Recall: FOL Definitions

- **Constants**: George, Monkey2, etc.
  - Name a specific object.
- **Variables**: X, Y.
  - Refer to an object without naming it.
- **Functions**: banana-of, grade-of, etc.
  - Mapping from objects to objects.
- **Terms**: George, grade-of(stdnt1)
  - Refer to objects
- **Relations**: Curious, PokesInTheEyes, etc.
  - State relationships between objects.
- **Atomic Sentences**: PokesInTheEyes(Moe, Curly)
  - Can be true or false
  - Correspond to propositional symbols P, Q
More Definitions

• **Logical connectives**: and, or, not, ⇒, ⇔

• **Quantifiers**:
  - ∀ For all (Universal quantifier)
  - ∃ There exists (Existential quantifier)

• **Examples**
  - Monkeys are curious
    ∀m: Monkey(m) ⇒ Curious(m)

  - There is a curious monkey
    ∃m: Monkey(m) ∧ Curious(m)

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Nested Quantifiers:
Order matters!

∀x∃y P(x,y) ≠ ∃y∀xP(x,y)

• **Examples**
  - Every monkey has a tail
    ∀m∃t has(m,t)

  - Every monkey shares a tail!
    ∃t∀m has(m,t)

  - Everybody loves somebody vs. Someone is loved by everyone
    ∀x∃y loves(x, y) vs. ∃y∀x loves(x, y)
Semantics

- **Semantics** = what the arrangement of symbols means in the world
- Propositional logic
  - Basic elements are **variables**
    - (refer to facts about the world)
  - Possible worlds: mappings from variables to T/F
- First-order logic
  - Basic elements are **terms**
    - (refer to objects)
  - **Interpretations**: mappings from terms to real-world elements.

Example: A World of Kings and Legs

- **Syntactic elements:**
  - **Constants:** Richard John
  - **Functions:** LeftLeg(p)
  - **Relations:** On(x,y) King(p)
Interpretation I

- Interpretations map syntactic tokens to model elements

Constants: Richard, John
Functions: LeftLeg(p)
Relations: On(x, y), King(p)

Interpretation II

Constants: Richard, John
Functions: LeftLeg(p)
Relations: On(x, y), King(p)
How Many Interpretations?

• **Two constants** (and 5 objects in world)
  Richard, John (R, J, crown, RL, JL)
  \[5^2 = 25\] object mappings

• **One unary relation**
  \(\text{King}(x)\)
  *Infinite* number of values for \(x\)  infinite mappings
  Even if we restricted \(x\) to: R, J, crown, RL, JL:
  \[2^5 = 32\] unary truth mappings

• **Two binary relations**
  \(\text{Leg}(x, y); \text{On}(x, y)\)
  Infinite. But even restricting \(x, y\) to five objects
  still yields \(2^{25}\) mappings *for each* binary relation

Satisfiability, Validity, & Entailment

• **S is valid** if it is true in all interpretations

• **S is satisfiable** if it is true in some interp

• **S is unsatisfiable** if it is false all interps

\[\vdash\]

• **S1 entails S2** if
  For all interps where \(S1\) is true,
  \(S2\) is also true
**Propositional Logic vs. First Order**

<table>
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<th>Ontology</th>
<th>Facts (P, Q, …)</th>
<th>Objects, Properties, Relations</th>
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<td>Syntax</td>
<td>Atomic sentences, Connectives</td>
<td>Variables &amp; quantification, Sentences have structure: terms father-of(mother-of(X))</td>
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**First-Order Wumpus World**

- **Objects**
  - Squares, wumpuses, agents, gold, pits, stinkiness, breezes
- **Relations**
  - Square topology (adjacency), Pits/breezes, Wumpus/stinkiness
Wumpus World: Squares

- Each square as an object:
  - Square\textsubscript{1,1}, Square\textsubscript{1,2}, ..., Square\textsubscript{3,4}, Square\textsubscript{4,4}
- Square topology relations?
  - Adjacent(Square\textsubscript{1,1}, Square\textsubscript{2,1})
  - Adjacent(Square\textsubscript{3,4}, Square\textsubscript{4,4})
- **Better:** Squares as lists:
  - [1, 1], [1, 2], ..., [4, 4]
- **Square topology relations:**
  - \(\forall x, y, a, b: \text{Adjacent}([x, y], [a, b]) \in \{[x+1, y], [x-1, y], [x, y+1], [x, y-1]\}\)

Wumpus World: Pits

- Each pit as an object:
  - Pit\textsubscript{1,1}, Pit\textsubscript{1,2}, ..., Pit\textsubscript{3,4}, Pit\textsubscript{4,4}
- **Problem?**
  - Not all squares have pits
- **List only the pits we have?**
  - Pit\textsubscript{3,1}, Pit\textsubscript{3,3}, Pit\textsubscript{4,4}
- **Problem?**
  - No reason to distinguish pits (same properties)
- **Better:** pit as unary predicate
  - Pit(x)
  - Pit([3,1]) \land Pit([3,3]) \land Pit([4,4]) will be true
Wumpus World: Breezes

• Represent breezes like pits, as unary predicates:
  Breezy(x)

• “Squares next to pits are breezy”:
\[
\forall x, y, a, b:\ 
Pit([x, y]) \land Adjacent([x, y], [a, b]) \Rightarrow Breezy([a, b])
\]

Wumpus World: Wumpuses

• Wumpus as object:
  Wumpus

• Wumpus home as unary predicate:
  WumpusIn(x)

• Better: Wumpus’s home as a function:
  Home(Wumpus) references the wumpus’s home square.
FOL Reasoning: Outline

- Basics of FOL reasoning
- Classes of FOL reasoning methods
  - Forward & Backward Chaining
  - Resolution
  - Compilation to SAT

Basics: Universal Instantiation

- Universally quantified sentence:
  \[ \forall x : \text{Monkey}(x) \Rightarrow \text{Curious}(x) \]

- Intuitively, \( x \) can be anything:
  - \( \text{Monkey}(\text{George}) \Rightarrow \text{Curious}(\text{George}) \)
  - \( \text{Monkey}(473\text{Student1}) \Rightarrow \text{Curious}(473\text{Student1}) \)
  - \( \text{Monkey}(\text{DadOf(George)}) \Rightarrow \text{Curious}(\text{DadOf(George)}) \)

- Formally:

  \[
  \begin{array}{c|c|c|c}
  \forall x & S & \forall x & \text{Monkey}(x) \\
  \text{Subst}\{\{x/p\}, S\} & \text{Monkey}(\text{George}) & \text{Curious}(\text{George}) \\
  \end{array}
  \]

  \( x \) is replaced with \( p \) in \( S \),
  and the quantifier removed

  \( x \) is replaced with \( \text{George} \) in \( S \),
  and the quantifier removed
**Basics: Existential Instantiation**

- **Existentially quantified sentence:**
  \[ \exists x: \text{Monkey}(x) \land \neg \text{Curious}(x) \]

- **Intuitively, \( x \) must name something. But what?**
  
  \[ \text{Monkey}(\text{George}) \land \neg \text{Curious}(\text{George}) \]
  No! \( S \) might not be true for \text{George}!

- **Use a \text{Skolem Constant}:**
  
  \[ \text{Monkey}(\text{K}) \land \neg \text{Curious}(\text{K}) \]
  ...where \( \text{K} \) is a \textbf{completely new symbol} (stands for the monkey for which the statement is true)

- **Formally:**
  \[
  \exists x \quad S \\
  Subst([x/K], S) \\
  \text{K is called a Skolem constant}
  \]

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**Basics: Generalized Skolemization**

- **What if our existential variable is nested?**
  
  \[ \forall x \exists y: \text{Monkey}(x) \Rightarrow \text{HasTail}(x, y) \]
  \[ \forall x: \text{Monkey}(x) \Rightarrow \text{HasTail}(x, \text{K_Tail}) \]

- **Existential variables can be replaced by \text{Skolem functions}**
  
  Argd to function are all surrounding \( \forall \) vars
  
  \[ \forall x: \text{Monkey}(x) \Rightarrow \text{HasTail}(x, \text{f}(x)) \]
  "tail-of" function
Next Time

• **Reasoning with FOL**
  - Unification
  - Chaining
  - Resolution