1 Question 1

1.1 Part A

Make each country into a graph node, and draw an edge between each pair of country nodes if those countries share an edge in the map.

1.2 Part B

F, because it’s the most constrained and constrained to one value.

1.3 Part C

A as green because it is the least constraining value.

1.4 Part D

A, C, D, E must be different colors, but they all touch each other and there are only three available colors. In the constraint graph this is represented as a fully connected subgraph involving more nodes (4) than values available (3).

2 Question 2

2.1 Part A

\[ \forall x, y, z \text{Mother}(y, x) \land \text{Father}(z, y) \Rightarrow \text{MaternalGrandfather}(z, x) \]

\[ \forall x, y \text{MaternalGrandfather}(y, x) \land \text{Bald}(y) \land \text{Male}(x) \Rightarrow \text{Bald}(x) \]

\[ \text{Bald}(Alan) \]

\[ \text{Father}(Alan, Brenda) \]
Mother(Brenda,Chris)

Male(Chris)

2.2 Part B

Substitute \{y/Brenda, x/Chris, z/Alan\}:

Mother(Brenda,Chris) ∧ Father(Alan, Brenda) ⇒ MaternalGrandfather(Alan,Chris)

Substitute \{y/Alan, x/Chris\}:

MaternalGrandfather(Alan,Chris) ∧ Bald(Alan) ⇒ Bald(Chris)

Given Mother(Brenda,Chris) ∧ Father(Alan, Brenda), we apply modus ponens to get MaternalGrandfather(Alan,Chris).

Given MaternalGrandfather(Alan,Chris) ∧ Bald(Alan), we apply modus ponens to get Bald(Chris).

2.3 Part C

Convert the first implication to conjunctive normal form.

\neg(Mother(y,x) ∧ Father(z,y)) ∨ MaternalGrandfather(z,x)

\neg Mother(y,x) ∨ \neg Father(z,y) ∨ MaternalGrandfather(z,x)

Convert the second implication to conjunctive normal form.

\neg(MaternalGrandfather(y,x) ∧ Bald(y) ∧ Male(x)) ∨ Bald(x)

\neg MaternalGrandfather(y,x) ∨ \neg Bald(y) ∨ \neg Male(x) ∨ Bald(x)
Perform the following substitutions on the first implication: \{y/Brenda, x/Chris, z/Alan\}

\(\neg\text{Mother}(Brenda, Chris) \lor \neg\text{Father}(Alan, Brenda) \lor \text{MaternalGrandfather}(Alan, Chris)\)

Perform the following substitutions on the second implication: \{y/Alan, x/Chris\}

\(\neg\text{MaternalGrandfather}(Alan, Chris) \lor \neg\text{Bald}(Alan) \lor \neg\text{Male}(Chris) \lor \text{Bald}(Chris)\)

Apply resolution between these two implications

\(\neg\text{Mother}(Brenda, Chris) \lor \neg\text{Father}(Alan, Brenda) \lor \neg\text{Bald}(Alan) \lor \neg\text{Male}(Chris) \lor \text{Bald}(Chris)\)

Use the following implications to finish the derivation:

\[true \lor \text{Mother}(Brenda, Chris)\]

to get

\(\neg\text{Father}(Alan, Brenda) \lor \neg\text{Bald}(Alan) \lor \neg\text{Male}(Chris) \lor \text{Bald}(Chris)\)

then

\[true \lor \text{Father}(Alan, Brenda)\]

to get

\(\neg\text{Bald}(Alan) \lor \neg\text{Male}(Chris) \lor \text{Bald}(Chris)\)

then

\[true \lor \text{Bald}(Alan)\]

to get

\(\neg\text{Male}(Chris) \lor \text{Bald}(Chris)\)

3
then

true \lor \text{Male}(\text{Chris})

to get

\text{Bald}(\text{Chris})

3 Question 3

3.1 Part A

\begin{equation}
P(A, B, C, D, E, F, G) = P(G|E, F)P(E|D)P(F|D)P(D|B, C)P(B|A)P(C|A)P(A)
\end{equation}

3.2 Part B

There are 7 variables.

\begin{align*}
cost_{\text{full}} & = 2^7 - 1 = 127 \\
cost_{\text{belief}} & = \sum_{i=1}^{n} 2^{\text{Parents}(X_i)} = 1 + 2 + 2 + 4 + 2 + 2 + 4 = 17
\end{align*}

3.3 Part C

No, because the path A-C-D-E-G is not blocked. C, D, and E are not evidence, and case three does not apply.

3.4 Part D

No, because G introduces a dependency between E and F.
4 Question 4

4.1 Part A

\[ P(W) = P(W|R)P(S)P(R) + P(W|\neg S,R)P(\neg S)P(R) + P(W|R)P(S)P(\neg R) + P(W|\neg S,\neg R)P(\neg S)P(\neg R) \]

\[ P(W) = 0.99(0.5)(0.5) + 0.9(0.5)(0.5) + 0.9(0.5)(0.5) + 0.01(0.5)(0.5) = 0.7 \]

4.2 Part B

\[ P(R|W) = \frac{P(R,W)}{P(W)} \]

\[ P(R|W) = \frac{P(W|R,\neg S)P(R)P(\neg S) + P(W|R,S)P(R)P(S)}{P(W)} \]

\[ P(R|W) = \frac{0.9(0.5)(1.0 - 0.5) + 0.99(0.5)(0.5)}{0.7} \]

\[ P(R|W) = \frac{0.4725}{0.7} = 0.675 \]

4.3 Part C

\[ P(R|W,S) = \frac{P(R,W,S)}{P(W,S)} \]

\[ P(R|W,S) = \frac{P(W|R,S)P(S)P(R)}{P(W|R,S)P(S)P(R) + P(W|R,\neg S)P(S)P(\neg R)} \]
\[ P(R|W,S) = \frac{0.99(0.5)(0.5)}{0.99(0.5)(0.5) + 0.90(0.5)(0.5)} \]

\[ P(R|W,S) = \frac{0.99}{0.99 + 0.90} = 0.524 \]

### 4.4 Part D

Now that we know the sprinkler is on, we have a probable cause for the wet grass, thereby making the chance that it rained smaller. The sprinkler explains away the wet grass.