world
- Deduce hidden properties of the world
- Deduce appropriate actions
A Typical Wumpus World

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<tr>
<th></th>
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<th>PIT</th>
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<tbody>
<tr>
<td>4</td>
<td>Stench</td>
<td>Breeze</td>
<td></td>
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<tr>
<td>3</td>
<td></td>
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<td>Breeze</td>
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<td>2</td>
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<tr>
<td>1</td>
<td>START</td>
<td>Breeze</td>
<td>PIT</td>
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Wumpus

You (Agent)
Wumpus World PEAS Description

Performance measure
- gold +1000, death -1000
- -1 per step, -10 for using the arrow

Environment
- Squares adjacent to wumpus are smelly
- Squares adjacent to pit are breezy
- Glitter iff gold is in the same square
- Shooting kills wumpus if you are facing it
- Shooting uses up the only arrow
- Grabbing picks up gold if in same square
- Climbing in [1,1] gets agent out of the cave

Sensors
- Stench, Breeze, Glitter, Bump, Scream

Actuators
- TurnLeft, TurnRight, Forward, Grab, Shoot, Climb
Wumpus World Characterization

Observable?
No, only local perception

Deterministic?
Yes, outcome exactly specified

Episodic?
No, sequential at the level of actions

Static?
Yes, Wumpus and pits do not move

Discrete?
Yes

Single-agent?
Yes, Wumpus is essentially a “natural” feature of the environment
Exploring the Wumpus World

[1, 1] KB initially contains the rules of the environment. First percept is \[\text{none, none, none, none, none, none}\], move to safe cell e.g. 2, 1

[2, 1] Breeze which indicates that there is a pit in [2,2] or [3,1], return to [1,1] to try next safe cell
Exploring the Wumpus World

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<tr>
<th>1,1</th>
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<td>3,3</td>
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A = Agent  
B = Breeze  
G = Glitter, Gold  
OK = Safe square  
P = Pit  
S = Stench  
V = Visited  
W = Wumpus

[1,2] Stench in cell which means that wumpus is in [1,3] or [2,2] but not in [1,1] 
YET … wumpus not in [2,2] or stench would have been detected in [2,1] 
THUS … wumpus must be in [1,3] 
ALSO [2,2] is safe because of lack of breeze in [1,2] 
THEREFORE pit must be in [3,1] 
Move to next safe cell [2,2]
Exploring the Wumpus World

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A = Agent
B = Breeze
G = Glitter, Gold
OK = Safe square
P = Pit
S = Stench
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W = Wumpus

[2,2] Move to [2,3]
[2,3] Detect glitter, smell, breeze
Grab gold
Breeze implies pit in [3,3] or [2,4]
How do we represent rules of the world and percepts encountered?

Why not use logic?
What is a logic?

A formal language

- **Syntax** – what expressions are legal (well-formed)
- **Semantics** – what legal expressions mean
  - In logic the truth of each sentence evaluated with respect to *each possible world*

E.g. the language of arithmetic

- **Syntax**: \( x+2 \geq y \) is a legal sentence
  \( x2y+= \) is not
- **Semantics**:
  - \( x+2 \geq y \) is true in a world where \( x=7 \) and \( y=1 \)
  - \( x+2 \geq y \) is false in a world where \( x=0 \) and \( y=6 \)
How do we draw conclusions and deduce new facts about the world using logic?
Entailment

Knowledge Base KB
Sentence $\alpha$

$KB \models \alpha$ (KB "entails" sentence $\alpha$)
if and only if $\alpha$ is true in all worlds (models) where KB is true.

E.g. $x>4$ entails $x>0$
(because $x>0$ is true for all values of $x$ for which $x>4$ is true)

But not vice versa! ($x>0$ does not entail $x>4$)
Models and Entailment

$m$ is a model of a sentence $\alpha$ if $\alpha$ is true in $m$

e.g. $m = \{x=5\}$ is a model for $\alpha = \text{“}x>4\text{”}$

$m = \{x=2\}$ is a model for $\alpha = \text{“}x>0\text{”}$ but not for the sentence “$x>4$”

$M(\alpha)$ is the set of all models of $\alpha$

Then $KB \models \alpha$ iff $M(KB) \subseteq M(\alpha)$

E.g. $KB = x>4$

$\alpha = x>0$
Wumpus world model

Situation after detecting nothing in \([1,1]\), moving right, breeze in \([2,1]\)

Consider possible models for ?s assuming only pits

3 Boolean choices \(\Rightarrow\) 8 possible models
8 possible models for pits

Models consistent with rules + observations?

Observations thus far
Models consistent with rules + observations

$KB = \text{wumpus-world rules} + \text{observations}$
Example of Entailment

**ASK:** Is [1,2] safe?

\[ \alpha_1 = "[1,2] is safe"; \text{ Does } KB \models \alpha_1 ? \]

Is \( M(KB) \subseteq M(\alpha_1) ? \)
Entailment by Model Checking

\[ \alpha_1 = "[1,2] \text{ is safe}" \]

\[ M(KB) \subseteq M(\alpha_1) \]

\[ KB = \text{wumpus-world rules + observations} \]

\[ \alpha_1 = "[1,2] \text{ is safe}" , KB \models \alpha_1, \text{ proved by model checking} \]
Another Example

Is [2,2] safe?

$KB = \text{wumpus-world rules + observations}$
Another Example

\[ M(KB) \not\subseteq M(\alpha_2) \]

\[ KB = \text{wumpus-world rules} + \text{observations} \]

\[ \alpha_2 = \text{“[2,2] is safe”}, \quad KB \models \alpha_2 \]
Inference Algorithms: Soundness and Completeness

If an inference algorithm only derives entailed sentences, it is called *sound* (or *truth preserving*).

- Otherwise it just makes things up
- Algorithm $i$ is sound if whenever $KB \models \alpha$ (i.e. $\alpha$ is derived by $i$ from $KB$) it is also true that $KB \models \alpha$

Completeness: An algorithm is complete if it can derive any sentence that is entailed.

\[ i \text{ is complete if whenever } KB \models \alpha \text{ it is also true that } KB \models \alpha \]
If a KB is true in the real world, then any sentence $\alpha$ derived from the KB by a sound inference procedure is also true in the real world.
Propositional Logic: Syntax

Propositional logic is the simplest logic - illustrates basic ideas

Atomic sentences = proposition symbols = A, B, P_{1,2}, P_{2,2} etc. used to denote properties of the world

• *Can be either True or False*

E.g. P_{1,2} = “There’s a pit in location [1,2]” is either true or false in the wumpus world
Propositional Logic: Syntax

Complex sentences constructed from simpler ones recursively using logical operators

If $S$ is a sentence, $\neg S$ is a sentence (negation)
If $S_1$ and $S_2$ are sentences, $S_1 \land S_2$ is a sentence (conjunction)
If $S_1$ and $S_2$ are sentences, $S_1 \lor S_2$ is a sentence (disjunction)
If $S_1$ and $S_2$ are sentences, $S_1 \Rightarrow S_2$ is a sentence (implication)
If $S_1$ and $S_2$ are sentences, $S_1 \Leftrightarrow S_2$ is a sentence (biconditional)
Propositional Logic: Semantics

A **model** specifies true/false for each proposition symbol.

E.g. \( P_{1,2} \) \( P_{2,2} \) \( P_{3,1} \)
true \( \text{false} \) \( \text{false} \)

Rules for evaluating truth *w.r.t. a model* \( m \):

\[-S\] is true iff \( S \) is false

\( S_1 \land S_2 \) is true iff \( S_1 \) is true and \( S_2 \) is true

\( S_1 \lor S_2 \) is true iff \( S_1 \) is true or \( S_2 \) is true

\( S_1 \Rightarrow S_2 \) is true iff \( S_1 \) is false or \( S_2 \) is true

\( S_1 \Leftrightarrow S_2 \) is true iff both \( S_1 \Rightarrow S_2 \) and \( S_2 \Rightarrow S_1 \) are true
Truth Tables for Connectives

Pssst...here's a useful tip:

P \Rightarrow Q is equivalent to \neg P \lor Q
(prove it!)

<table>
<thead>
<tr>
<th>P</th>
<th>Q</th>
<th>\neg P</th>
<th>P \land Q</th>
<th>P \lor Q</th>
<th>P \Rightarrow Q</th>
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Propositional Logic: Semantics

Simple recursive process can be used to evaluate an arbitrary sentence

E.g., Model: \( P_{1,2} \quad P_{2,2} \quad P_{3,1} \)
false true false

\(-P_{1,2} \land (P_{2,2} \lor P_{3,1})\)
\(= true \land (true \lor false)\)
\(= true \land true\)
\(= true\)
Next Time: Inference using Propositional Logic

To Do:
Project #2 (Multi-Agent PacMan) assigned today!
Read Chapter 7 in textbook