Question 1

(8 points)

Recall that the heuristic function in best-first search is $f(n) = g(n) + h(n)$, where $g(n)$ is the exact cost of getting to the current node n, and $h(n)$ is the estimated minimum cost of getting from n to a goal state.

a (4 points) Suppose we run a greedy search algorithm with $h(n) = -g(n)$. What sort of search will the greedy search emulate?

The best nodes (those with the lowest scores), will be those with the longest paths, so this emulates depth first search. A common mistake was to confuse greedy search with best first, or A*. Greedy search uses $f(n) = h(n)$. Since in this case $h(n) = -g(n)$, the deeper a node is in the tree, the better it's f-cost will be.

A common mistake was to use $f(n) = g(n) + h(n)$, which is the formula for best-first, but not greedy search. In this case, $f(n)$ becomes 0, so the search is effectively random (actually, it depends on the details of the queueing function.)

b (4 points) Prove that if the heuristic function h obeys the triangle inequality, then the f-cost along any path in the search tree is nondecreasing. (The triangle inequality says that the sum of the costs from A to B and B to C must not be less than the cost from A to C directly.)

Think of the triangle inequality as meaning that the direct route from A to C is faster than the indirect route through B.

Nondecreasing f-cost along a path means that f of a successor is always at least as large as that of the node:

$$f(n) \leq f(n') \text{if} n' \in S(n)$$

Substituting $f(n) = g(n) + h(n)$ we get:

$$g(n) + h(n) \leq g(n') + h(n') \text{if} n' \in S(n)$$

Our goal is to show that this is implied by the triangle inequality. The triangle inequality applied to a heuristic $h(n)$ says that

$$h(n) \leq k(n, n') + h(n')$$

for any nodes n, n', where $k(n, n')$ is the cost of the shortest path from n to n'. Adding $g(n)$ to both sides we get

$$g(n) + h(n) \leq g(n) + k(n, n') + h(n')$$

But if n' is a successor of n, then $g(n) + k(n, n')$ is equal to $g(n')$.

So

$g(n) + h(n) \leq g(n') + h(n')$

Question 2

(8 + 4 bonus points)

Consider the following map coloring problem:

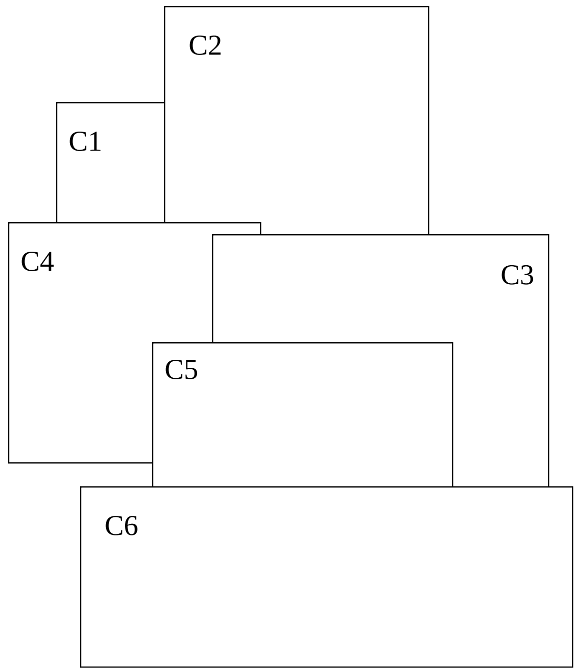


Figure 1: Assign each region one of the three colors (Red, Green or Blue) so that no two adjacent regions have the same colors.

- **a** (8 points) In class, we studied two heuristics for CSPs, *least-constraining-value* and *most-constrained-variable*. Solve the graph coloring problem above using these two heuristics and forward checking. (Show work on back)
- **b** (BONUS 4 points) Describe an additional heuristic that would be useful in solving this problem.

  If there is a tie for most-constrained-variable, as in the first choice, use the *most-constraining-variable*, that is, the variable whose assignment will add the most new constraints. For example, using this heuristic it makes sense to select C4 first, since it limits the choices for all other variables except C6. Several people suggested using MOMS, this was OK, if you discuss converting the problem to SAT.

C4 = Red	Red	Green	Blue
C1	X		
C2	X		
C3	X		
C4	$\surd$		
C5	X		
C6			

C5 = Green	Red	Green	Blue
C1	X		
C2	X		
C3	X	X	
C4	$\surd$	X	
C5	X	$\surd$	
C6		X	

C3 = Blue	Red	Green	Blue
C1	X		
C2	X		X
C3	X	X	$\surd$
C4	$\surd$	X	X
C5	X	$\surd$	X
C6		X	X

C2 = Green	Red	Green	Blue
C1	X	X	
C2	X	$\surd$	X
C3	X	X	$\surd$
C4	$\surd$	X	X
C5	X	$\surd$	X
C6		X	X

C1 = Blue	Red	Green	Blue
C1	X	X	$\surd$
C2	X	$\surd$	X
C3	X	X	$\surd$
C4	$\surd$	X	X
C5	X	$\surd$	X
C6		X	X

C6 = Red	Red	Green	Blue
C1	X	X	$\surd$
C2	X	$\surd$	X
C3	X	X	$\surd$
C4	$\surd$	X	X
C5	X	$\surd$	X
C6	$\surd$	X	X

Table 1: CSP solution

Question 3

(6 points)

Let us consider the problem of search in a *three-player* game. (You can assume no alliances are allowed.) We will call the players 0, 1, and 2 for convenience. Assume you have an evaluation function that returns a list of three values, indicating (say) the likelihood of winning for players 0, 1 and 2, respectively. Complete the following game tree by filling in the backed-up values for all remaining nodes including the root.

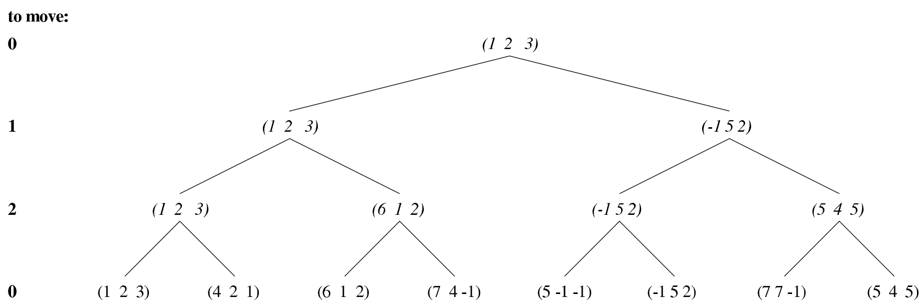


Figure 2: The first three ply of a game tree with three players (0, 1, and 2).

The important thing to understand is that each player will act so as to maximize their score from the choices presented to them, and the the score vectors are moved up the tree as units (no mixing) because they represent the value to each player of a particular game.

Question 4

(4 points)

Given the following, can you prove that the unicorn is mythical? How about magical? Horned?

> If the unicorn is mythical, then it is immortal, but if it is not mythical, then it is a mortal mammal. If the unicorn is either immortal or a mammal, then it is horned. The unicorn is magical if it is horned.

First a proof by counterexample that you can't prove that the unicorn is mythical:

It is consistent with the above sentences for the unicorn to be a non-mythical, magical, mortal, horned mammal.

Now for the tedious version, let us define the following propositions:

$MYTHICAL$**: The unicorn is mythical.**
$MORTAL$**: The unicorn is mortal (Which means that** $\neg MORTAL$ **translates as "The unicorn is immortal.")**
$MAMMAL$**: The unicorn is a mammal.**
$HORNED$**: The unicorn is horned.**
$MAGICAL$**: The unicorn is magical.**

We can now translate the statements above as:

$MYTHICAL \rightarrow \neg MORTAL$
$\neg MYTHICAL \rightarrow MORTAL \wedge MAMMAL$
$(\neg MORTAL \vee MAMMAL) \rightarrow HORNED$
$HORNED \rightarrow MAGICAL$
First, we'll look at the last two questions, which are easy. Since either $MYTHICAL$ **or** $\neg MYTHICAL$ **must be true, either** $\neg MORTAL$ **or** $MORTAL \wedge MAMMAL$ **must be true, which means that** $(\neg MORTAL \vee MAMMAL)$ **is true. By the third premise, this means that** $HORNED$ **is true, and by the fourth premise,** $MAGICAL$ **is also true. So we can prove** $MAGICAL$ **and** $HORNED$**.**

To prove $MYTHICAL$**, we would have to to show that** $MORTAL \wedge MAMMAL$ **is false. We could show this by denying either conjunct. If we wanted to show** $MORTAL$ **was false, we'd need to prove** $MYTHICAL$ **and use premise 1, but that would involve a circularity. If we try to deny** $MAMMAL$**, the only way to do it would be to deny** $HORNED$**. The only rule which could achieve that is the last, and we would have to show** $\neg MAGICAL$**. Since there is no way to show that, this also**

fails. This exhausts the possibilities for proving $MYTHICAL$, so it is unprovable from this set of sentences.

Question 5

(6 points)

Consider a world in which there are only four propositions, A, B, C, and D. How many models are there for the following sentences?

a (2 points) $A \wedge B$

4 - A and B must be true, but C and D are unconstrained

A	B	C	D
T	T	T	T
T	T	T	F
T	T	F	T
T	T	F	F

Table 2: Models for $A \wedge B$

b (2 points) $A \vee B$

12 - There are three ways to satisfy $A \vee B$ and for each one, four models corresponding to all possible truth values for C and D.

c (2 points) $A \wedge B \wedge C$

2 - One where D is true and one where D is false.

Question 6

(10 + 2 bonus points)

Here are two sentences in the language of first-order logic:

(A) $\forall x \exists y (x \geq y)$

(B) $\exists y \forall x (x \geq y)$

a (2 point) Assume that the variables range over all natural numbers $0, 1, 2, \ldots, \infty$, and that the "$\geq$" predicate means "greater than or equal to." Under this interpretation, translate the these sentences into English.

The first sentence translates as "For every natural number, there is some (other) natural number that it is greater than or equal to." The second, as "There is a specific natural number that is less than than or equal to every natural number."

b (1 point) Is (A) true under this interpretation?

Yes - for any natural number, you can pick itself as the "other" number.

c (1 points) Is (B) true under this interpretation?

Yes - The number 0 has this property.

d (2 points) Does (A) logically entail (B)?

No, (A) does not logically entail (B). (Counterexample: Consider the integers, A is true, but B is not.)

e (2 points) Does (B) logically entail (A)?

Yes, (B) logically entails (A)

f (2 points) Try to prove that (A) follows from (B) using resolution. Do this even if you think that (B) does not logically entail (A); continue until the proof breaks down and you cannot proceed (if it does break down). Show the unifying substitution for each resolution step. If the proof fails, explain exactly where, how and why it breaks down.

We set the knowledge base to the negation of (A) and (B). Again, we convert both sentences to canonical form (which requires introducing a Skolem constant for (A) and a Skolem function for (B)):

($\neg$A): $\neg(F_1 \geq y)$
(B): $x \geq F_2$
Resolving these clauses we use the substitution: $\{x/F_1, y/F_2\}$**. This gives us:**

$\neg(F_1 \geq F_2)$ **and** $F_1 \geq F_2$

Which does resolve, giving us *False* and proving that (B) entails (A)

g (BONUS 2 points) Now try to prove that (B) follows from (A).

We set the knowledge base to (A) and the negation of (B). First we convert both sentences to canonical form (which requires introducing Skolem functions):

(A): $x \geq F_1(x)$
(¬B): $\neg F_2(y) \geq y$
Now we try to derive a contradiction. There are only two clauses, so we try to unify them. The obvious unification would be: $\{x/F_2(y), y/F_1(x)\}$**, but this is equivalent to** $\{x/F_2(y), y/F_1(F_2(y))\}$**, which fails because an expression containing** y **is being substituted for** y**. The resolution fails, and there are no other clauses or unifications to try, so the proof fails.**

Question 7

(15 + 5 bonus points)

Two astronomers, in different parts of the world, make measurements M_1 and M_2 of the number of stars N in some small region of the sky, using their telescopes. Normally, there is a small possibility of error by up to one star. Each telescope can also (with a slightly smaller probability) be badly out of focus (events F_1 and F_2), in which case, the scientist will undercount by three or more stars. Consider the three networks shown below.

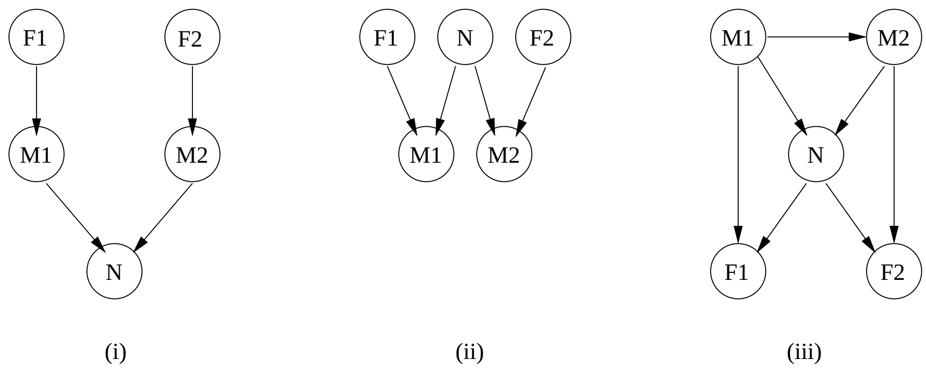


Figure 3: Three possible networks for the telescope problem.

a (5 points) Which of these belief networks correctly (but not necessarily efficiently) represent the above information?

Althrough (i) in some sense depicts the "flow of information" during calculation, it is clearly incorrect as a network, since it says that given the measurements M_1 and M_2, the number of stars is independent of the focus. (ii) correctly represents the causal structure: each measurement is influenced by the actual number of stars and the focus, and the two telescopes are independent of each other. (iii) shows a correct but more complicated network — the one obtained by ordering the nodes M_1, M_2, N, F_1, F_2. If you order M_2 before M_1 you would get the same network except with the arrow from M_1 to M_2 reversed.

Many people confused the notion of correctly representing the causal relationship with correctly representing the conditional independence relation.

b (5 points) Which is the best network?

(ii) requires fewer parameters and is therefore better than (iii).

(Many people wrote that (ii) is better because it is "more causally accurate." While this is true, it's only part of the picture.

c (5 points) Give a reasonable conditional probability table for the values of $\mathbf{P}(M_1|N)$. (For simplicity, consider only the possible values 1, 2, and 3 in this part.)

To compute $\mathbf{P}(M_1|N)$, we will need to condition on F_1 (that is, consider both possible cases for F_1, weighted by their probabilities.)

$\mathbf{P}(M_1|N) = \mathbf{P}(M_1|N, F_1)\mathbf{P}(F_1|N) + \mathbf{P}(M_1|N, \neg F_1)\mathbf{P}(\neg F_1|N)$
$\mathbf{P}(M_1|N) = \mathbf{P}(M_1|N, F_1)\mathbf{P}(F_1) + \mathbf{P}(M_1|N, \neg F_1)\mathbf{P}(\neg F_1)$
Let f be the probability that the telescope is out of focus. The problem states that this will cause an "undercount of three or more starts." For N=3 or less stars, we assume this means the count will be 0 if the telescope is out of focus. If it is in focus, then we will assume there is a probability of e of counting one too few, and e of counting one too many. The rest of the time $(1-2e)$, the count will be accurate. Then the table is as follows:

	$N = 1$	$N = 2$	$N = 3$
$M_1 = 0$	$f + e(1-f)$	f	f
$M_1 = 1$	$(1-2e)(1-f)$	$e(1-f)$	0.0
$M_1 = 2$	$e(1-f)$	$(1-2e)(1-f)$	$e(1-f)$
$M_1 = 3$	0.0	$e(1-f)$	$(1-2e)(1-f)$
$M_1 = 4$	0.0	0.0	$e(1-f)$

Table 3: Conditional probabilities of $M_1|N$

Notice that each column has to add up to 1. Reasonable values for e and f might be 0.05 and 0.02.

d (BONUS 5 points) Suppose $M_1 = 1$ and $M_2 = 3$. What are the possible numbers of stars?

Consider all the possible values of the focus and off-by-one variables, and the implications each has on the resulting possible values of N.

- **If neither F_1 nor F_2 are true, then the only possible value for N is 2 (astronomer 1 undercounts by 1 and astronomer 2 overcounts by 1).**
- **If F_1 is true and F_2 is false, then the only possible value for N is 4 (astronomer 1 undercounts by 3, astronomer 2 undercounts by 1).**

- **If F_1 if false, and F_2 is true, then there is no consistent value for N, so this can't be the case.**
- **Finally, if both F_1 and F_2 are true, then the possible values are $N \geq 6$, with astronomer 2 undercounting by $i \geq 3$ and astronomer 1 undercounting by $i + 2$.**

Question 8

(10 + 3 bonus points)

Let the instance space be $X = \{0, 1\}^4$, the training set be $D = \{(< 0, 0, 0, 0 >, 1)\}$, and the hypothesis space H be the set of conjunctions over X.

Notation: Let x_i be the ith attribute, and $\neg x$ denote "not x".

a (4 points) Compute the cardinality of the version space of H over D, $|VS_{H,D}|$.

$|VS| = 2^4 = 16$ **(VS = the set of conjunctions with all negated literals**

b (3 points) Derive the S and G frontiers using the candidate elimination algorithm.

$S = \{\neg x_1 \wedge \neg x_2 \wedge \neg x_3 \wedge \neg x_4\}$ **(singleton set),** $G = \{True\}$ **(the null conjunction)**

c (3 points) Suppose you see the additional example $(< 1, 1, 1, 1 >, 0)$. Derive the new S and G frontiers.

$S = \{\neg x_1 \wedge \neg x_2 \wedge \neg x_3 \wedge \neg x_4\}$**,** $G = \{\neg x_1, \neg x_2, \neg x_3, \neg x_4\}$

d (BONUS 3 points) Suppose you see one more example, $(< 0, 1, 1, 1 >, 1)$. Derive the new S and G frontiers.

$S = \{\neg x_1\}$**,** $G = \{\neg x_1\}$

Question 9

(4 points)

Suppose a training set is made up of 16 examples of class A, 8 examples of class B, 32 examples of class C, and 8 examples of class D. When growing a decision tree from this training set, what is the maximum information gain that any attribute can have?

Consider a single attribute x that is perfectly correlated with the class. I.e. $x = 1 \equiv C = A$, $x = 2 \equiv C = B$, $x = 3 \equiv C = C$, and $x = 4 \equiv C = D$. In this case, the entropy after splitting on this attribute will be 0 (all subsets are pure.) So the maximum information gain is the entropy of the training set - 0.

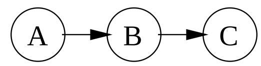
The entropy of the training set is:

$$
\begin{aligned}
H(D) &= -\frac{16}{64}\log_2(\frac{16}{64}) - \frac{8}{64}\log_2(\frac{8}{64}) - \frac{32}{64}\log_2(\frac{32}{64}) - \frac{8}{64}\log_2(\frac{8}{64}) \\
&= \frac{1}{4}\cdot 2 + \frac{1}{8}\cdot 3 + \frac{1}{2}\cdot 1 + \frac{1}{8}\cdot 3 \\
&= \frac{4}{8} + \frac{3}{8} + \frac{4}{8} + \frac{3}{8} \\
&= \frac{14}{8} = 1.75
\end{aligned}
$$

Question 10

(5 points)

Consider the following Bayesian network, in which variables A, B and C are Boolean:



Suppose you want to learn the parameters for this network using the training set $\{<0,1,1>, <1,0,0>, <1,1,1>, <1,?,0>\}$, where examples are in the form $<A,B,C>$, and "?" indicates a missing value. Show the sequence of filled-in values and parameters produced by the EM algorithm, assuming the parameters are initialized by ignoring missing values. (Hint: EM converges very quickly on this problem.)

Initialization:
$P(A) = 0.75$
$P(B|A) = 0.5, P(B|-A) = 1$
$P(C|B) = 1, P(C|-B) = 0$

First iteration:
E step:

$$
\begin{aligned}
P(?=1) &= P(B|A,-C) = \frac{P(A,B,-C)}{P(A,-C)} \\
&= \frac{P(A)P(B|A)P(-C|B)}{P(A)P(B|A)P(-C|B) + P(A)P(-B|A)P(-C|-B)} \\
&= \frac{(0.75 \cdot 0.5 \cdot 0)}{(0 + 0.75 \cdot 0.5 \cdot 1)} \\
&= 0
\end{aligned}
$$

So ? $=0$ with probability 1. Compute conditional probabilities with this substitution.
M step:
$P(A) = 0.75$
$P(B|A) = 0.333\ldots, P(B|-A) = 1$
$P(C|B) = 1, P(C|-B) = 0$

Second iteration:

E step:

$$\begin{aligned}
P(? = 1) &= P(B|A,-C) = \frac{P(A,B,-C)}{P(A,-C)} \\
&= \frac{P(A)P(B|A)P(-C|B)}{P(A)P(B|A)P(-C|B) + P(A)P(-B|A)P(-C|-B)} \\
&= \frac{(0.75 \cdot 0.333\ldots \cdot 0)}{(0 + 0.75 \cdot 0.666\ldots \cdot 1)} \\
&= 0
\end{aligned}$$

M step: Same result as first iteration (converged).

Question 12

(5 points)

Suppose you want to learn to recognize digits in a 7-segment LED display from noisy examples (i.e., each segment has been flipped with 10% probability). Which of the learning algorithms you studied would you use?

The best choice is naive Bayes, because the attributes (i.e., whether each segment is on or off) are independent given the class (i.e., the digit), so naive Bayes is the optimal classifier for this problem. Nearest-neighbor with overlap distance is also a reasonable solution, and can be given half-credit. A general Bayes net can also be given half-credit (it will have zero bias, like naive Bayes, but more variance, and will also be slower). Likewise for a neural net with one output per class and a "max" function. Decision trees and rules are the least appropriate.