Chapter 13

Outline

Methods for handling uncertainty

Uncertainty

Chapter 13 1

Uncertainty

Chapter 13 2

Chapter 13 3

Chapter 13 4

Chapter 13 5

Chapter 13 6
Every probability must correspond to a set or countable set of sample points.

Probability distribution is a countable set of sample points, e.g., every sample point of a finite or at most countable discrete set. Every probability distribution is countable set of sample points.

Where the proposition is true.

Where the proposition is false.

A random variable is a function from sample points to some range. The random variable is a function from sample points to some range.

The de Finetti theorem states that every probability distribution can be represented by a countable set of sample points.

A random variable is a function from sample points to some range.
For any proposition \( \phi \), with the joint distribution one has:

\[
\text{Inference by enumeration}
\]

\[
(p(X|\phi))^{\text{loothac}e} = (\phi)\rho
\]

The kind of inference sanctioned by domain knowledge is crucial but is not always used. Note: the less specific belief remains valid after evidence appears.

If we know more info. is also given, then we have

\[
\text{Inference for conditional distributions}
\]

\[
\text{Conditional probability}
\]

\[
\text{Gaussian density}
\]

\[
\text{Probability for continuous variables}
\]
Normalization

Obvious problems:

The same independence holds if I haven't got a cavity:

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Absolute independence powerful but rare

The independence of Toothache and Catch, given Cavity, is conditional on whether I have a cavity.

Inference by enumeration

<table>
<thead>
<tr>
<th>Toothache</th>
<th>Catch</th>
<th>Cavity</th>
</tr>
</thead>
<tbody>
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Chapter 13 24

Chapter 13 20

Normalization

By having observed variables and summing over hidden variables

General idea: compute distribution on joint variables

Dentistry is a large field with hundreds of variables, none of which are independent. What to do?

Equivalent statements:

[Equation]

Other programs:

Currently, the set of worn variables

The same independence holds if I haven't got a cavity:

Let the hidden variable be H = E - A - X

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Inference by enumeration, cont'd.

Inference by enumeration, cont'd.

Inference by enumeration, cont'd.
Conditional Independence is our most basic and robust form of knowledge about uncertain environments.

### Bayesian Rule and Conditional Independence

When positing probabilities of meaningful small size:

\[
0.000000001 = \frac{s}{s|a|d} = \frac{s}{s|a|d|b|c\text{etc.}}
\]

The joint distribution is

\[
\prod_i p_i = \prod_i p_i(q_i|\text{etc.})
\]

We have already seen two examples of naive Bayes model:

\[
\begin{align*}
\mathcal{A} \mathcal{B} | \mathcal{A} \mathcal{B} & = \mathcal{A} \mathcal{B} |
\text{(classical conditional independence)} \\
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For inferences, p(\text{Cavity}) = p(\text{Cavity} | \text{Toothache}) p(\text{Toothache})

Second term: pits are placed randomly, probability 0.2 per square:

Apply product rule:

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We know the following facts:

- Cavity
- Toothache
- Second term: pits are placed randomly, probability 0.2 per square:
- Apply product rule:
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### Specifying the Probability Model

Include only those entries in the probability model

\[
\begin{array}{c|ccc}
\hline
\text{Toothache} & \text{Yes} & \text{No} \\
\hline
\text{Cavity} & \text{Yes} & \frac{1}{2} & \frac{1}{2} \\
\text{No} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\
\hline
\end{array}
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Wumpus World

### Specifying the Probability Model

In the probability model, the joint distribution from enumeration is

A joint distribution from enumeration is in the form:

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Independence and conditional independence provide the tools for modeling domains, where we must find a way to reduce the joint size. Queries can be answered by summing over atomic events. Given probability distribution vectors, probability of every atomic event is a rigorous formalism for uncertain knowledge.

\[
\begin{align*}
\langle \text{Her} \rangle_0 &\approx \langle q' \text{Forest} \rangle_0 d \\
\langle \text{Blim} \rangle_0 &\approx \langle q' \text{Forest} \rangle_0 \tau = \langle q' \text{Forest} \rangle_0 d
\end{align*}
\]

Using conditional independence contd.

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