2D Transformations

Introduction

- A 2D transform is a "useful" mapping of the 2D plane onto itself.
  - Homeomorphism
  - Preserves some amount of geometric information
- Many different functions are possible under this definition, but we'll see that some in particular are useful for computer graphics.

Applications

- Sometimes, it's easier to specify an object using a transform than directly.
- Instancing
  - Transforms can be a more compact representation than copies of an object.
  - A transformed instance of an object preserves more information about the object than a copy.
- Hierarchical modeling
  - We can group a collection of geometry under a transform node and manipulate it all at once.

Linearity

- One important class of transforms are the linear transforms.
  - They correspond to change-of-basis or change-of-coordinate-system transforms.
- How can we represent linear transforms?
- Which transforms are linear?

Linear Isn't Enough

- What important transform isn't linear?
  - Why not?
- How can we unify all of the transforms we've discussed?

Affine Geometry

- Treating the Euclidean plane as a vector space ignores the distinction between points and vectors.
- The affine plane is a set of points together with a set of vectors.
  - No distinguished origin
  - The difference between two points is a vector.
  - A vector added to a point yields a new point.
  - Vectors and points are not interchangeable.
- How can we represent the affine plane?
- How can we represent transforms in the affine plane?
- What geometric properties do affine transforms preserve?

Properties of Transforms

- Compact representation
- Fast implementation
- Easy to invert
- Easy to compose

Examples

- Rotation about a point other than the origin
- Reflection through a line other than an axis
- Viewport transform

Summary

- A small number of 2D transforms are especially useful in computer graphics: translation, rotation, scale, shear, reflection, and of course identity.
- Some of these transforms are linear and can be represented using a 2x2 matrix.
- Some are affine and can be represented using a 3x3 matrix acting on homogeneous coordinates.
- Affine transforms have certain properties which make them very practical for computer graphics.

Reading

The reading here covers 2D and 3D transformations.

- Angel, chapter 4
- Foley et al., chapter 5 and Appendix A
- Hearn & Baker, chapter 5 (2D), chapter 11 (3D)