Hierarchical Modeling

Brian Curless
CSE 457
Spring 2015
Reading

Required:

- Angel, sections 8.1 – 8.6, 8.8

Optional:

- *OpenGL Programming Guide*, chapter 3
Symbols and instances

Most graphics APIs support a few geometric primitives:

- spheres
- cubes
- cylinders

These symbols are instanced using an instance transformation.

Q: What is the matrix for the instance transformation above?

\[ M = \begin{bmatrix} S & R & T \end{bmatrix} \]

\[ M = \begin{bmatrix} T & R \end{bmatrix} \]
3D Example: A robot arm

Consider this robot arm with 3 degrees of freedom:

- Base rotates about its vertical axis by $\theta$
- Upper arm rotates in its $xy$-plane by $\phi$
- Lower arm rotates in its $xy$-plane by $\psi$

(Note that the angles are set to zero in the figure; i.e., the parts are shown in their “default” positions.)

**Q:** What matrix do we use to transform the base?

**Q:** What matrix for the upper arm?

**Q:** What matrix for the lower arm?

$$R_x(\cdot), R_y(\cdot), R_z(\cdot)$$

$$T(\cdot, \cdot, \cdot)$$

$$R_z(\phi) T(0, h_1, 0)$$

$$R_y(\theta) T(0, h_1, 0) R_z(\phi) T(0, h_2, 0) R_z(\psi)$$

[Angel, 2011]
3D Example: A robot arm

An alternative interpretation is that we are taking the original coordinate frames...

...and translating and rotating them into place:
From parts to model to viewer

Model or object space

Model space

World space

M_{model}

Eye or camera space

M_{view}
Robot arm implementation

The robot arm can be displayed by keeping a global matrix and computing it at each step:

Matrix $M$, $M_{\text{model}}$, $M_{\text{view}}$;

```c
main()
{
    ... 
    M_{\text{view}} = \text{compute\_view\_transform}();
    \text{robot\_arm}();
    ... 
}
```

```c
\text{robot\_arm}()
{
    \hspace{1em} M_{\text{model}} = R_y(\theta);
    \hspace{1em} M = M_{\text{view}}*M_{\text{model}};
    \hspace{1em} \text{base}();
    \hspace{1em} M_{\text{model}} = R_y(\theta)*T(0,h_1,0)*R_z(\phi);
    \hspace{1em} M = M_{\text{view}}*M_{\text{model}};
    \hspace{1em} \text{upper\_arm}();
    \hspace{1em} M_{\text{model}} = R_y(\theta)*T(0,h_1,0)
    \hspace{2.5em} *R_z(\phi)*T(0,h_2,0)*R_z(\psi);
    \hspace{1em} M = M_{\text{view}}*M_{\text{model}};
    \hspace{1em} \text{lower\_arm}();
}
```

Do the matrix computations seem wasteful?
Robot arm implementation, better

Instead of recalculating the global matrix each time, we can just update it \textit{in place} by concatenating matrices on the right:

\begin{verbatim}
Matrix M_modelview;

main()
{
    ...
    M_modelview = compute_view_transform();
    robot_arm();
    ...
}

robot_arm()
{
    M_modelview *= R_y(theta);
    base();
    M_modelview *= T(0,h1,0)*R_z(phi);
    upper_arm();
    M_modelview *= T(0,h2,0)*R_z(psi);
    lower_arm();
}
\end{verbatim}
Robot arm implementation, OpenGL

OpenGL maintains a global state matrix called the **model-view matrix**, which is updated by concatenating matrices on the *right*.

```c
main()
{
    ...
    glMatrixMode( GL_MODELVIEW );
    Matrix M = compute_view_xform();
    glLoadMatrixf( M );
    robot_arm();
    ...
}

robot_arm()
{
    glRotatef( theta, 0.0, 1.0, 0.0 );
    base();
    glTranslatef( 0.0, h1, 0.0 );
    glRotatef( phi, 0.0, 0.0, 1.0 );
    lower_arm();
    glTranslatef( 0.0, h2, 0.0 );
    glRotatef( psi, 0.0, 0.0, 1.0 );
    upper_arm();
}
```
Hierarchical modeling

Hierarchical models can be composed of instances using trees or DAGs:

- edges contain geometric transformations
- nodes contain geometry (and possibly drawing attributes)

How might we draw the tree for the robot arm?
A complex example: human figure

Q: What’s the most sensible way to traverse this tree?

depth-first
Human figure implementation, OpenGL

```c
figure()
{
    torso();
    glPushMatrix();
        glTranslate( ... );
        glRotate( ... );
    head();
    glPopMatrix();
    glPushMatrix();
        glTranslate( ... );
        glRotate( ... );
    left_upper_arm();
    glPushMatrix();
        glTranslate( ... );
        glRotate( ... );
        left_lower_arm();
    glPopMatrix();
    glPopMatrix();
    ...
}
```
Animation

The above examples are called *articulated models*:

- rigid parts
- connected by joints

They can be animated by specifying the joint angles (or other display parameters) as functions of time.
Key-frame animation

The most common method for character animation in production is **key-frame animation**.

- Each joint specified at various **key frames** (not necessarily the same as other joints)
- System does interpolation or **in-betweening**

Doing this well requires:

- A way of smoothly interpolating key frames: **splines**
- A good interactive system
- A lot of skill on the part of the animator
Scene graphs

The idea of hierarchical modeling can be extended to an entire scene, encompassing:

- many different objects
- lights
- camera position

This is called a scene tree or scene graph.
Summary

Here’s what you should take home from this lecture:

- All the **boldfaced terms**.
- How primitives can be instanced and composed to create hierarchical models using geometric transforms.
- How the notion of a model tree or DAG can be extended to entire scenes.
- How OpenGL transformations can be used in hierarchical modeling.
- How keyframe animation works.