Reinforcement Learning

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Many slides over the course adapted from either Dan Klein, Stuart Russell or Andrew Moore
Outline

- Reinforcement Learning
  - Passive Learning
  - TD Updates
  - Q-value iteration
  - Q-learning
  - Linear function approximation
Recap: MDPs

- **Markov decision processes:**
  - States $S$
  - Actions $A$
  - Transitions $P(s'|s,a)$ (or $T(s,a,s')$)
  - Rewards $R(s,a,s')$ (and discount $\gamma$)
  - Start state $s_0$

- **Quantities:**
  - Policy = map of states to actions
  - Utility = sum of discounted rewards
  - Values = expected future utility from a state
  - Q-Values = expected future utility from a q-state
What is it doing?

Step Delay: 0.10000
Discount: 0.800
Epsilon: 0.500
Learning Rate: 0.800

Step: 75  Position: 63  Velocity: -6.04  100-step Avg Velocity: 0.68
Reinforcement Learning

- Reinforcement learning:
  - Still have an MDP:
    - A set of states \( s \in S \)
    - A set of actions (per state) \( A \)
    - A model \( T(s,a,s') \)
    - A reward function \( R(s,a,s') \)
  - Still looking for a policy \( \pi(s) \)
  - New twist: don’t know \( T \) or \( R \)
    - I.e. don’t know which states are good or what the actions do
    - Must actually try actions and states out to learn
Example: Animal Learning

- RL studied experimentally for more than 60 years in psychology
  - Rewards: food, pain, hunger, drugs, etc.
  - Mechanisms and sophistication debated

- Example: foraging
  - Bees learn near-optimal foraging plan in field of artificial flowers with controlled nectar supplies
  - Bees have a direct neural connection from nectar intake measurement to motor planning area
Example: Backgammon

- Reward only for win / loss in terminal states, zero otherwise
- TD-Gammon learns a function approximation to $V(s)$ using a neural network
- Combined with depth 3 search, one of the top 3 players in the world

- You could imagine training Pacman this way…
- … but it’s tricky! (It’s also P3)
Passive Learning

- **Simplified task**
  - You don’t know the transitions $T(s,a,s')$
  - You don’t know the rewards $R(s,a,s')$
  - You are given a policy $\pi(s)$
  - **Goal:** learn the state values (and maybe the model)
  - I.e., policy evaluation

- **In this case:**
  - Learner “along for the ride”
  - No choice about what actions to take
  - Just execute the policy and learn from experience
  - We’ll get to the active case soon
  - This is NOT offline planning!
Detour: Sampling Expectations

- Want to compute an expectation weighted by $P(x)$:

$$E[f(x)] = \sum_x P(x) f(x)$$

- Model-based: estimate $P(x)$ from samples, compute expectation

$$x_i \sim P(x)$$

$$\hat{P}(x) = \text{count}(x)/k$$

$$E[f(x)] \approx \sum_x \hat{P}(x) f(x)$$

- Model-free: estimate expectation directly from samples

$$x_i \sim P(x)$$

$$E[f(x)] \approx \frac{1}{k} \sum_i f(x_i)$$

- Why does this work? Because samples appear with the right frequencies!
Example: Direct Estimation

- **Episodes:**

  - (1,1) up -1  
  - (1,2) up -1  
  - (1,2) up -1  
  - (1,3) right -1  
  - (2,3) right -1  
  - (3,3) right -1  
  - (3,2) up -1  
  - (3,3) right -1  
  - (4,2) exit -100  
  - (3,3) right -1  
  - (4,3) exit +100  
  - (done)  

\[
V(1,1) \sim \frac{(92 + -106)}{2} = -7
\]

\[
V(3,3) \sim \frac{(99 + 97 + -102)}{3} = 31.3
\]

\[
\gamma = 1, R = -1
\]
Model-Based Learning

- **Idea:**
  - Learn the model empirically (rather than values)
  - Solve the MDP as if the learned model were correct
  - Better than direct estimation?

- **Empirical model learning**
  - Simplest case:
    - Count outcomes for each \( s,a \)
    - Normalize to give estimate of \( T(s,a,s') \)
    - Discover \( R(s,a,s') \) the first time we experience \( (s,a,s') \)
  - More complex learners are possible (e.g. if we know that all squares have related action outcomes, e.g. “stationary noise”)
Example: Model-Based Learning

Episodes:

- (1,1) up -1
- (1,2) up -1
- (1,2) up -1
- (1,3) right -1
- (2,3) right -1
- (3,3) right -1
- (3,2) up -1
- (3,2) up -1
- (3,3) right -1
- (3,3) right -1
- (3,2) up -1
- (4,2) exit -100
- (3,3) right -1
- (done)
- (4,3) exit +100
- (done)

\[ T(<3,3>, \text{right}, <4,3>) = \frac{1}{3} \]

\[ T(<2,3>, \text{right}, <3,3>) = \frac{2}{2} \]
Recap: Model-Based Policy Evaluation

- Simplified Bellman updates to calculate $V$ for a fixed policy:
  - New $V$ is expected one-step-look-ahead using current $V$
  - Unfortunately, need $T$ and $R$

\[
V_0^\pi(s) = 0
\]

\[
V_{i+1}^\pi(s) \leftarrow \sum_{s'} T(s, \pi(s), s')[R(s, \pi(s), s') + \gamma V_i^\pi(s')]
\]
Sample Avg to Replace Expectation?

\[ V_{i+1}^\pi(s) \leftarrow \sum_{s'} T(s, \pi(s), s') [R(s, \pi(s), s') + \gamma V_i^\pi(s')] \]

- Who needs T and R? Approximate the expectation with samples (drawn from T!)

\[
\begin{align*}
\text{sample}_1 &= R(s, \pi(s), s'_1) + \gamma V_i^\pi(s'_1) \\
\text{sample}_2 &= R(s, \pi(s), s'_2) + \gamma V_i^\pi(s'_2) \\
&\vdots \\
\text{sample}_k &= R(s, \pi(s), s'_k) + \gamma V_i^\pi(s'_k) \\
V_{i+1}^\pi(s) &\leftarrow \frac{1}{k} \sum_i \text{sample}_i
\end{align*}
\]
Detour: Exp. Moving Average

- Exponential moving average
  - Makes recent samples more important

\[ \bar{x}_n = \frac{x_n + (1 - \alpha) \cdot x_{n-1} + (1 - \alpha)^2 \cdot x_{n-2} + \ldots}{1 + (1 - \alpha) + (1 - \alpha)^2 + \ldots} \]

- Forgets about the past (distant past values were wrong anyway)
- Easy to compute from the running average

\[ \bar{x}_n = (1 - \alpha) \cdot \bar{x}_{n-1} + \alpha \cdot x_n \]

- Decreasing learning rate can give converging averages
Model-Free Learning

- Big idea: why bother learning $T$?
  - Update $V$ each time we experience a transition
- Temporal difference learning (TD)
  - Policy still fixed!
  - Move values toward value of whatever successor occurs: running average!

\[
V^\pi(s) \leftarrow \sum_{s'} T(s, \pi(s), s')[R(s, \pi(s), s') + \gamma V^\pi(s')] 
\]

\[
sample = R(s, \pi(s), s') + \gamma V^\pi(s') \\
V^\pi(s) \leftarrow (1 - \alpha)V^\pi(s) + (\alpha)sample \\
V^\pi(s) \leftarrow V^\pi(s) + \alpha(sample - V^\pi(s))
\]
Example: TD Policy Evaluation

\[ V^\pi(s) \leftarrow (1 - \alpha)V^\pi(s) + \alpha \left[ R(s, \pi(s), s') + \gamma V^\pi(s') \right] \]

(1,1) up -1  
(1,2) up -1  
(1,2) up -1  
(1,3) right -1  
(2,3) right -1  
(2,3) right -1  
(3,3) right -1  
(3,3) right -1  
(3,2) up -1  
(4,2) exit -100  
(3,3) right -1  
(4,3) exit +100  
(done)

Take \( \gamma = 1 \), \( \alpha = 0.5 \)
Problems with TD Value Learning

- TD value learning is model-free for policy evaluation (passive learning)
- However, if we want to turn our value estimates into a policy, we’re sunk:
  \[
  \pi(s) = \arg \max_a Q^*(s, a)
  \]
  \[
  Q^*(s, a) = \sum_{s'} T(s, a, s') \left[ R(s, a, s') + \gamma V^*(s') \right]
  \]
- Idea: learn Q-values directly
- Makes action selection model-free too!
Active Learning

- Full reinforcement learning
  - You don’t know the transitions $T(s,a,s’)$
  - You don’t know the rewards $R(s,a,s’)$
  - You can choose any actions you like
  - Goal: learn the optimal policy
  - … what value iteration did!

- In this case:
  - Learner makes choices!
  - Fundamental tradeoff: exploration vs. exploitation
  - This is NOT offline planning! You actually take actions in the world and find out what happens…
Detour: Q-Value Iteration

- Value iteration: find successive approx optimal values
  - Start with $V_0^*(s) = 0$
  - Given $V_i^*$, calculate the values for all states for depth $i+1$:
    \[
    V_{i+1}(s) \leftarrow \max_a \sum_{s'} T(s, a, s') \left[ R(s, a, s') + \gamma V_i(s') \right]
    \]

- But Q-values are more useful!
  - Start with $Q_0^*(s,a) = 0$
  - Given $Q_i^*$, calculate the q-values for all q-states for depth $i+1$:
    \[
    Q_{i+1}(s, a) \leftarrow \sum_{s'} T(s, a, s') \left[ R(s, a, s') + \gamma \max_{a'} Q_i(s', a') \right]
    \]
Q-Learning Update

- Q-Learning: sample-based Q-value iteration
  \[ Q^*(s, a) = \sum_{s'} T(s, a, s') \left[ R(s, a, s') + \gamma \max_{a'} Q^*(s', a') \right] \]

- Learn \( Q^*(s,a) \) values
  - Receive a sample \((s,a,s',r)\)
  - Consider your old estimate: \( Q(s, a) \)
  - Consider your new sample estimate:
    \[ \text{sample} = R(s, a, s') + \gamma \max_{a'} Q(s', a') \]
  - Incorporate the new estimate into a running average:
    \[ Q(s, a) \leftarrow (1 - \alpha)Q(s, a) + (\alpha) [\text{sample}] \]
Q-Learning: Fixed Policy
Q-Learning Properties

- Amazing result: Q-learning converges to optimal policy
  - If you explore enough
  - If you make the learning rate small enough
  - ... but not decrease it too quickly!
  - Not too sensitive to how you select actions (!)

- Neat property: off-policy learning
  - Learn optimal policy without following it (some caveats)
Exploration / Exploitation

- Several schemes for action selection
  - Simplest: random actions ($\varepsilon$ greedy)
    - Every time step, flip a coin
    - With probability $\varepsilon$, act randomly
    - With probability $1-\varepsilon$, act according to current policy
  - Problems with random actions?
    - You do explore the space, but keep thrashing around once learning is done
    - One solution: lower $\varepsilon$ over time
    - Another solution: exploration functions
Q-Learning: $\varepsilon$ Greedy

CURRENT Q-VALUES
Exploration Functions

- **When to explore**
  - Random actions: explore a fixed amount
  - Better idea: explore areas whose badness is not (yet) established

- **Exploration function**
  - Takes a value estimate and a count, and returns an optimistic utility, e.g. \( f(u, n) = u + k/n \) (exact form not important)
  - Exploration policy \( \pi(s') = \)

\[
\max_{a'} Q_i(s', a') \quad \text{vs.} \quad \max_{a'} f(Q_i(s', a'), N(s', a'))
\]
Q-Learning Final Solution

- Q-learning produces tables of q-values:
Q-Learning Properties

- Amazing result: Q-learning converges to optimal policy
  - If you explore enough
  - If you make the learning rate small enough
  - … but not decrease it too quickly!
  - Not too sensitive to how you select actions (!)

- Neat property: off-policy learning
  - learn optimal policy without following it (some caveats)
Q-Learning

- In realistic situations, we cannot possibly learn about every single state!
  - Too many states to visit them all in training
  - Too many states to hold the q-tables in memory

- Instead, we want to generalize:
  - Learn about some small number of training states from experience
  - Generalize that experience to new, similar states
  - This is a fundamental idea in machine learning, and we’ll see it over and over again
Example: Pacman

- Let’s say we discover through experience that this state is bad:

- In naïve q learning, we know nothing about related states and their q values:

- Or even this third one!
Feature-Based Representations

- Solution: describe a state using a vector of features (properties)
  - Features are functions from states to real numbers (often 0/1) that capture important properties of the state
  - Example features:
    - Distance to closest ghost
    - Distance to closest dot
    - Number of ghosts
    - $1 / (\text{dist to dot})^2$
    - Is Pacman in a tunnel? (0/1)
    - ...... etc.
    - Is it the exact state on this slide?
  - Can also describe a q-state $(s, a)$ with features (e.g. action moves closer to food)
Linear Feature Functions

- Using a feature representation, we can write a q function (or value function) for any state using a few weights:

\[ V(s) = w_1 f_1(s) + w_2 f_2(s) + \ldots + w_n f_n(s) \]

\[ Q(s, a) = w_1 f_1(s, a) + w_2 f_2(s, a) + \ldots + w_n f_n(s, a) \]

- Advantage: our experience is summed up in a few powerful numbers

- Disadvantage: states may share features but actually be very different in value!
Function Approximation

\[
Q(s, a) = w_1 f_1(s, a) + w_2 f_2(s, a) + \ldots + w_n f_n(s, a)
\]

- **Q-learning with linear q-functions:**
  
  \[
  \text{transition} = (s, a, r, s')
  \]
  
  \[
  \text{difference} = \left[ r + \gamma \max_{a'} Q(s', a') \right] - Q(s, a)
  \]
  
  \[
  Q(s, a) \leftarrow Q(s, a) + \alpha \text{[difference]}
  \]
  
  \[
  w_i \leftarrow w_i + \alpha \text{[difference]} f_i(s, a)
  \]

- **Intuitive interpretation:**
  - Adjust weights of active features
  - E.g. if something unexpectedly bad happens, disprefer all states with that state’s features

- **Formal justification:** online least squares

- Exact Q’s
- Approximate Q’s
Example: Q-Pacman

\[ Q(s, a) = 4.0 f_{DOT}(s, a) - 1.0 f_{GST}(s, a) \]

\[ f_{DOT}(s, \text{NORTH}) = 0.5 \]

\[ f_{GST}(s, \text{NORTH}) = 1.0 \]

\[ Q(s, a) = +1 \]

\[ R(s, a, s') = -500 \]

\[ \text{correction} = -501 \]

\[ w_{DOT} \leftarrow 4.0 + \alpha [-501] 0.5 \]

\[ w_{GST} \leftarrow -1.0 + \alpha [-501] 1.0 \]

\[ Q(s, a) = 3.0 f_{DOT}(s, a) - 3.0 f_{GST}(s, a) \]
Linear Regression

Prediction

\[ \hat{y} = w_0 + w_1 f_1(x) \]

Prediction

\[ \hat{y}_i = w_0 + w_1 f_1(x) + w_2 f_2(x) \]
Ordinary Least Squares (OLS)

\[
\text{total error} = \sum_i (y_i - \hat{y}_i)^2 = \sum_i \left( y_i - \sum_k w_k f_k(x_i) \right)^2
\]

Observation $y$

Prediction $\hat{y}$

Error or "residual"
Minimizing Error

Imagine we had only one point $x$ with features $f(x)$:

$$\text{error}(w) = \frac{1}{2} \left( y - \sum_k w_k f_k(x) \right)^2$$

$$\frac{\partial \text{error}(w)}{\partial w_m} = - \left( y - \sum_k w_k f_k(x) \right) f_m(x)$$

$$w_m \leftarrow w_m + \alpha \left( y - \sum_k w_k f_k(x) \right) f_m(x)$$

Approximate q update:

"target"  "prediction"

$$w_m \leftarrow w_m + \alpha \left[ r + \gamma \max_a Q(s', a') - Q(s, a) \right] f_m(s, a)$$
Overfitting

Degree 15 polynomial
Which Algorithm?

Q-learning, no features, 50 learning trials:

SCORE: 0
Which Algorithm?

Q-learning, no features, 1000 learning trials:

SCORE: 0
Which Algorithm?

Q-learning, simple features, 50 learning trials:
Policy Search*
Policy Search*

- Problem: often the feature-based policies that work well aren’t the ones that approximate $V$ / $Q$ best
  - E.g. your value functions from project 2 were probably horrible estimates of future rewards, but they still produced good decisions
  - We’ll see this distinction between modeling and prediction again later in the course

- Solution: learn the policy that maximizes rewards rather than the value that predicts rewards

- This is the idea behind policy search, such as what controlled the upside-down helicopter
Policy Search*

- Simplest policy search:
  - Start with an initial linear value function or q-function
  - Nudge each feature weight up and down and see if your policy is better than before

- Problems:
  - How do we tell the policy got better?
  - Need to run many sample episodes!
  - If there are a lot of features, this can be impractical
Advanced policy search:

- Write a stochastic (soft) policy:

\[ \pi_w(s) \propto e^{\sum_i w_i f_i(s,a)} \]

- Turns out you can efficiently approximate the derivative of the returns with respect to the parameters \( w \) (details in the book, optional material)

- Take uphill steps, recalculate derivatives, etc.