19. Subdivision surfaces

Chaikin’s use of subdivision for curves inspired similar techniques for subdivision surfaces.

Iteratively refine a control polyhedron (or control mesh) to produce the limit surface

\[ \sigma = \lim_{j \to \infty} M^j \]

using splitting and averaging steps.

Reading

Recommended:

Triangular subdivision

There are a variety of ways to subdivide a polygon mesh.

A common choice for triangle meshes is 4:1 subdivision – each triangular face is split into four subfaces:
Loop averaging step

Once again we can use masks for the averaging step:

\[
Q \leftarrow \frac{\alpha(n)Q + Q_1 + \cdots + Q_n}{\alpha(n) + n}
\]

where

\[
\alpha(n) = \frac{n(1 - \beta(n))}{\beta(n)} \quad \beta(n) = \frac{5}{4} \frac{(3 + 2\cos(2\pi / n))^2}{32}
\]

These values, due to Charles Loop, are carefully chosen to ensure smoothness – namely, tangent plane or normal continuity.

Note: tangent plane continuity is also known as \(G^1\) continuity.

Loop evaluation and tangent masks

As with subdivision curves, we can split and average a number of times and then push the points to their limit positions.

\[
Q^n = \frac{\epsilon(n)Q + Q_1 + \cdots + Q_n}{\epsilon(n) + n}
\]

\[
\tau_1(n) = \tau_1(n)Q_1 + \tau_2(n)Q_2 + \cdots + \tau_n(n)Q_n
\]

\[
\tau_2(n) = \tau_1(n)Q_1 + \tau_2(n)Q_2 + \cdots + \tau_{n-1}(n)Q_n
\]

where

\[
\epsilon(n) = \frac{3n}{\beta(n)} \quad \tau_i(n) = \cos(2\pi i / n)
\]

How do we compute the normal?

Recipe for subdivision surfaces

As with subdivision curves, we can now describe a recipe for creating and rendering subdivision surfaces:

- Subdivide (split+average) the control polyhedron a few times. Use the averaging mask.
- Compute two tangent vectors using the tangent masks.
- Compute the normal from the tangent vectors.
- Push the resulting points to the limit positions. Use the evaluation mask.
- Render!

Adding creases without trim curves

In some cases, we want a particular feature such as a crease to be preserved. With NURBS surfaces, this required the use of trim curves.

For subdivision surfaces, we can just modify the subdivision mask:

This gives rise to \(G^0\) continuous surfaces (i.e., having positional but not tangent plane discontinuity)
Creases without trim curves, cont.

Here’s an example using Catmull-Clark surfaces (based on subdividing quadrilateral meshes):

Summary

What to take home:

- The meanings of all the **boldfaced** terms.
- How to construct and render Loop subdivision surfaces from the averaging masks, evaluation masks, and tangent masks.