

## Announcements

- Midterm due Friday, beginning of lecture
- Guest lecture on Friday: Antonio Criminisi, Microsoft Research

## Mosaics



<http://www.destination360.com/start1.htm>  
[http://www.vrsatlia.com/html/vrview.php?cat\\_id=E11&vs\\_id=vs380](http://www.vrsatlia.com/html/vrview.php?cat_id=E11&vs_id=vs380)

### Today's Readings

- Szeliski and Shum paper (sections 1 and 2, skim the rest)
  - <http://www.acm.org/publications/proceedings/qr/qrh/258734/pz251-szeliski/>

## Image Mosaics



### Goal

- Stitch together several images into a seamless composite

## How to do it?

### Basic Procedure

- Take a sequence of images from the same position
  - Rotate the camera about its optical center
- Compute transformation between second image and first
  - Lucas & Kanade registration
- Shift the second image to overlap with the first
- Blend the two together to create a mosaic
- If there are more images, repeat

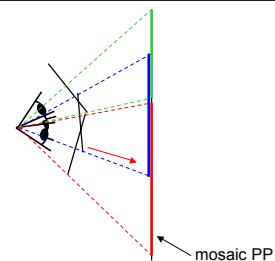
## Aligning images



### How to account for warping?

- Translations are not enough to align the images
- [Photoshop demo](#)

## Image reprojection



### The mosaic has a natural interpretation in 3D

- The images are reprojected onto a common plane
- The mosaic is formed on this plane

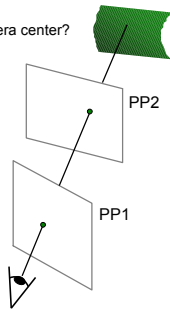
## Image reprojection

### Basic question

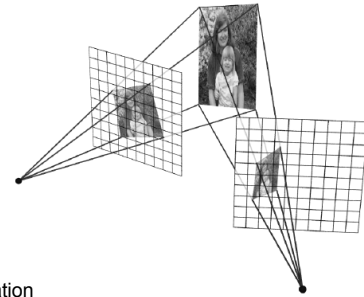
- How to relate two images from the same camera center?
  - how to map a pixel from PP1 to PP2

### Answer

- Cast a ray through each pixel in PP1
- Draw the pixel where that ray intersects PP2



## Image reprojection



### Observation

- Rather than thinking of this as a 3D reprojection, think of it as a 2D image warp from one image to another

## Homographies

### Perspective projection of a plane

- Lots of names for this:
  - homography, texture-map, colineation, planar projective map
- Modeled as a 2D warp using homogeneous coordinates

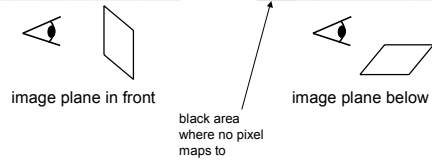
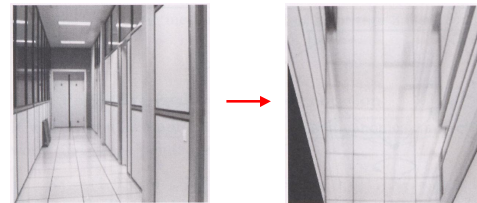
$$\begin{bmatrix} sx' \\ sy' \\ s \end{bmatrix} = \begin{bmatrix} * & * & * \\ * & * & * \\ * & * & * \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$\mathbf{p}' \quad \mathbf{H} \quad \mathbf{p}$

### To apply a homography $\mathbf{H}$

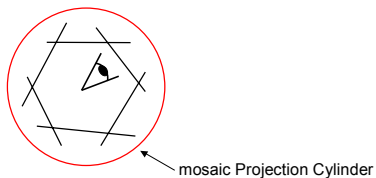
- Compute  $\mathbf{p}' = \mathbf{H}\mathbf{p}$  (regular matrix multiply)
- Convert  $\mathbf{p}'$  from homogeneous to image coordinates
  - divide by  $w$  (third) coordinate

## Image warping with homographies

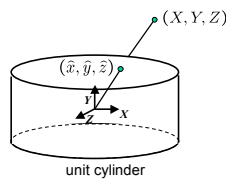


## Panoramas

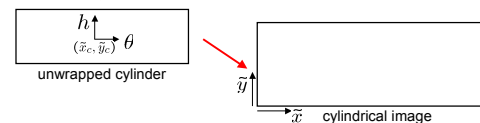
What if you want a 360° field of view?



## Cylindrical projection

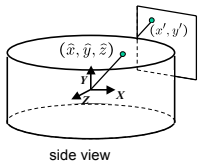


- Map 3D point  $(X, Y, Z)$  onto cylinder
 
$$(\hat{x}, \hat{y}, \hat{z}) = \frac{1}{\sqrt{X^2 + Z^2}}(X, Y, Z)$$
- Convert to cylindrical coordinates
 
$$(\sin\theta, h, \cos\theta) = (\hat{x}, \hat{y}, \hat{z})$$
- Convert to cylindrical image coordinates
 
$$(\tilde{x}, \tilde{y}) = (f\theta, fh) + (\tilde{x}_c, \tilde{y}_c)$$

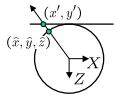


## Cylindrical reprojection

How to map from a cylinder to a planar image?



side view



top-down view

- Apply camera projection matrix
  - for project 1, account for focal length and assume principle point is at center of image
    - $x'_c = \frac{1}{2}$  image width,  $y'_c = \frac{1}{2}$  image height

$$\begin{bmatrix} wx' \\ wy' \\ w \end{bmatrix} = \begin{bmatrix} -f & 0 & w/2 & 0 \\ 0 & -f & h/2 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \hat{x} \\ \hat{y} \\ \hat{z} \\ 1 \end{bmatrix}$$

- Convert to image coordinates
  - divide by third coordinate ( $w$ )

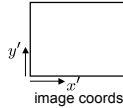
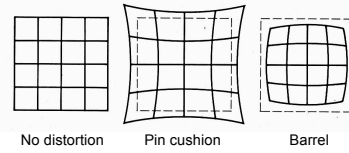


image coords

## Distortion



No distortion

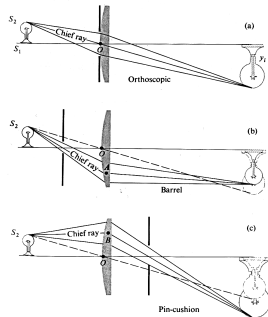
Pin cushion

Barrel

Radial distortion of the image

- Caused by imperfect lenses
- Deviations are most noticeable for rays that pass through the edge of the lens

## Distortion



## Modeling distortion

Project  $(\hat{x}, \hat{y}, \hat{z})$  to "normalized" image coordinates

$$x'_n = \hat{x}/\hat{z}$$

$$y'_n = \hat{y}/\hat{z}$$

Apply radial distortion

$$r^2 = x'^2_n + y'^2_n$$

$$x'_d = x'_n(1 + \kappa_1 r^2 + \kappa_2 r^4)$$

$$y'_d = y'_n(1 + \kappa_1 r^2 + \kappa_2 r^4)$$

Apply focal length translate image center

$$x' = fx'_d + x_c$$

$$y' = fy'_d + y_c$$

To model lens distortion

- Use above projection operation instead of standard projection matrix multiplication

## Cylindrical reprojection



Image 384x300

f = 180 (pixels)

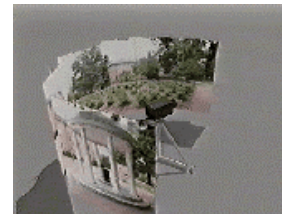
f = 280

f = 380

Map image to cylindrical coordinates

- need to know the focal length

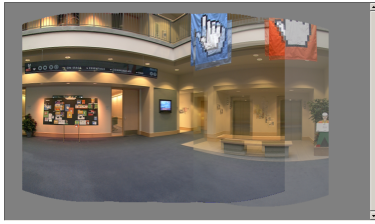
## Cylindrical panoramas



Steps

- Reproject each image onto a cylinder
- Blend
- Output the resulting mosaic

## Cylindrical image stitching



What if you don't know the camera rotation?

- Solve for the camera rotations
  - Note that a rotation of the camera is a **translation** of the cylinder!
  - Use Lukas-Kanade to solve for translations of cylindrically-warped images

## Full-view Panorama



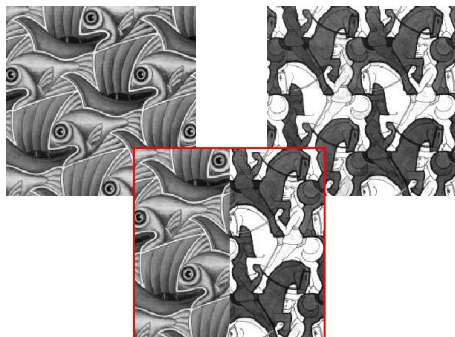
## Different projections are possible



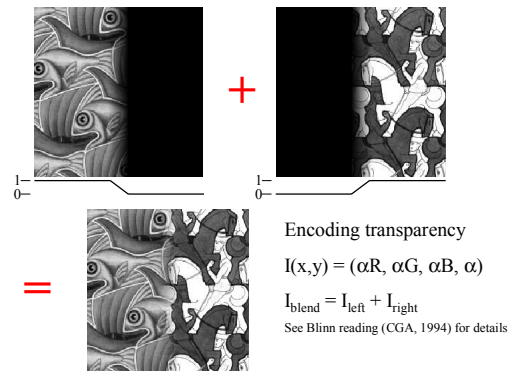
## Project 2 (out on Friday)

1. Take pictures on a tripod (or handheld)
2. Warp to cylindrical coordinates
3. Automatically compute pair-wise alignments
4. Correct for drift
5. Blend the images together
6. Crop the result and import into a viewer

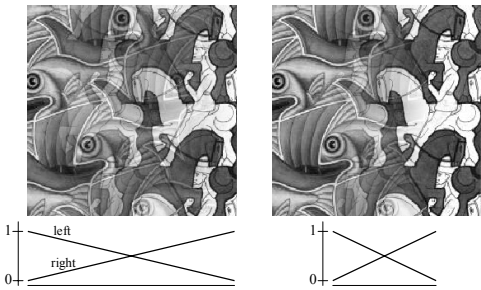
## Image Blending



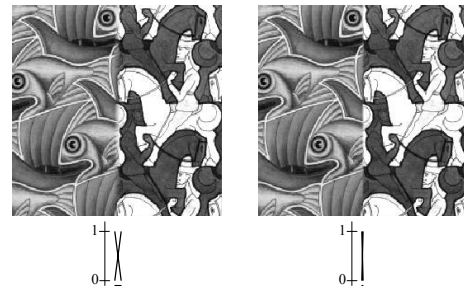
## Feathering



### Effect of window size



### Effect of window size

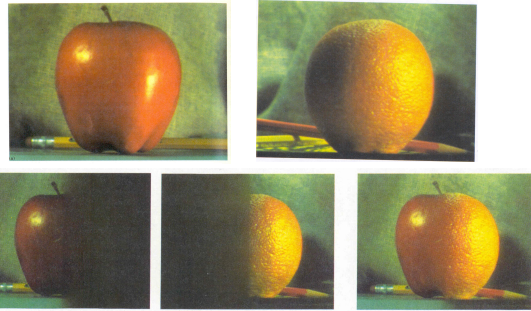


### Good window size



"Optimal" window: smooth but not ghosted  
 • Doesn't always work...

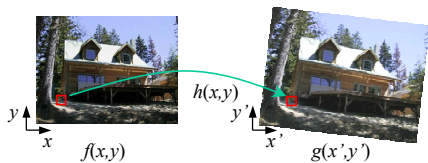
### Pyramid blending



Create a Laplacian pyramid, blend each level

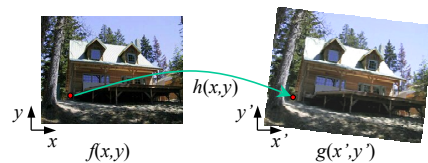
• Burt, P. J. and Adelson, E. H., [A multiresolution spline with applications to image mosaics](#), ACM Transactions on Graphics, 42(4), October 1983, 217-236.

### Image warping



Given a coordinate transform  $(x',y') = h(x,y)$  and a source image  $f(x,y)$ , how do we compute a transformed image  $g(x',y') = f(h(x,y))$ ?

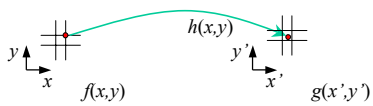
### Forward warping



Send each pixel  $f(x,y)$  to its corresponding location  $(x',y') = h(x,y)$  in the second image

Q: what if pixel lands "between" two pixels?

## Forward warping

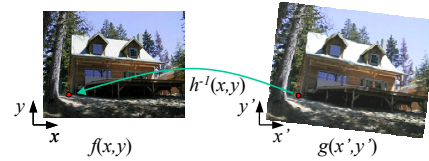


Send each pixel  $f(x, y)$  to its corresponding location  $(x', y') = h(x, y)$  in the second image

Q: what if pixel lands "between" two pixels?

A: distribute color among neighboring pixels  $(x', y')$   
 - Known as "splatting"

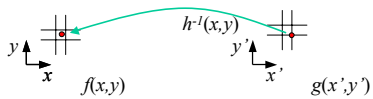
## Inverse warping



Get each pixel  $g(x', y')$  from its corresponding location  $(x, y) = h^{-1}(x', y')$  in the first image

Q: what if pixel comes from "between" two pixels?

## Inverse warping



Get each pixel  $g(x', y')$  from its corresponding location  $(x, y) = h^{-1}(x', y')$  in the first image

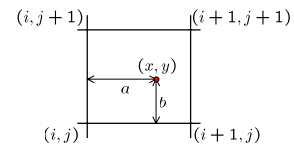
Q: what if pixel comes from "between" two pixels?

A: *resample* color value

- We discussed resampling techniques before
  - nearest neighbor, bilinear, Gaussian, bicubic

## Bilinear interpolation

A common method for resampling images



$$F(x, y) = (1-a)(1-b) F(i, j) + a(1-b) F(i+1, j) + ab F(i+1, j+1) + (1-a)b F(i, j+1)$$

## Forward vs. inverse warping

Q: which is better?

A: usually inverse—eliminates holes

- however, it requires an invertible warp function—not always possible...

## Other types of mosaics



Can mosaic onto *any* surface if you know the geometry

- See NASA's [Visible Earth project](http://earthobservatory.nasa.gov/Newsroom/BlueMarble/) for some stunning earth mosaics
- <http://earthobservatory.nasa.gov/Newsroom/BlueMarble/>

## Summary

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### Things to take home from this lecture

- Image alignment
- Image reprojection
  - homographies
  - cylindrical projection
- Radial distortion
- Creating cylindrical panoramas
- Image blending
- Image warping
  - forward warping
  - inverse warping
  - bilinear interpolation