Rational Decisions
Chapter 16

Outline

Rational preferences
Utilities
Money
Multiattribute utilities
Decision networks
Value of information

Preferences

An agent chooses among prizes (A, B, etc.) and lotteries, i.e., situations with uncertain prizes.
Lottery \( L = [p; A; (1-p); B] \)

Notation:
- \( A \) preferred to \( B \)
- \( A \) indierence between \( A \) and \( B \)
- \( A \) not preferred to \( A \)

Rational preferences

Idea: preferences of a rational agent must obey constraints.

Constraints:
- Orderability: \( A \sim B \) \( \Rightarrow \) \( B \sim A \) \( \Rightarrow \) \( A \sim B \)
- Transitivity: \( A \sim B \) \( \land \) \( B \sim C \) \( \Rightarrow \) \( A \sim C \)
- Continuity: \( A \sim X \sim C \) \( \Rightarrow \) there exists a real-valued function \( U \) such that there exist \( p \) and \( q \) such that \( U(p; A) \leq U(q; B) \leq U(p; C) \)
- Substitutability: \( A \sim B \) \( \Rightarrow \) \( p; A \sim (1-p; B) \) \( \land \) \( q; A \giml (1-q; B) \)
- Monotonicity: \( A \sim B \) \( \Rightarrow \) \( p \leq q \) \( \land \) \( U(p; A) \leq U(q; B) \)

Maximizing expected utility

Theorem (Ramsey, 1931; von Neumann and Morgenstern, 1944):
Given preferences satisfying the constraints, there exists a real-valued function \( U \) such that for all lotteries \( L = [p; A; (1-p); B] \), the agent chooses the action that maximizes expected utility.

Value of information

Decision networks
Multiattribute utilities
Money
Utilities
Rational preferences
Utilities map states to real numbers. Which numbers?

Standard approach to assessment of human utilities:

- Compare a given state $A$ to a standard lottery $L$.

  - The best possible prize
  - $u$ with probability $p$
  - The worst possible catastrophe
  - $u$? with probability $(1-p)$

Adjust lottery probability $p$ until $A \approx L$

$0.000001$ to $0.999999$

Continue as before

*Instant death*

Pay $30$

**Chapter 16 7**

**Utility scales**

- **Normalized utilities**:
  - $u > 0.5$
  - $u < 0.5$

- **Micromorts**:
  - One-millionth chance of death
  - Useful for Russian roulette, paying to reduce product risks, etc.

- **QALYs**:
  - Quality-adjusted life years
  - Useful for medical decisions involving substantial risk

Note: behavior is invariant w.r.t. +ve linear transformation

$U_0(x) = k_1U(x) + k_2$ where $k_1 > 0$

With deterministic prizes only (no lottery choices), only ordinal utility can be determined, i.e., total order on prizes

**Chapter 16 8**

**Money**

Money does not behave as a utility function

- Given a lottery $L$ with expected monetary value $EMV(L)$,
  - Usually $U(L) < U(EMV(L))$, i.e., people are risk-averse

**Chapter 16 9**

Utility curve: for what probability $p$ am I indifferent between a prize $x$ and a lottery $[p; M; (1-p); 0]$ for large $M$?

Typical empirical data, extrapolated with risk-prone behavior:

<table>
<thead>
<tr>
<th>$x$</th>
<th>$-150,000$</th>
<th>$800,000$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p$</td>
<td>$0$</td>
<td>$500$</td>
</tr>
</tbody>
</table>

**Chapter 16 10**

**Decision networks**

Add action nodes and utility nodes to belief networks to enable rational decision making

**Algorithm**

For each value of action node compute expected value of utility node given action, evidence

Return MEU action

**Chapter 16 11**

**Multiattribute utility**

- How can we handle utility functions of many variables $X_1 : \ldots : X_n$?
  - E.g., what is $U(Deaths; Noise; Cost)$?

- How can complex utility functions be assessed from preference behavior?

  **Idea 1**: identify conditions under which decisions can be made without complete identification of $U(x_1; \ldots ; x_n)$
  **Idea 2**: identify various types of independence in preferences and derive consequent canonical forms for $U(x_1; \ldots ; x_n)$

**Chapter 16 12**

**Student group utility**

For each value of action node, adjust $p$ until half the class votes for lottery (M=10,000)

**Utility scale**

- Weight deterministic gains only (no lottery choices), only
  - $X < 0$ $X > 0$

- Note behavior is invariant w.r.t. linear transformation

- Used for medical decisions involving subjective risk

- Money scales are multiattribute scales of utility

- Money scales to real numbers, which enumerate

**Utility**
Strict dominance typically defines attributes such that $$U$$ is monotonic in each. Strict dominance: choice $$B$$ strictly dominates choice $$A$$ if
$$\forall i \ X_i(B) > X_i(A) \quad \text{and hence} \quad U(B) > U(A).$$

Deterministic attributes Uncertain attributes

Strict dominance seldom holds in practice.

Stochastic dominance

Can model belief networks with stochastic dominance information:

For every event $$e$$, $$\omega$$ other parents that influence $$e$$.

E.g., construction cost increases with distance from city.
$$S_1$$ is closer to the city than $$S_2$$.
$$S_1$$ stochastically dominates $$S_2$$ on cost.

E.g., injury increases with collision speed.

Can annotate belief networks with stochastic dominance information:

For every value $$z$$ of $$Y$$'s other parents $$Z$$,
$$\mathbb{P}(Y|X_1; Z)$$ stochastically dominates $$\mathbb{P}(Y|X_2; Z)$$.

Stochastic dominance can often be determined without exact distributions using qualitative reasoning.

Stochastic dominance contd.

Multiattribute case: stochastic dominance on all attributes.

Chapter 16 14

Stochastic dominance contd.

Chapter 16 15

Chapter 16 16

Chapter 16 17

Chapter 16 18
Preference structure: Deterministic

X1 and X2 preferentially independent of X3

Preference between h_{x1;x2;x3} and h_{x0;x0;x3}
does not depend on x3

E.g., h_{Noise; Cost; Safety}:

h_{20,000 suer, $4.6 billion, 0.06 deaths/mpm} vs. h_{70,000 suer, $4.2 billion, 0.06 deaths/mpm}

Theorem
(Leontief, 1947): if every pair of attributes is P.I. of its complement, then every subset of attributes is P.I. of its complement: mutual P.I.

Theorem
(Debreu, 1960): mutual P.I. 

\[ V(S) = \sum_{i} V_i(X_i(S)) \]

Hence assess n single-attribute functions; obtain a good approximation

\[ A(S) = \sum_{i} A_i(X_i(S)) \]

Preference structure: Stochastic

Need to consider preferences over lotteries:

X is utility-independent of Y
preferences over lotteries in X do not depend on y

Mutual U.I.: each subset is U.I of its complement

\[ U = k_1 U_1 + k_2 U_2 + k_3 U_3 + k_1 k_2 U_1 U_2 + k_2 k_3 U_2 U_3 + k_3 k_1 U_3 U_1 \]

Routine procedures and software packages for generating preference tests to identify various canonical families of utility functions

Value of information

Idea: compute value of acquiring each possible piece of evidence
Can be done directly from decision network

Example: buying oil drilling rights
Two blocks A and B, exactly one has oil, worth k=2
Prior probabilities 0.5 each, mutually exclusive
Current price of each block is $2

Consultant offers accurate survey of A for $7

E.g., P(A|Oil)=0.8 and P(A|No Oil)=0.2

Load no depend on X

Preference over lotteries are dependent on X

X is strictly independent of Y

Preference over lotteries are independent of X

Hence assess n single-attribute functions; obtain a good approximation

\[ A(S) = \sum_{i} A_i(X_i(S)) \]

Theorem
(Chebychev, 1960): mutual P.I.

Theorem
(Debreu, 1958): utility-independent

\[ 700,000 suer, 2.2 billion 0.6 deaths/km² \]

\[ 200,000 suer, 0.6 deaths/km² \]

E.g., \( P(A|Oil) = 0.8 \) and \( P(A|No Oil) = 0.2 \)

Load no depend on X and X is strictly independent of Y

Preference over lotteries are dependent on X
General formula

Current evidence, current best action

Possible action outcomes

Suppose we knew \( E_j = e_{jk} \), then we would choose \( e_{jk} \) s.t.

\[
EU(e_{jk}) = \max_a i U(S_i) P(S_{ij}E; a; E_j = e_{jk})
\]

\( E_j \) is a random variable whose value is currently unknown

must compute expected gain over all possible values:

\[
VPI = \sum_{E_j} EU(e_{jk})
\]

\( VPI = \text{value of perfect information} \)

Qualitative behaviors

a) Choice is obvious, information worth little

b) Choice is nonobvious, information worth a lot

c) Choice is nonobvious, information worth little

Nonadditive — consider, e.g., obtaining \( E_j \) twice

Order-independent

\[
VPI(E_j; E_k) = VPI(E_j) + VPI(E_k)
\]

Note: when more than one piece of evidence can be gathered, maximizng VPI for each to select one is not always optimal, evidence-gathering becomes a sequential decision problem