Chapter 9
Reasoning with First-Order Logic

What’s on our menu today?

- Reasoning with FOL
  - Unification
  - Forward/Backward Chaining
  - Resolution
  - Compilation to SAT
Motivation for Unification

• What if we want to use modus ponens?
  Propositional Logic:
  \( a \land b, \ a \land b \Rightarrow c \)
  \( c \)

• In First-Order Logic?
  \( \text{Monkey}(x) \Rightarrow \text{Curious}(x) \)
  \( \text{Monkey}(\text{George}) \)
  \( \text{????} \)

• Must “unify” \( x \) with George:
  Need to substitute \( \{x/\text{George}\} \) in \( \text{Monkey}(x) \Rightarrow \text{Curious}(x) \)
  to infer \( \text{Curious}(\text{George}) \)

What is Unification?

Not this kind of unification...
What is Unification?

- **Match up expressions by finding variable values that make the expressions identical**
- **Unify**(x, y) returns most general unifier (MGU). Examples:
  - `Unify(city(x), city(kent))` returns `{x/kent}`
  - `Unify(PokesInTheEyes(Moe,x), PokesInTheEyes(y,z))` returns `{y/Moe,x/z}`
  - `{y/Moe,x/Moe,z/Moe}` possible but not MGU
  - **MGU** places fewest restrictions on values of variables

Unification and Substitution

**Unification** produces a mapping from variables to values (e.g., `{x/kent,y/seattle}`)

**Substitution:** `Subst(mapping,sentence)` returns new sentence with variables replaced by values

  - `Subst({x/kent,y/seattle }, connected(x, y))` returns `connected(kent, seattle)`
Unification Examples I

- Unify(road(x, kent), road(seattle, y))
  Returns \{x / seattle, y / kent\}
  When substituted in both expressions, the resulting expressions match:
  Each is (road(seattle, kent))

- Unify(road(x, x), road(seattle, kent))
  Not possible - Fails!
  x can’t be seattle and kent at the same time!

Unification Examples II

- Unify(f(g(x, dog), y)), f(g(cat, y), dog)
  \{x / cat, y / dog\}

- Unify(f(g(x)), f(x))
  Fails: no substitution makes them identical.
  E.g. \{x / g(x)\} yields f(g(g(x))) and f(g(x))
  which are not identical!

- Thus: A variable value may not contain itself in a substitution
  Directly or indirectly
Unification Examples III

• Unify(f(g(cat, y), y), f(x, dog))
  \{ x / g(cat, dog), y / dog \}
• Unify(f(g(y)), f(x))
  \{ x / g(y) \}

• Back to curious monkeys:
  \[
  \begin{array}{l}
  \text{Monkey}(x) \quad \text{Curious}(x) \\
  \text{Monkey}(\text{George}) \\
  \hdashline
  \text{Curious}(\text{George})
  \end{array}
  \]
  Unify and then use modus ponens = 
  \textit{generalized modus ponens} 
  \textit{("Lifted" version of modus ponens)}

Inference I: Forward Chaining

• The algorithm:
  \[
  \begin{align*}
  &\text{Start with the KB} \\
  &\text{Add any fact you can generate with GMP (i.e.,} \\
  &\quad \text{unify expressions and use modus ponens) } \\
  &\text{Repeat until: goal reached or generation halts.}
  \end{align*}
  \]

• Sound? Complete? Decidable?
  Yes; yes for definite KB; no (see p. 283 in text)

• Speed concerns? Inefficiencies due to:
  Unification via exhaustive pattern matching; premise
  rechecking; irrelevant fact generation.
  (see p. 283-287 for strategies to increase speed)
Inference II: Backward Chaining

- The algorithm:
  - Start with KB and goal.
  - Find all rules whose results unify with goal:
    - Add the premises of these rules to the goal list
    - Remove the corresponding result from the goal list
  - Stop when:
    - Goal list is empty (SUCCEED) or
    - Progress halts (FAIL)

Inference III: Resolution

[Robinson 1965]

\{ (p \lor q), (\neg p \lor r \lor s) \} \vdash_R (q \lor r \lor s)

Recall Propositional Case:
- Literal in one clause
- Its negation in the other
- Result is disjunction of other literals
**First-Order Resolution**  
[Robinson 1965]

\[
\{ (p(x) \lor q(A), \neg p(B) \lor r(x) \lor s(y)) \} \\
\vdash R \\
(q(A) \lor r(B) \lor s(y))
\]

- Literal in one clause
- Negation of *something which unifies* in other
- Result is disjunction of all other literals with substitution based on MGU

Substitute MGU \{x/B\} in all literals

**Inference using First-Order Resolution**

- As before, use "proof by contradiction"  
  To show \( KB \models a \), show \( KB \land \neg a \) unsatisfiable

- Method
  
  Let \( S = KB \land \neg \text{goal} \)  
  Convert \( S \) to clausal form
  
  - Standardize apart variables
  
  - Move quantifiers to front, skolemize to remove \( \exists \)
  
  - Replace \( \Rightarrow \) with \( \lor \) and \( \neg \)
  
  - Demorgan's laws to get CNF (ands-of-ors)
  
  Resolve clauses in \( S \) until empty clause (unsatisfiable) or no new clauses added
First-Order Resolution Example

• Given
  \( \forall x \text{ man}(x) \Rightarrow \text{human}(x) \)
  \( \forall x \text{ woman}(x) \Rightarrow \text{human}(x) \)
  \( \forall x \text{ singer}(x) \Rightarrow \text{man}(x) \lor \text{woman}(x) \)
  \( \text{singer}(\text{Diddy}) \)

• Prove
  \( \text{human}(\text{Diddy}) \)

CNF representation (list of clauses):
\[ [\neg \text{m}(x), \text{h}(x)] [\neg \text{w}(y), \text{h}(y)] [\neg \text{s}(z), \text{m}(z), \text{w}(z)] [\text{s}(D)] [\neg \text{h}(D)] \]

FOL Resolution Example

\[ [\neg \text{m}(x), \text{h}(x)] [\neg \text{w}(y), \text{h}(y)] [\neg \text{s}(z), \text{m}(z), \text{w}(z)] [\text{s}(D)] [\neg \text{h}(D)] \]

\[ [\text{m}(D), \text{w}(D)] \]
\[ [\text{w}(D), \text{h}(D)] \]
\[ [\text{h}(D)] \]
\[ [] \]

Eh yo homies, dis proves \( \text{human}(\text{Diddy}) \)
FOL Resolution Example

- Much More Difficult Exercise:
  Prove human(MJ)

What about me?

FOL Resolution Example 2

Given
\( \forall x \exists y \text{Twin}(x) \Rightarrow \text{Twin}(y) \)
\( \text{Twin}(Ashley) \)

Prove
\( \text{Twin}(Diddy) \)

Skolemization

\[ [ (\neg T(x), T(F(x))) \text{ (T(A)) } (\neg T(D)) ] \]

\( T(F(A))) \)
\( T(F(F(A)))) \)
\( T(F(F(F(A)))) \)

May not terminate!
Inference IV: Compilation to Prop. Logic

- Sentence $S$:
  $\forall_{\text{city}} \, a, b \, \text{Connected}(a, b)$
- Universe
  Cities: seattle, tacoma, enumclaw
- Equivalent propositional formula?

$$Cst \land Cse \land Cts \land Cte \land Ces \land Cet$$

Compilation to Prop. Logic (cont)

- Sentence $S$:
  $\exists_{\text{city}} \, c \, \text{Biggest}(c)$
- Universe
  Cities: seattle, tacoma, enumclaw
- Equivalent propositional formula?

$$Bs \lor Bt \lor Be$$
Compilation to Prop. Logic
(cont again)

- Universe
  - Cities: seattle, tacoma, enumclaw
  - Firms: IBM, Microsoft, Boeing
- First-Order formula
  \[ \forall_{\text{firm}} f \exists_{\text{city}} c \ \text{HeadQuarters}(f, c) \]
- Equivalent propositional formula
  \[
  \begin{align*}
  & (\text{HQis} \lor \text{HQit} \lor \text{HQie}) \land \\
  & (\text{HQms} \lor \text{HQmt} \lor \text{HQme}) \land \\
  & (\text{HQbs} \lor \text{HQbt} \lor \text{HQbe})
  \end{align*}
  \]

Hey!

- You said FO Inference is semi-decidable
- But you compiled it to SAT
  Which is NP Complete
- So now we can always do the inference?!?
  (might take exponential time but still decidable?)
- Something seems wrong here....????
  Something to ponder over the weekend...