# Image Processing

# Forget Object Space

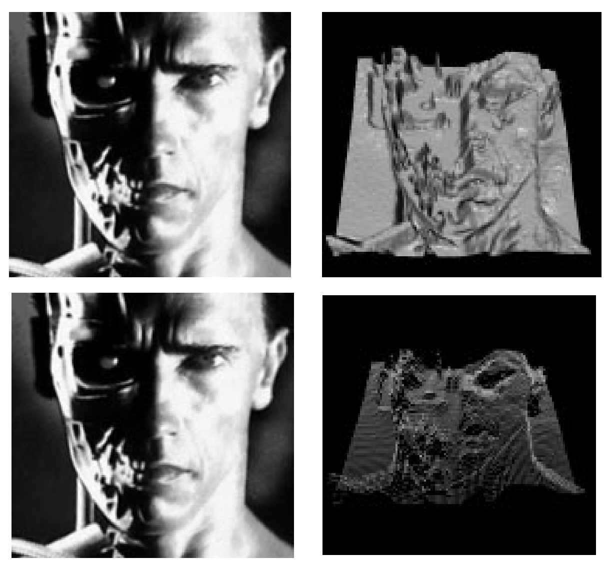
- So far, most of what we've done assumes we have some underlying abstract representation of what's in a scene.
- But there are many graphics techniques that operate only on images
- Image processing: operations that take images as input, produce images as output

- Is 3D graphics more important than image processing? *No way!*

# Images

- In its most general form, an image is a function $f$ from $\mathrm{R}^2$ to $\mathrm{R}$
  - $f( x, y )$ gives the intensity of a channel at position $( x, y )$
  - Realistically, we expect the image only to be defined over a rectangle, with a finite range:
    - $f: [a,b] \times [c,d] \rightarrow [0,1]$
  - A colour image is just three functions pasted together:
    - $F( x, y ) = (f_r( x, y ), f_g( x, y ), f_b( x, y ))$
- In computer graphics, we usually operate on digital (discrete) images
  - Quantize space into units (pixels)
  - Image is constant over each unit
  - A kind of step function
  - $f: \{0 \ldots m\text{-}1\} \times \{0 \ldots n\text{-}1\} \rightarrow [0,1]$
- An image processing operation typically defines a new image $f'$ in terms of an existing image $f$

# Images as Functions



# Pixel-to-pixel Operations

- The simplest operations are those that transform each pixel in isolation
  - $f'(x, y) = g(f(x,y))$
- Example: threshold, RGB → greyscale

# Pixel Movement

- Some operations preserve intensities, but move pixels around in the image
  - $f'(x, y) = f(g(x,y), h(x,y))$
- Examples: many amusing warps of images

# Noise

Original

Salt and pepper noise

Impulse noise

Gaussian noise

- Common types of noise:
  - Salt and pepper noise: contains random occurences of black and white pixels
  - Impulse noise: contains random occurences of white pixels
  - Gaussian noise: variations in intensity drawn from a Gaussian normal distribution

# Noise Reduction

- How can we "smooth" away noise?

- Is there a more abstract way to represent this sort of operation? Of course there is*!*

# Convolution

- Convolution is a fancy way to combine two functions.
  - Think of *f* as an image and *g* as a "smear" operator
  - *g* determines a new intensity at each point in terms of intensities of a neighbourhood of that point

$$h(x,y) = f(x,y) * g(x,y)$$
$$= \int_{-\infty}^{\infty} f(x',y')g(x-x',y-y')dx'dy'$$

- The computation at each point $(x,y)$ is like the computation of cone responses

# Discrete Convolution

- Since digital images are like step functions, integration becomes summation. We can express convolution as a two-dimensional sum:

$$h[i,j] = f[i,j] * g[i,j]$$
$$= \sum_k \sum_l f[k,l]g[i-k,j-l]$$

# Convolution Representation

- Since *f* and *g* are defined over finite regions, we can write them out in two-dimensional arrays:

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 62 | 79 | 23 | 119 | 120 | 105 | 4 | 0 |
| 10 | 10 | 9 | 62 | 12 | 78 | 34 | 0 |
| 10 | 58 | 197 | 46 | 46 | 0 | 0 | 48 |
| 176 | 135 | 5 | 188 | 191 | 68 | 0 | 49 |
| 2 | 1 | 1 | 29 | 26 | 37 | 0 | 77 |
| 0 | 89 | 144 | 147 | 187 | 102 | 62 | 208 |
| 255 | 252 | 0 | 166 | 123 | 62 | 0 | 31 |
| 166 | 63 | 127 | 17 | 1 | 0 | 99 | 30 |

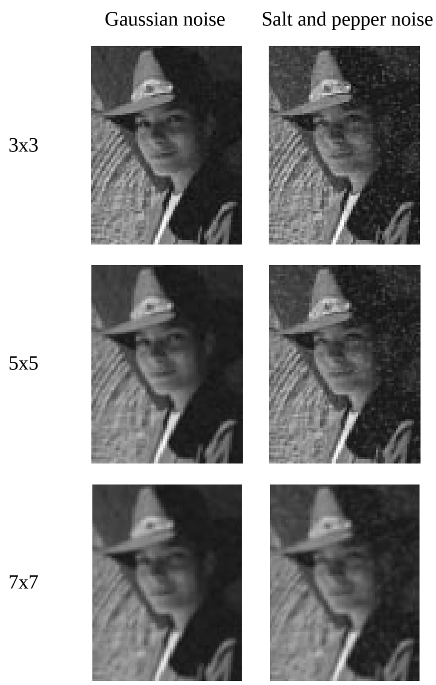
| | | |
|---|---|---|
| X .2 | X 0 | X .2 |
| X 0 | X .2 | X 0 |
| X .2 | X 0 | X .2 |

- *Note*: This is not matrix multiplication*!*

# Mean Filters

- How can we represent our noise-reducing averaging filter as a convolution diagram?

# Using Mean Filters



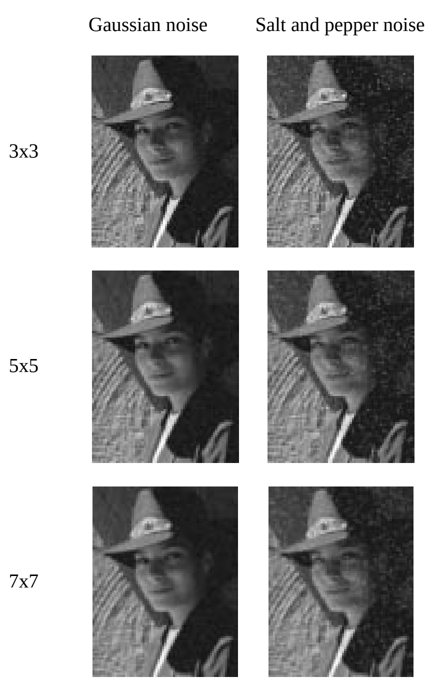

# Gaussian Filters

- Gaussian filters weigh pixels based on their distance to the location of convolution.

$$g[i,j] = e^{-(i^2+j^2)/(2\sigma^2)}$$

- This does a good job of blurring noise while preserving features of the image.

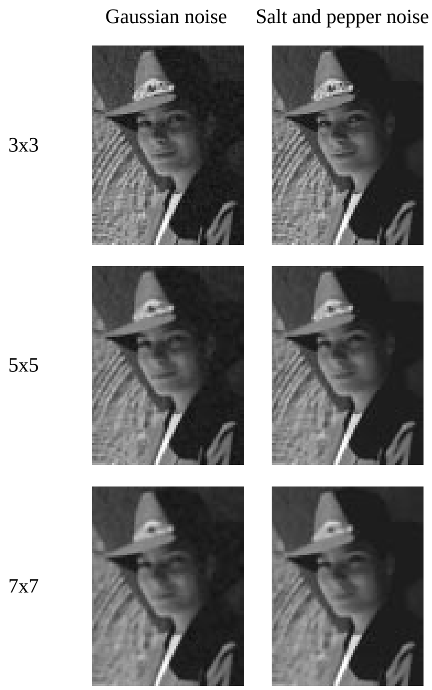
# Using Gaussian Filters




# Median Filters

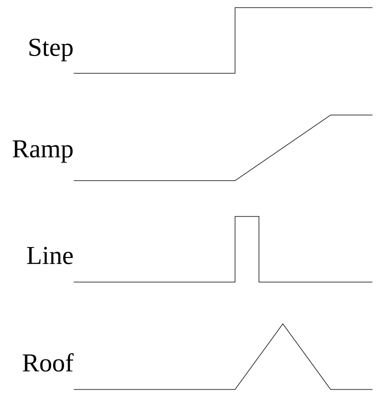
- A Median Filter operates over a *k*x*k* region by selecting the median intensity in the region.
- What advantage does a median filter have over a mean filter?
- Is a median filter a kind of convolution?

# Using Median Filters




# Edge Detection

- One of the most important uses of image processing is edge detection
  - Really easy for humans
  - Really difficult for computers
  - Fundamental in computer vision
  - Important in many graphics applications
- What defines an edge?




# Gradient

- The gradient is the 2D equivalent of the derivative:

$$\nabla f(x,y) = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right)$$

- Properties of the gradient
  - It's a vector
  - Points in the direction of maximum increase of $f$
  - Magnitude is rate of increase
- How can we approximate the gradient in a discrete image?

# Edge Detection Algorithms

- Edge detection algorithms typically proceed in three or four steps:
  - Filtering: cut down on noise
  - Enhancement: amplify the difference between edges and non-edges
  - Detection: use a threshold operation
  - Localization (optional): estimate geometry of edges beyond pixels

# Edge Enhancement

- A popular gradient magnitude computation is the Sobel operator:

$$s_x = \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix}$$

$$s_y = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{bmatrix}$$

- We can then compute the magnitude of the vector $(s_x, s_y)$

# Using Sobel Operators


Original


Smoothed


Sx + 128


Sy + 128


Magnitude


Threshold = 64


Threshold = 128

# Second Derivative Operators

- The Sobel operator can produce thick edges. Ideally, we're looking for infinitely thin boundaries.
- An alternative approach is to look for local extrema in the first derivative: places where the change in the gradient is highest.
- We can find these by looking for zeroes in the second derivative
- Using similar reasoning as above, we can derive a Laplacian filter, which approximates the second derivative:

$$\Delta^2 = \begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

- Zero values in the convoluted image correspond to extreme gradients, i.e. edges.

# Summary

- Formal definitions of image and image processing
- Kinds of image processing: pixel-to-pixel, pixel movement, convolution, others
- Types of noise and strategies for noise reduction
- Definition of convolution and how discrete convolution works
- The effects of mean, median and Gaussian filtering
- How edge detection is done
- Gradients and discrete approximations