Reasoning about Protocols

A Replay Attack

Key Server

1: A, B, N

Alice

2: \{ N, B, K_{ab}, K_{ab} A \} K_{bs} \{ K_{ab} \} K_{as}

3: K_{ab} \{ K_{as} \}

4: \{ N \} K_{ab}

5: \{ N_{b-1} \} K_{ab}

Bob

Sniffing Sam

3: \{ K_{ab}, A \} K_{bs}

… several days later...

4: \{ N_{b} \} K_{ab}

5: \{ N_{b-1} \} K_{ab}

Bob
A Logic of Authentication

- Seminal paper published in 1991 SOSP by Burrows, Abadi and Needham
  - BAN Logic
- Simple idea
  - make explicit assumptions in an authentication protocol
  - describe protocol by formal algebra
    - make explicit initial states
    - derive belief relationships through state transitions
    - final state tells us what we can know

Example Questions

- What does this protocol achieve?
- Does this protocol need more assumptions than another one?
- Does this protocol do anything unnecessary that could be left out without weakening it?
- Does this protocol encrypt something that could be sent in the clear
**Main Principles**

- Freshness
  - if you’ve sent Joe a number (nonce) you’ve never sent him before, and if you receive back from Joe something that depends on the number, then you ought to believe that Joe’s message is fresher than yours

- Shared secrets
  - if you believe that only you and Joe know X, then you ought to believe that any encrypted message you receive containing X originally comes from Joe

- Private key validity
  - if you believe that you and Joe know K, then you ought to believe that anything you receive encrypted with K comes from Joe.

- Public validity
  - if you believe that K is Joe’s public key, than you should believe that any message you can decrypt with K comes from Joe.

**Principles are not the same as facts**

- Joe could reveal his secret to someone.
- A bad guy could deduce a public/private key.
- A nonce may not actually be so fresh or random.
The Approach

- Describe the messages sent in a protocol
  - A -> B: {A, Kab}\text{Kbs}
    - “A sends to B a message containing \{A, Kab\} encrypted with the private key Kbs, where Kab is a key suggested by some server S.”
- Transform each message into a idealized message that can lead to beliefs
  - A -> B: {A \text{<-Kab->} B}\text{Kbs}
    - “A says to B that Kab is a good key for communicating between A and B according to S.”
  - Generally omit cleartext components since they can be forged and can not lead to new beliefs.
- Beliefs yield assertions about the system
  - B believes that S once said that Kab is a good key for communication between A and B.

Beliefs, Past and Present

- Parties only say what they believe.
- Present
  - begins with the start of the protocol
- Past
  - anything before the present
- If you believe something in the present, then you believe it for the run of the protocol.
- A belief held in the past (before the current run of the protocol) is not necessarily valid in the present.
  - beware of old beliefs
Encrypted Messages

- An encrypted message is a logical state concealed by an encryption key
  - \((A \leftrightarrow\Rightarrow B)_{Kbs}\)
  - “K is a good key for use between A and B”
- An encrypted message can not be understood by a principal who does not have the key

The Logic

- A, B, S denote principles
- Kab, Kas, Kbs denote shared (secret) keys
- Ka, Kb, Ks denote public keys
  - \(1/Ka, 1/Kb, 1/Ks\) denote matching private keys
- Na, Nb, Ns denote specific statements
  - eg, nonces, which can be used to establish freshness
- Conjunction (,) is only propositional connective
Some Constructs

- \( P \models X \)
  - \( P \) believes \( X \)
  - \( P \) may act as though \( X \) were true
- \( P \not\models X \)
  - \( P \) sees \( X \)
  - someone has sent \( P \) a message containing \( X \). \( P \) may repeat \( X \) in other messages
- \( P \models X \)
  - \( P \) once said \( X \)
  - \( P \) at one time sent a message containing the statement \( X \). No one knows how long ago.
- \( P \models X \)
  - \( P \) has jurisdiction over \( X \)
  - \( P \) is an authority on \( X \) and should be trusted on this matter. Used for delegation, eg, servers that generate keys.
- \#(\( X \))
  - The formula \( X \) is fresh
  - never been sent before this run of the protocol

- \( P \models -P \)

Message Meaning Rules

- How to derive beliefs from the contents of messages

\[
P \models Q \iff \text{K \rightarrow P, P \not\models \{X\}K} \\
P \models -Q \iff \{X\}K
\]

**SHARED KEYS**: If \( P \) believes that \( K \) is a good key for \( P \) and \( Q \), and \( P \) sees \( X \) encrypted with \( K \), then \( P \) believes that \( Q \) once said \( X \).

\[
P \models K \rightarrow Q, P \models \{X\}1/K \\
P \models -Q \iff \{X\}K
\]

**PUBLIC KEYS**: If \( P \) believes that \( K \) is \( Q \)'s public key, and \( P \) sees \( X \) encrypted with \( K \)'s private key, then \( P \) believes that \( Q \) once said \( X \).

\[
P \models Q = X \iff P, P \models <X>Y \\
P \models -Q \iff <X>Y
\]

**SHARED SECRETS**: If \( P \) believes that \( Y \) is a secret shared between \( Q \) and \( P \), and \( P \) see’s \( <X>Y \), then \( P \) believes that \( Q \) once said \( X \).
Nonce Verification

- Decryption of a message only says that it was uttered at some point, possibly in the past.
  - does not say if the sender still believes it
    - eg, could be result of a replay

\[
\frac{P \vdash \#(X), P \vdash Q \vdash \neg X}{P \vdash Q \vdash X}
\]

If P believes that X was said recently, and that Q said X, then P believes that Q believes X

*This is the only formula that promotes* \(\neg\) to \(\vdash\).
It reflects essence of challenge/response protocols.
Fresh statement is challenge.
Any message containing challenge is also fresh.

Jurisdiction Rule

- If P believes that Q has jurisdiction over X, then P trusts Q on the truth of X

\[
\frac{P \vdash Q \Rightarrow X, P \vdash Q \vdash X}{P \vdash X}
\]

This rule gets used a lot when thinking about key servers.
Some Other Rules

P believes a set of statements iff P believes each individual statement.

\[ P \models X, \quad P \models Y \quad \text{iff} \quad P \models (X,Y) \quad \text{iff} \quad P \models (X,Y) \quad \text{iff} \quad P \models X \]

If a principal sees a formula, then he can see its components (provided keys are known).

\[ P \lessdot (X,Y) \quad P \lessdot \langle X \rangle Y \quad P \models Q \leftarrow K \rightarrow P, \quad P \models \models (X)K \]

\[ P \lessdot X \quad P \lessdot \langle X \rangle \quad P \models X \]

If a part of a formula is known to be fresh, the entire formula is fresh (freshness distributes)

\[ P \models \#(X) \quad P \models \#(X,Y) \]

Idealized Protocols

- Transform a message step into an idealized protocol step
  - include only information that contributes to the beliefs of the recipient
  - eliminate hints
  - make explicit beliefs

- Example
  - Protocol Step
    \[ A \rightarrow B : \{A, \text{Kab}\}Kbs \]
    Intended to tell B, who knows Kbs, that Kab is a good key for communicating with A.
  - Idealized Protocol Step
    \[ A \rightarrow B : \{A \leftarrow \text{Kab} \rightarrow B\} \text{Kbs} \]
    Allows us to deduce
    \[ B \lessdot \langle A \leftarrow \text{Kab} \rightarrow B\}Kbs \]
    If B \models B \leftarrow Kbs \rightarrow S and S \rightarrow \text{Kab}, then
    \[ B \models S \models \langle A \leftarrow \text{Kab} \rightarrow B\} \]
    "If B believes that Kbs is a good key for B and S, and S has jurisdiction over Kab, then B believes that S once said that Kab is a good key for use between A and B."
    - is it still a good key?
      - who knows??
How to reason about a protocol

- Derive idealized protocol from original one
- Write assumptions about initial system state
- Attach logical formulas to the statements of the protocol
- Attach assertions to the statements of the protocol
- Apply postulates to assumptions and assertions to derive new beliefs
  - first assertion contains assumptions
  - last assertion contains the conclusions.
- Repeat until convinced.

Formal Goals of Authentication

- Initial assumptions state what keys are shared between principles, which principles are trusted, and which statements are fresh
- Authentication then means we can conclude
  - $A \vdash A \leftarrow K \rightarrow B$
  - $B \vdash A \leftarrow K \rightarrow B$
- Could also mean in addition
  - $A \vdash B \vdash A \leftarrow K \rightarrow B$
  - $B \vdash A \vdash A \leftarrow K \rightarrow B$
Needham and Schroeder Revisited

1: A->S: A, B, Na
2: S->A: [Na, B, Kab], [Kab, A|Kbs]|Kbs
3: A->B: [Kab, A]Kbs
4: B->A: [Nb]Kab
5: A->B: [Nb-1]Kab

The actual protocol
(messages sent)

The idealized protocol
(statements made)
**Analyzing the protocol: Initial Assumptions**

\[
\begin{align*}
A & \models A \langle\text{Kas}\rangle \rightarrow S \\
S & \models A \langle\text{Kas}\rangle \rightarrow S \\
S & \models A \langle\text{Kab}\rangle \rightarrow B \\
A & \models (S \Rightarrow A \langle\text{Kab}\rangle \rightarrow B) \\
A & \models (S \Rightarrow \#(A \langle\text{Kab}\rangle \rightarrow B)) \\
A & \models \#(Na) \\
S & \models \#(A \langle\text{Kab}\rangle \rightarrow B) \\
B & \models B \langle\text{Kbs}\rangle \rightarrow S \\
S & \models B \langle\text{Kbs}\rangle \rightarrow S \\
B & \models (S \Rightarrow A \langle\text{Kab}\rangle \rightarrow B) \\
B & \models \#(\text{Nb}) \\
B & \models \#(A \langle\text{Kab}\rangle \rightarrow B)
\end{align*}
\]

Initial keys
Key server
Freshness
This is that hidden initial assumption

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**What the idealized protocol says**

2: S\rightarrow A: [Na, (A\langle\text{Kab}\rangle \rightarrow B), \#(A\langle\text{Kab}\rangle \rightarrow B), \{A\langle\text{Kab}\rangle \rightarrow B\}Kbs]Kas

First,

\[
A \models \{Na, (A\langle\text{Kab}\rangle \rightarrow B), \#(A\langle\text{Kab}\rangle \rightarrow B), \{A\langle\text{Kab}\rangle \rightarrow B\}Kbs\}Kas
\]

which A decrypts using Kas. Since A knows Na to be fresh, we can apply:

\[
P \models \#(X), P \models Q \models X \\
\text{(nonce verification)}
\]

Leading to

\[
A \models S \models A \langle\text{Kab}\rangle \rightarrow B \\
A \models S \models \#(A \langle\text{Kab}\rangle \rightarrow B) \\
\text{(good key)} \\
\text{(fresh key)}
\]

Applying

\[
P \models Q \rightarrow X, P \models Q \models X \\
\text{(jurisdiction)}
\]

Gives

\[
A \models A \langle\text{Kab}\rangle \rightarrow B \\
A \models \#(A \langle\text{Kab}\rangle \rightarrow B)
\]
A repeats what it sees

A \models \{ A \leftarrow K \rightarrow B \} K b s

3: A \rightarrow B: \{ A \leftarrow K \rightarrow B \} K b s

We apply the message meaning postulate:

\[
\begin{align*}
P \models Q \leftarrow K \rightarrow P, & \quad P \models \{ X \} K \\
P \models Q \models X
\end{align*}
\]

To obtain

B \models S \models A \leftarrow K \rightarrow B

In order to obtain \( B \models A \leftarrow K \rightarrow B \), we need to rely on nonce verification (recall, only N.V. promotes \(~ \) to \( = \))

Relying on an assumption now explicit

Since we assumed initially that

B \models \#(A \leftarrow K \rightarrow B)

We can promote

B \models S \models A \leftarrow K \rightarrow B

to

B \models A \leftarrow K \rightarrow B

using

\[
\begin{align*}
P \models \#(X), & \quad P \models Q \models X \\
P \models Q \models X
\end{align*}
\]

\textit{(nonce verification)}
Getting A to believe that B is on board

4: B -> A: \{ Nb, (A <- K_a B) \} K_b

Since
A < | \{ Nb, (A <- K_a B) \} K_b
and
A |= A <- K_a B
then
A |= B |= (A <- K_a B)
(since B said it, B believes it)

The final step

5: A -> B: \{ Nb, (A <- K_a B) \} K_b

Allows
B |= A |~ \{ Nb, (A <- K_a B) \}

Freshness distributes, so we can apply

\[ \frac{P \models \#(X), P \models Q |- X}{P \models Q |- X} \quad \text{nonce verification} \]

to get

B |= A |= A <- K_a B
At the end we have

\[
\begin{align*}
A &\models A \leftarrow \text{Kab} \rightarrow B \\
B &\models A \leftarrow \text{Kab} \rightarrow B \\
A &\models B \models A \leftarrow \text{Kab} \rightarrow B \\
B &\models A \models A \leftarrow \text{Kab} \rightarrow B
\end{align*}
\]

which is the goal of an authentication protocol.

Had we not made the freshness assumption, we would have been stuck and could not have gotten here.

Conclusions

- We need to make an awful lot of assumptions in designing authentication protocols.
- The assumptions are there, whether you state them or not.
- Only by stating them explicitly can we enter into a final acceptable state of mutual authentication.
Authenticity and Integrity

- Sometimes we care about knowing messages authentic, but don’t care about privacy.
- If only sender and receiver knew the keys we would be done … but that’s often not the case
  - A pair of keys for each pair of communicating parties?
- In public key (RSA) systems the “encryption” key is potentially known by everyone
  - anyone could have sent us a confidential message by encrypting with our public key

A Faster “RSA Signature”

- Encryption can be expensive, e.g., RSA 1Kbps
- To speed up, let’s sign just the checksum instead!
  - Check that the encrypted bit is a signature of the checksum
- Problem: Easy to alter data without altering checksum
- Answer: Cryptographically strong “checksums” called message digests where it’s computationally difficult to choose data with a given checksum
  - But they still run much more quickly than encryption
  - MD5 (128 bits) is the most common example
Message Digests (MD5, SHA)

- Act as a cryptographic checksum or hash
  - Typically small compared to message (MD5 128 bits)
  - “One-way”: infeasible to find two messages with same digest

Cryptography in Protocols

- These techniques can be applied at different levels:
  - IP packets (IPSEC)
  - Web transfers or other transports (SSL/TLS, Secure HTTP)
  - Email (PGP)

- Next time ..
Key Concepts

- Privacy, integrity, and authenticity
- Cryptographic mechanisms are used to support these properties: private key, public key and digests