Hierarchical Modeling

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CSE 457
Autumn 2010

Symbols and instances

Most graphics APIs support a few geometric primitives:
- spheres
- cubes
- cylinders

These symbols are **instantiated** using an instance transformation.

Q: What is the matrix for the instance transformation above?

\[ M \neq SRT = TRS \]

3D Example: A robot arm

Consider this robot arm with 3 degrees of freedom:
- Base rotates about its vertical axis by \( \theta \)
- Upper arm rotates in its xy-plane by \( \phi \)
- Lower arm rotates in its xy-plane by \( \psi \)

(Note that the angles are set to zero in the figure; i.e., the parts are shown in their “default” positions.)

Q: What matrix do we use to transform the base?
Q: What matrix for the upper arm?
Q: What matrix for the lower arm?

\[ R_y(\phi) \ T(0, h_1, 0) \ R_x(\psi) \ T(0, h_2, 0) \ R_z(\theta) \]

Optional:
- OpenGL Programming Guide, chapter 3
3D Example: A robot arm

An alternative interpretation is that we are taking the original coordinate frames...

...and translating and rotating them into place.

Robot arm implementation

The robot arm can be displayed by keeping a global matrix and computing it at each step:

Matrix $M_{\text{model}}$

```c
main()
{
    ...
    robot_arm();
    ...
}
```

```c
robot_arm()
{
    $M_{\text{model}} = R_y(\theta)$;
    base();
    $M_{\text{model}} = R_y(\theta)^{-1} R_z(\phi1)$;
    upper_arm();
    $M_{\text{model}} = R_y(\theta)^{-1} R_z(\phi1)^{-1} R_z(\psi1)$;
    lower_arm();
}
```

Do the matrix computations seem wasteful?

Robot arm implementation, better

Instead of recalculating the global matrix each time, we can just update it in place by concatenating matrices on the right:

```c
Matrix $M_{\text{model}}$

```c
main()
{
    ...
    $M_{\text{model}} = \text{Identity}();$
    robot_arm();
    ...
}
```

```c
robot_arm()
{
    $M_{\text{model}} = R_y(\theta)$;
    base();
    $M_{\text{model}} = T(0, h1, 0) R_z(\phi1)$;
    upper_arm();
    $M_{\text{model}} = T(0, h2, 0) R_z(\psi1)$;
    lower_arm();
}
```

Robot arm implementation, OpenGL

OpenGL maintains a global state matrix called the **model-view matrix**, which is updated by concatenating matrices on the **right**.

```c
main()
{
    ...
    glMatrixMode( GL_MODELVIEW );
    glLoadIdentity();
    robot_arm();
    ...
}
```

```c
robot_arm()
{
    glTranslatef( theta, 0.0, 1.0, 0.0 );
    base();
    glTranslatef( 0.0, h1, 0.0 );
    glRotatef( phi1, 0.0, 0.0, 1.0 );
    lower_arm();
    glTranslatef( 0.0, h2, 0.0 );
    glRotatef( psi1, 0.0, 0.0, 1.0 );
    upper_arm();
}
```
Hierarchical modeling

Hierarchical models can be composed of instances using trees or DAGs:

- nodes contain geometry and possibly drawing attributes
- edges contain geometric transformations

How might we draw the tree for the robot arm?

Human figure implementation, OpenGL

```c
figure()
{
    term();
    glPushMatrix();
    glTranslatef(...);
    glRotatef(...);
    head();
    glPopMatrix();
    glTranslatef(...);
    glRotatef(...);
    left_upper_arm();
    glPushMatrix();
    glTranslatef(...);
    glRotatef(...);
    left_lower_arm();
    glPopMatrix();
    glPopMatrix();
}```

A complex example: human figure

Q: What’s the most sensible way to traverse this tree?
   Depth first
   Stack

Animation

The above examples are called articulated models:
- rigid parts
- connected by joints

They can be animated by specifying the joint angles (or other display parameters) as functions of time.
Key-frame animation

The most common method for character animation in production is key-frame animation.

- Each joint specified at various key frames (not necessarily the same as other joints)
- System does interpolation or in-betweening

Doing this well requires:

- A way of smoothly interpolating key frames: splines
- A good interactive system
- A lot of skill on the part of the animator

Scene graphs

The idea of hierarchical modeling can be extended to an entire scene, encompassing:

- many different objects
- lights
- camera position

This is called a scene tree or scene graph.

Summary

Here's what you should take home from this lecture:

- All the **bold**ed terms.
- How primitives can be instanced and composed to create hierarchical models using geometric transforms.
- How the notion of a model tree or DAG can be extended to entire scenes.
- How OpenGL transformations can be used in hierarchical modeling.
- How keyframe animation works.