1 Question 1

1.1 Part A

Make each country into a graph node, and draw an edge between each pair of country nodes if those countries share an edge in the map.

1.2 Part B

F, because it's the most constrained and constrained to one value.

1.3 Part C

A as green because it is the least constraining value.

1.4 Part D

A, C, D, E must be different colors, but they all touch each other and there are only three available colors. In the constraint graph this is represented as a fully connected subgraph involving more nodes (4) than values available (3).

2 Question 2

2.1 Part A

$$\forall x, y, z\, Mother(y, x) \wedge Father(z, y) \Rightarrow MaternalGrandfather(z, x)$$

$$\forall x, y\, MaternalGrandfather(y, x) \wedge Bald(y) \wedge Male(x) \Rightarrow Bald(x)$$

$$Bald(Alan)$$

$$Father(Alan, Brenda)$$

$$Mother(Brenda, Chris)$$

$$Male(Chris)$$

2.2 Part B

Substitute $\{y/Brenda, x/Chris, z/Alan\}$:

$$Mother(Brenda, Chris) \land Father(Alan, Brenda) \Rightarrow MaternalGrandfather(Alan, Chris)$$

Substitute $\{y/Alan, x/Chris\}$:

$$MaternalGrandfather(Alan, Chris) \land Bald(Alan) \Rightarrow Bald(Chris)$$

Given $Mother(Brenda, Chris) \land Father(Alan, Brenda)$, we apply modus ponens to get $MaternalGrandfather(Alan, Chris)$.

Given $MaternalGrandfather(Alan, Chris) \land Bald(Alan)$, we apply modus ponens to get $Bald(Chris)$.

2.3 Part C

Convert the first implication to conjunctive normal form.

$$\neg(Mother(y, x) \land Father(z, y)) \lor MaternalGrandfather(z, x)$$

$$\neg Mother(y, x) \lor \neg Father(z, y) \lor MaternalGrandfather(z, x)$$

Convert the second implication to conjunctive normal form.

$$\neg(MaternalGrandfather(y, x) \land Bald(y) \land Male(x)) \lor Bald(x)$$

$$\neg MaternalGrandfather(y, x) \lor \neg Bald(y) \lor \neg Male(x) \lor Bald(x)$$

Perform the following substitutions on the first implication: $\{y/Brenda, x/Chris, z/Alan\}$

$\neg Mother(Brenda, Chris) \vee \neg Father(Alan, Brenda) \vee MaternalGrandfather(Alan, Chris)$

Perform the following substitutions on the second implication: $\{y/Alan, x/Chris\}$

$\neg MaternalGrandfather(Alan, Chris) \vee \neg Bald(Alan) \vee \neg Male(Chris) \vee Bald(Chris)$

Apply resolution between these two implications

$\neg Mother(Brenda, Chris) \vee \neg Father(Alan, Brenda) \vee \neg Bald(Alan) \vee \neg Male(Chris) \vee Bald(Chris)$

Use the following implications to finish the derivation:

$$true \vee Mother(Brenda, Chris)$$

to get

$$\neg Father(Alan, Brenda) \vee \neg Bald(Alan) \vee \neg Male(Chris) \vee Bald(Chris)$$

then

$$true \vee Father(Alan, Brenda)$$

to get

$$\neg Bald(Alan) \vee \neg Male(Chris) \vee Bald(Chris)$$

then

$$true \vee Bald(Alan)$$

to get

$$\neg Male(Chris) \vee Bald(Chris)$$

then

$$true \vee Male(Chris)$$

to get

$$Bald(Chris)$$

3 Question 3

3.1 Part A

$$P(A, B, C, D, E, F, G) = P(G|E, F)P(E|D)P(F|D)P(D|B, C)P(B|A)P(C|A)P(A)$$

3.2 Part B

There are 7 variables.

$$cost_{full} = 2^7 - 1 = 127$$

$$cost_{belief} = \sum_{i=1}^{n} 2^{|Parents(X_i)|} = 1 + 2 + 2 + 4 + 2 + 2 + 4 = 17$$

3.3 Part C

No, because the path A-C-D-E-G is not blocked. C, D, and E are not evidence, and case three does not apply.

3.4 Part D

No, because G introduces a dependency between E and F.

4 Question 4

4.1 Part A

$$\begin{aligned} P(W) &= P(W|S,R)P(S)P(R) \\ &+ P(W|\neg S,R)P(\neg S)P(R) \\ &+ P(W|S,\neg R)P(S)P(\neg R) \\ &+ P(W|\neg S,\neg R)P(\neg S)P(\neg R) \end{aligned}$$

$$P(W) = 0.99(0.5)(0.5) + 0.9(0.5)(0.5) + 0.9(0.5)(0.5) + 0.01(0.5)(0.5) = 0.7$$

4.2 Part B

$$P(R|W) = \frac{P(R,W)}{P(W)}$$

$$P(R|W) = \frac{P(W|R,\neg S)P(R)P(\neg S) + P(W|R,S)P(R)P(S)}{P(W)}$$

$$P(R|W) = \frac{0.9(0.5)(1.0-0.5) + 0.99(0.5)(0.5)}{0.7}$$

$$P(R|W) = \frac{0.4725}{0.7} = 0.675$$

4.3 Part C

$$P(R|W,S) = \frac{P(R,W,S)}{P(W,S)}$$

$$P(R|W,S) = \frac{P(W|S,R)P(S)P(R)}{P(W|S,R)P(S)P(R) + P(W|S,\neg R)P(S)P(\neg R)}$$

$$P(R|W,S) = \frac{0.99(0.5)(0.5)}{0.99(0.5)(0.5) + 0.90(0.5)(0.5)}$$

$$P(R|W,S) = \frac{0.99}{0.99 + 0.90} = 0.524$$

4.4 Part D

Now that we know the sprinker is on, we have a probable cause for the wet grass, thereby making the chance that it rained smaller. The sprinkler explains away the wet grass.