Hidden Surfaces

The Quest for 3D

- Construct a 3D hierarchical geometric model
- Define a virtual camera
- Map points in 3D space to points in an image
- Produce a wireframe drawing in 2D from a 3D object
- Of course, there’s more work to be done…

Introduction

- Not every part of every 3D object is visible to a particular viewer. We need an algorithm to determine what parts of each object should get drawn.
- Known as “hidden surface elimination” or “visible surface determination”.
- Hidden surface elimination algorithms can be categorized in three major ways:
  - Object space vs. image space
  - Object order vs. image order
  - Sort first vs. sort last

Reading

- Foley et al, Chapter 15
- Optional
Object Space Algorithms

- Operate on geometric primitives
  - For each object in the scene, compute the part of it which isn’t obscured by any other object, then draw.
  - Must perform tests at high precision
  - Resulting information is resolution-independent

- Complexity
  - Must compare every pair of objects, so \( O(n^2) \) for \( n \) objects
  - For an \( mxm \) display, have to fill in colors for \( m^2 \) pixels.
  - Overall complexity can be \( O(k_{obj} n^2 + k_{disp} m^2) \).
  - Best for scenes with few polygons or resolution-independent output

- Implementation
  - Difficult to implement!
  - Must carefully control numerical error

Image Space Algorithms

- Operate on pixels
  - For each pixel in the scene, find the object closest to the COP which intersects the projector through that pixel, then draw.
  - Perform tests at device resolution, result works only for that resolution

- Complexity
  - Must do something for every pixel in the scene, so at least \( O(m^2) \).
  - Easiest solution is to test projector against every object, giving \( O(n m^2) \).
  - Better approaches check only the objects that could be visible at each pixel. Let’s say, on average, \( d \) objects are visible at each pixel (a.k.a., depth complexity). Then, \( O(d m^2) \).

- Implementation
  - Usually very simple!

Object Order vs. Image Order

- Object order
  - Consider each object only once - draw its pixels and move on to the next object
  - Might draw the same pixel multiple times

- Image order
  - Consider each pixel only once - draw part of an object and move on to the next pixel
  - Might compute relationships between objects multiple times

Sort First vs. Sort Last

- Sort first
  - Find some depth-based ordering of the objects relative to the camera, then draw from back to front
  - Build an ordered data structure to avoid duplicating work

- Sort last
  - Sort implicitly as more information becomes available
Important Algorithms
• Ray casting
• Z-buffer
• Binary space partitioning
• Back face culling

Ray Casting
• Partition the projection plane into pixels to match screen resolution:
1. For each pixel \( p_i \), construct ray from COP through PP at that pixel and into scene
2. Intersect the ray with every object in the scene
3. Color the pixel according to the object with the closest intersection

Aside: Definitions
• An algorithm exhibits coherence if it uses knowledge about the continuity of the objects on which it operates
• An online algorithm is one that doesn’t need all the data to be present when it starts running
  – Example: insertion sort

Ray Casting Analysis
Categorization:
• Easy to implement?
• Hardware implementation?
• Incremental drawing calculations (uses coherence)?
• Pre-processing required?
• On-line (doesn’t need all objects before drawing begins)?
• Memory intensive?
• Handles transparency?
• Handles refraction?
• Polygon-based?
• Extra work for moving objects?
• Extra work for moving viewer?
• Efficient shading?
• Handles cycles and self-intersections?
Z-buffer

- Idea: along with a pixel’s red, green and blue values, maintain some notion of its depth
  - An additional channel in memory, like alpha
  - Called the depth buffer or Z-buffer

   ```c
   void draw_mode_setup( void ) {
     GlEnable( GL_DEPTH_TEST );
     ...}
   }
   ```

- When the time comes to draw a pixel, compare its depth with the depth of what’s already in the framebuffer. Replace only if it’s closer
- Very widely used
- History
  - Originally described as “brute-force image space algorithm”
  - Written off as impractical algorithm for huge memories
  - Today, done easily in hardware

Visibility tricks for Z-buffers

Z-buffering is the algorithm of choice for hardware rendering

What is the complexity of the Z-buffer algorithm?

What can we do to decrease the constants?

Z-buffer Implementation

```c
for each pixel p_i
  Z-buffer[ p_i ] = FAR
  Fb[ p_i ] = BACKGROUND_COLOUR
}
```

for each polygon P

```c
  for each pixel p_i in the projection of P
    Compute depth z and shade s of P at p_i
    if z < Z-buffer[ p_i ]
      Z-buffer[ p_i ] = z
      Fb[ p_i ] = s
}
```
Depth Preserving Conversion to Parallel Projection

Z value interpolation

\[ z_0 = z_0 - (z_t - z_p) \frac{y - y_t}{y_1 - y_2} \]
\[ z_1 = z_1 - (z_t - z_p) \frac{y - y_1}{y_1 - y_2} \]
\[ z_2 = z_0 - (z_0 - z_p) \frac{x - x_0}{x_0 - x_2} \]

Z-buffer Analysis

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Binary Space Partitioning

**Goal:** build a tree that captures some relative depth information between objects. Use it to draw objects in the right order.
- Tree doesn’t depend on camera position, so we can change viewpoint and redraw quickly
- Called the binary space partitioning tree, or BSP tree

**Key observation:** The polygons in the scene are painted in the correct order if for each polygon \( P \),
- Polygons on the far side of \( P \) are painted first
- \( P \) is painted next
- Polygons in front of \( P \) are painted last
Building a BSP Tree (in 2D)

BSP Tree Construction

```c
BSPtree makeBSP( L: list of polygons )
{
    if L is empty
        { return the empty tree }
    Choose a polygon P from L to serve as root
    Split all polygons in L according to P
    return new TreeNode( P, makeBSP( polygons on negative side of P ),
                          makeBSP( polygons on positive side of P ) )
}

• Splitting polygons is expensive! It helps to choose P wisely at each step.
  – Example: choose five candidates, keep the one that splits the fewest polygons
```

Alternate BSP Tree

BSP Tree Display

```c
showBSP( v: Viewer, T: BSPtree )
{
    if T is empty then return
    P := root of T
    if viewer is in front of P
        { showBSP( back subtree of T )
          draw P
          showBSP( front subtree of T )
        }
    else
        { showBSP( front subtree of T )
          draw P
          showBSP( back subtree of T )
        }
}
```
**BSP Tree Analysis**

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- Extra work for moving viewer?
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**Back Face Culling**

- Can be used in conjunction with polygon-based algorithms
- Often, we don’t want to draw polygons that face away from the viewer. So test for this and eliminate (cull) back-facing polygons before drawing
- How can we test for this?

**Summary**

- Classification of hidden surface algorithms
- Understanding of Z-buffer
- Familiarity with BSP trees and back face culling