Lecture 19:
Tensor Product Surfaces
Reading

- Hearn and Baker, sections 10.8, 10.9, 10.14.

Recommended:

Given a grid of control points $V_{ij}$, forming a control net, construct a surface $S(u,v)$ by:

- treating rows of $V$ as control points for curves $V_0(u),..., V_n(u)$.
- treating $V_0(u),..., V_n(u)$ as control points for a curve parameterized by $v$. 

Tensor product Bézier surfaces
Building surfaces from curves

Let the geometry vector vary by a second parameter $v$:

$$S(u,v) = U \cdot M \cdot \begin{bmatrix} G_1(v) \\ G_2(v) \\ G_3(v) \\ G_4(v) \end{bmatrix}$$

$$G_i(v) = V \cdot M \cdot g_i$$

$$g_i = [g_{i1} \quad g_{i2} \quad g_{i3} \quad g_{i4}]^T$$
Geometry matrices

By transposing the geometry curve we get:

\[ G_i(v)^T = (V \cdot M \cdot g_i)^T \]

\[ = g_i^T \cdot M^T \cdot V^T \]

\[ = [g_{i1} \ g_{i2} \ g_{i3} \ g_{i4}] \cdot M^T \cdot V^T \]
Geometry matrices

Combining

\[ G_i(v) = \begin{bmatrix} g_{i1} & g_{i2} & g_{i3} & g_{i4} \end{bmatrix} \cdot M^T \cdot V^T \]

And

\[ S(u, v) = U \cdot M \cdot \begin{bmatrix} G_1(v) \\ G_2(v) \\ G_3(v) \\ G_4(v) \end{bmatrix}^T \]

We get

\[ S(u, v) = U \cdot M \cdot \begin{bmatrix} g_{11} & g_{12} & g_{13} & g_{14} \\ g_{21} & g_{22} & g_{23} & g_{24} \\ g_{31} & g_{32} & g_{33} & g_{34} \\ g_{41} & g_{42} & g_{43} & g_{44} \end{bmatrix} \cdot M^T \cdot V^T \]
Tensor product surfaces, cont.

Let’s walk through the steps:

Which control points are interpolated by the surface?
Matrix form

Tensor product surfaces can be written out explicitly:

\[ S(u, v) = \sum_{i=0}^{n} \sum_{j=0}^{n} V_{ij} B_i^m(u) B_j^n(v) \]

\[ = \begin{bmatrix} v^3 & v^2 & v & 1 \end{bmatrix} \begin{bmatrix} u^3 \\ u^2 \\ u \\ 1 \end{bmatrix} \]
Tensor product B-spline surfaces

As with spline curves, we can piece together a sequence of Bézier surfaces to make a spline surface. If we enforce C2 continuity and local control, we get B-spline curves:

- treat rows of $B$ as control points to generate Bézier control points in $u$.
- treat Bézier control points in $u$ as B-spline control points in $v$.
- treat B-spline control points in $v$ to generate Bézier control points in $u$. 
Tensor product B-splines, cont.

Which B-spline control points are interpolated by the surface?
Tensor product B-splines, cont.

Another example:
Trimmed NURBS surfaces

Uniform B-spline surfaces are a special case of NURBS surfaces.

Sometimes, we want to have control over which parts of a NURBS surface get drawn.

For example:

![Image of trimmed NURBS surface]

We can do this by **trimming** the $u$-$v$ domain.

- Define a closed curve in the $u$-$v$ domain (a **trim curve**)
- Do not draw the surface points inside of this curve.

It’s really hard to maintain continuity in these regions, especially while animating.
Summary

What to take home:

- How to construct tensor product Bézier surfaces
- How to construct tensor product B-spline surfaces