Computer Science and Engineering 457

Introduction to Computer Graphics

Homework 1

Spring Quarter, 2012; University of Washington

Due Wednesday, April 18 at the beginning of class

Instructions: Do this assignment individually (NOT in partnerships). Legibly write your answers on sheets of paper. When giving explanations, make sure they are clear. If there are corrections or clarifications to these problems, they will be posted on the course website, linked from the homepage.

1. (4 points) Double buffering is a technique designed to reduce certain display artifacts when the graphical contents of a frame buffer need to be changed. Explain how the timing of updating and buffer switching should be arranged to minimize visual inconsistencies.

2. (3 points) Consider the following convolution kernel.

1	4	1
4	6	4
1	4	1

   If this is used as-is (without postprocessing, for example) for noise filtering, there will be a deleterious effect on the images. Explain that effect and fix the kernel to avoid it.

3. (3 points) Suppose that you wish to filter a monochrome image by considering a 7 by 7 neighborhood around each pixel p and then replacing p by the average of those pixels whose values lie within the range $[v - 10, v + 10]$ where v is the value of p. Is it possible for this to be expressed as a convolution filter? Explain.

4. (8 points) Computing normal vectors for a surface is an important part of some shading algorithms.
   (a) Explain how to find a vector that is normal to the 3D triangle having corners P, Q, and R. Assuming that P, Q, and R are in clockwise order, your normal should point down, by the right-hand rule.
   (b) Explain how to make the length of your normal vector equal to 1.
   (c) What will happen if P, Q and R all line in the same line?

5. (8 points) In the following vector dot and cross products problem, let u, v, and w be 3-dimensional vectors.
   (a) Simplify: $(((u \times v) \bullet u)\, w)$.
   (b) Simplify: $(u \bullet v) + (v \bullet u)$.
   (c) Simplify: $\sqrt{(u \bullet u)(v \bullet v)} \cos \theta$, where θ is the angle formed by u and v.
   (d) Simplify: $(u \bullet v) + (u \bullet w)$.

6. (20 points)

   In a liquid-crystal display, the light entering each crystal passes through a first fixed polarizing filter having direction of polarization α. Then each crystal rotates the direction of polarization of its incoming light from α by a controlled amount θ. The light coming out of the crystal has linear polarization with angle $\alpha + \theta$. When this light gets to the second fixed polarizing filter, whose angle of polarization is β, only a fraction of the light passes through.

   In a typical LCD, $\beta = \alpha + \pi/2$, and when no voltage is applied to the crystal, the light rotates by $\theta = \pi/2$ and the light, now having the same direction β of polarization as the second filter, passes through. If a relatively large voltage is applied, then the molecules within the nematic crystal line up and no longer rotate the light's polarization; i.e., $\theta = 0$. This results in no light passing through the second filter, since its orientation is still α which is orthogonal to β. If the voltage is between the extremes, then the angle of rotation θ is somewhere between 0 and $\pi/2$, and some fraction of the light gets through the second filter. The color of the light will depend on another layer of filters over the crystals, with some being red, some green, and some blue.

   In the simpler case of two polarizing filters without a liquid crystal in between, the amount of light that exits the second fixed polarizing filter is given by:

$$I = I(o) \cdot \cos^2 q$$

   where $I(o)$ is the intensity of incoming light falling on the first filter, and q is the difference in angle between the directions of polarization of the two filters.

(see http://micro.magnet.fsu.edu/primer/java/polarizedlight/filters/ for more details).

Extending this formula to our case in which there is a liquid crystal between the polarizers is simple under the assumption that changing the nematic twist in the crystal (by applying the electric field) is equivalent to changing the angle of the second polarizing filter.

Now, suppose that due to an error in manufacturing, the plane of polarization of the second filter, normally β, varies by the color of the subpixel it covers, and it is wrong at each red pixel: it's $\beta + \pi/2$. Explain each of your answers below.

(a) Suppose the computer sends to the display an image that should be all white: RGB = (255, 255, 255). What color will be seen? (And what is the equivalent RGB triple?)

(b) Suppose an additional polarizing filter is held up to the screen and that it is oriented at the angle $\beta + \pi/2$, which is the same direction as the second filters over the red pixels. What color will be seen? (You may assume that the color filters do not affect the polarization of the light.)

(c) Suppose that the computer sends to display the color RGB = (0, 255, 255). What color is seen? Suppose the additional polarizing filter is again held up to the screen, but that it is rotated by $\pi/4$, and its orientation is now $\beta + \pi/4$. What color is seen?

7. (20 points) (Convolution)

In this problem, you will consider several convolution filtering operations and their behaviors. You do not need to worry about flipping filters before sliding them across images; i.e., assume filters are pre-flipped. In addition, assume that the y-axis points up, the x-axis points to the right, and the lower left corner of the image is at (0,0). For each sub-problem, justify your answer.

(a) (2 points) The image you're editing is too dark, and you decide you need to amplify the value of each pixel by a factor of 8. Suggest a convolution filter that will scale, by a factor of 8, the value at each pixel of the image without changing it in any other way. (Technically, after scaling pixel values, they could be out of range; assume that any needed clamping will be taken care of later, after filtering).

(b) (3 points) While taking a photograph with your digital camera, you fail to hold the camera steady, and it effectively causes of vertical translation of the image while the shutter is open. You discover this later when you see that horizontal edges, in particular, have been blurred a bit (an effect called motion blur). You decide to filter the image so that horizontal edges are sharpened, but vertical edges are unchanged. Suggest a single convolution filter that does this.

(c) (4 points) After thinking a little more about the previous picture, you decide that motion blur is cool, and you want to apply it to another image. In this case, though, you want to simulate the effect of a camera translating diagonally along the $y = x/2$ direction while the shutter is open. Suggest a convolution filter that would accomplish some diagonal blurring along that direction by averaging across m pixels.

(d) (4 points) Describe a non-constant image that, when convolved with your diagonal blur filter from (c), would be unchanged by the filter. (You may ignore the boundaries.)

(e) (7 points) Suppose you pad the boundary of an image in some way that allows you to compute output values for every pixel being filtered by a convolution filter. For an image of dimensions n by n and a filter of dimensions m by m, how many output pixels will be influenced by input pixels "hallucinated" beyond the boundary of the image? For simplicity, assume that m is odd. However, m and n may otherwise have arbitrary positive values.

8. (12 points) The basic scaling matrix discussed in lecture scales only with respect to the x, y, and/or z axes. Using the basic translation, scaling, and rotation matrices, specify how to build a transformation matrix that scales along any ray in 3D space. This new transformation is determined by the ray origin $p = (p_x, p_y, p_z)$ and direction vector $v = (v_x, v_y, v_z)$, and the amount of scaling s_v. For clarity, a diagram has been provided, showing a box being scaled with respect to a given ray. Your answer should work for any ray, not just the case shown in the picture.

9. (20 points) Hierarchical modeling. Suppose you wish to create a model for the "Octopus" fun-fair ride. This consists of several parts.

(1) There is a main hub (a wheel) that corresponds to the octopus' body center. The wheel can rotate on an axis A_0.

(2) The hub has 8 spokes that protrude out from the center and past the rim of the wheel. (Each spoke can be thought of as a steel beam.)

(3) There is an axle along A_0 that is mounted in a tiltable platform. This platform is hinged to a fixed ground platform at the left side of the ride, at a distance d from A_0.

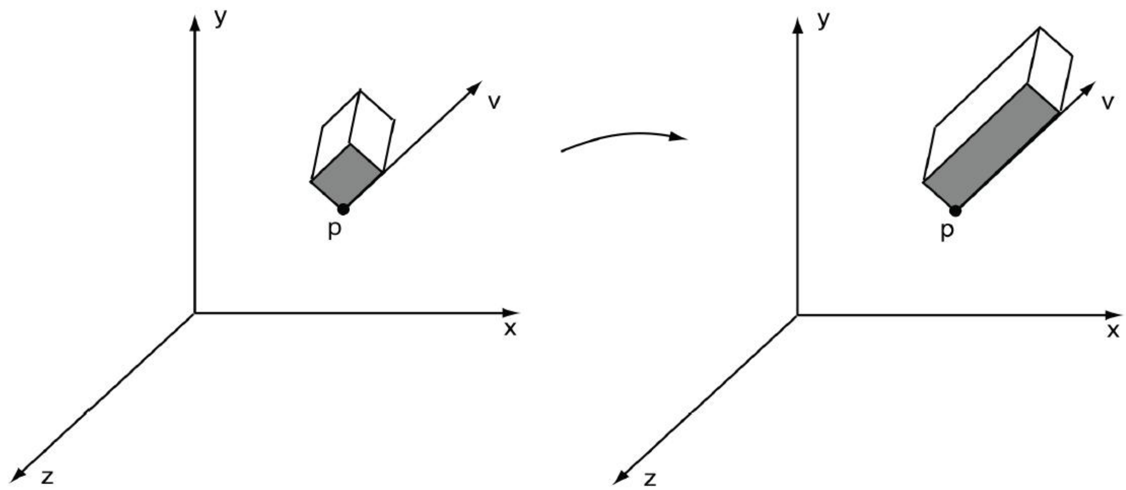

Figure 1: Scaling along arbitrary rays.

(4) At the outer end of each spoke a rotatable seat assembly S_i is attached. It is attached with an axle on an axis A_i parallel to A_0 but displaced from A_0 by radius r_1.

(5) There is a (human?) rider (you?) seated in one of the seat assemblies. You are seated in seat assembly S_8, with your head at height h meters above the base of the seat assembly, and out from its axis A_8 a distance r_2. You are facing the center of the seat assembly.

Draw a tree that represents the model and yourself as positioned in your seat. For one of the eight leaf nodes of the tree (the one corresponding to you at S_8), provide the sequence of transformations that expresses the location and orientation of the seat assembly, also indicating the subsequences that represent its axle position and orientation, and the position and orientation of the spoke on which it rests.

Show each transformation matrix using a combination of numbers and symbols (e.g., $\sin\theta$). You don't have to perform numerical or symbolic matrix multiplication; just show a matrix product as a concatenation of the symbols representing the matrices. Assume the following: α is the angle by which the platform holding the octopus is tilted; β is the angle by which spoke number 8 has been rotated counterclockwise away from 0, which we'll assume is at the right side of the octopus. (When the octopus is tilted, as it is in the illustration, your seat is at the highest it gets, which happens when you are as far to the right as possible. There you are at $\beta = 0$. θ is angle by which your seat assembly is rotated relative to the spoke on which it sits. For example if you are facing the octopus' center, then $\theta = 0$, and if you are facing out, then

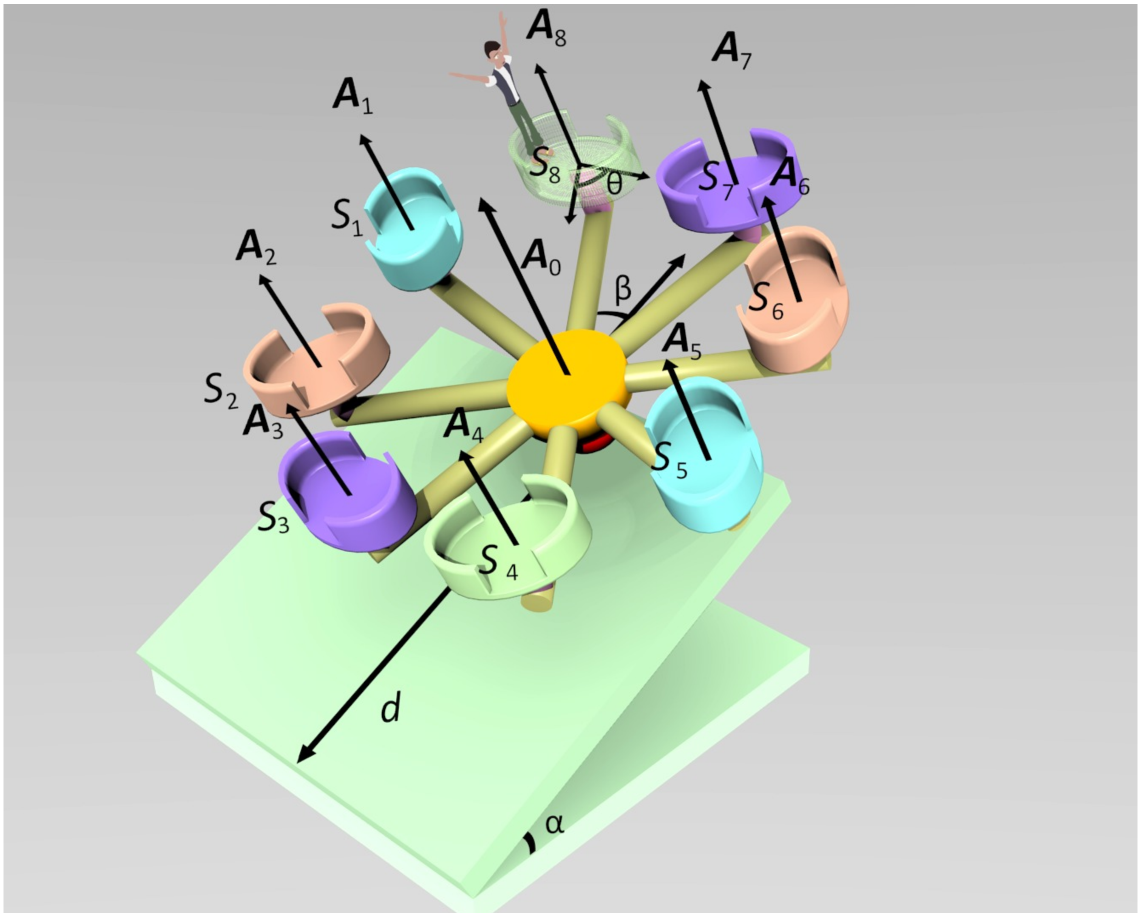

Figure 2: The Octopus Fun-Fair Ride (illustration created by Leeran Raphaely).

$\theta = \pi$. The net direction in which you are facing is $\beta + \theta$ (within the tilted plane).