Solving Problems by Searching
Terminology

- State
- State Space
- Goal
- Action
- Cost
- State Change Function
- Problem-Solving Agent
- State-Space Search
Formal State-Space Model

Problem = \((S, s, A, f, g, c)\)

- **S** = state space
- **s** = initial state
- **A** = actions
- **f** = state change function \(f: S \times A \rightarrow S\)
- **g** = goal test function \(g: S \rightarrow \{\text{true}, \text{false}\}\)
- **c** = cost function \(c: S \times A \times S \rightarrow \mathbb{R}\)

• How do we define a solution?
• How about an optimal solution?
3 Coins Problem
A Very Small State Space Problem

• There are 3 (distinct) coins: coin1, coin2, coin3.

• The initial state is: $H\ H\ T$

• The legal operations are to turn over exactly one coin.
  – 1 (flip coin1), 2 (flip coin2), 3 (flip coin3)

• There are two goal states: $H\ H\ H$
  $T\ T\ T$

What are $S, s, A, f, g, c$ ?
What are some solutions?
What if the problem is changed to allow only 3 actions?
Modified State-Space Problem

• How would you define a state for the new problem?

• How do you define the operations (1, 2, 3) with this new state definition?

• What do the paths to the goal states look like now?
How do we build a search tree for the modified 3 coins problem?
## The 8-Puzzle Problem

<table>
<thead>
<tr>
<th>one initial state</th>
<th>goal state</th>
<th>B=blank</th>
</tr>
</thead>
<tbody>
<tr>
<td>7 2 4</td>
<td>B 1 2</td>
<td></td>
</tr>
<tr>
<td>5 B 6</td>
<td>3 4 5</td>
<td></td>
</tr>
<tr>
<td>8 3 1</td>
<td>6 7 8</td>
<td></td>
</tr>
</tbody>
</table>

1. Formulate a state as a data structure.
2. Show how start and goal states are represented.
3. How many possible states are there?
4. How would you specify the state-change function?
5. What is the goal test?
6. What is the path cost function?
7. What is the complexity of the search?
Search Tree Example:
Fragment of 8-Puzzle Problem Space
Another Example: N Queens

• Input:
  – Set of states
  – Operators [and costs]
  – Start state
  – Goal state (test)

• Output
Example: Route Planning

- **Input:**
  - Set of states
  - Operators [and costs]
  - Start state
  - Goal state (test)

- **Output:**
Search Strategies

• Blind Search (Ch 3)
  – Depth first search
  – Breadth first search
  – Depth limited search
  – Iterative deepening search

• Informed Search (Ch 4)

• Constraint Satisfaction (Ch 5)
Depth First Search

• Maintain stack of nodes to visit
• Evaluation
  – Complete? *Not for infinite spaces*
  – Time Complexity? \(O(b^d)\)
  – Space? \(O(d)\)
Breadth First Search

- Maintain queue of nodes to visit
- Evaluation
  - Complete? Yes
  - Time Complexity? $O(b^d)$
  - Space? $O(b^d)$
The Missionaries and Cannibals Problem
(from text problem 3.9)

- Three missionaries and three cannibals are on one side of a river, along with a boat that can hold one or two people.

- If there are ever more cannibals than missionaries on one side of the river, the cannibals will eat the missionaries. (We call this a “dead” state.)

- Find a way to get everyone to the other side, without anyone getting eaten.
Missionaries and Cannibals Problem

The chief said to the first missionary...

Death? or Bunga-Bunga?

And the missionary said...

Well, I guess nothing's worse than death. I'll take Bunga-Bunga.
Missionaries and Cannibals Problem
Missionary and Cannibals Notes

- Define your state as (M,C,S)
  - M: number of missionaries on left bank
  - C: number of cannibals on left bank
  - S: side of the river that the boat is on

- When the boat is moving, we are in between states. When it arrives, everyone gets out.
When is a state considered “DEAD”?

1. There are more cannibals than missionaries on the left bank.  (Bunga-Bunga)

2. There are more cannibals than missionaries on the right bank.  (Bunga-Bunga)

3. There is an ancestor state of this state that is exactly the same as this state.  (Why?)
Assignment (problem 3.9b, which is part of the first homework set)

• Implement and solve the problem
  – First with a blind depth-first search using a stack and/or recursion.
  – Second with a blind breadth-first search.
  – Definitely avoid repeated states.
  – Keep track of how many states are searched in each.

• Use the computer language of your choice for this assignment.
  – Java
  – C++
  – Lisp or Lisp variant
Is memory a limitation in search?

• Suppose:
  • 2 GHz CPU
  • 1 GB main memory
  • 100 instructions / expansion
  • 5 bytes / node

• 200,000 expansions / sec
• Memory filled in 100 sec … < 2 minutes
Iterative Deepening Search

• DFS with limit; incrementally grow limit
• Evaluation
  – Complete?  
    Yes
  – Time Complexity?  
    $O(b^d)$
  – Space Complexity?  
    $O(d)$
## Cost of Iterative Deepening

<table>
<thead>
<tr>
<th>b</th>
<th>ratio ID to DFS</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>5</td>
<td>1.5</td>
</tr>
<tr>
<td>10</td>
<td>1.2</td>
</tr>
<tr>
<td>25</td>
<td>1.08</td>
</tr>
<tr>
<td>100</td>
<td>1.02</td>
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</tbody>
</table>
vs. Bidirectional
Problem

• All these methods are too slow for real applications (blind)

• Solution → add guidance

• “informed search”