CSE446 Machine Learning, Winter 2015: Homework 1

Due: Monday, January 28^{th}, beginning of class

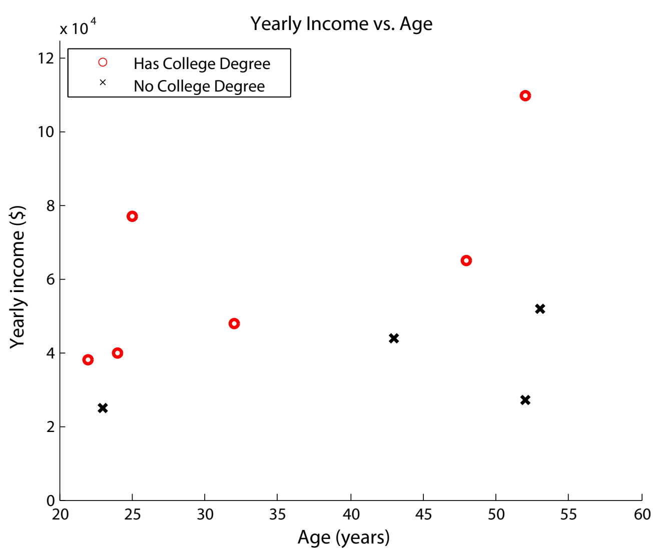
Start Early! Also, typed solutions (specifically those in LaTeX) are preferred to hand-written solutions. Any illegible solutions will be counted wrong at the sole discretion of the grader. Please feel free to use the homework document as a template, putting your solutions inline. Please include your code for Questions 1 and 4.

1 Decision Trees [32 points]

For the first two problems, it would be helpful for you to draw the decision boundary of your learned tree in the figure.

1. *(16 points)* Consider the problem of predicting if a person has a college degree based on age and salary. The table and graph below contain training data for 10 individuals.

Age	Salary ($)	College Degree
24	40,000	Yes
53	52,000	No
23	25,000	No
25	77,000	Yes
32	48,000	Yes
52	110,000	Yes
22	38,000	Yes
43	44,000	No
52	27,000	No
48	65,000	Yes



Program a decision tree for classifying whether a person has a college degree by greedily choosing threshold splits that maximize information gain, as described in class. Draw your tree and include the information gain at each split. How did you decide how to prune your tree? Include your code with the submission.

2. (12 points) A multivariate decision tree is a generalization of univariate decision trees, where more than one attribute can be used in the decision rule for each split. That is, splits need not be orthogonal to a feature's axis.

   For the same data, learn a multivariate decision tree where each decision rule is a linear classifier that makes decisions based on the sign of $\alpha x_{age} + \beta x_{income} - 1$.

   Draw your tree, including the α, β and the information gain for each split. Include your code with the submission.

3. (4 points) Multivariate decision trees have practical advantages and disadvantages. List an advantage and a disadvantage multivariate decision trees have compared to univariate decision trees.

2 MLE [20 points]

This question uses a discrete probability distribution known as the Poisson distribution. A discrete random variable X follows a Poisson distribution with parameter λ if

$$\Pr(X = k) = \frac{\lambda^k}{k!} e^{-\lambda} \qquad k \in \{0, 1, 2, \dots\}$$

You are a warrior in Peter Jackson's The Hobbit: Battle of the Five Armies. Because Peter decided to make his battle scenes as legendary as possible, he's decided that the number of orcs that will die with one swing of your sword is Poisson distributed (i.i.d) with parameter λ. You swing your sword eight times in the scene. Later, you go see the movie in theaters and record the number of orcs slain during each swing of your sword:

Sword Swing	1	2	3	4	5	6	7	8
Orcs Slain	6	4	2	7	5	1	2	5

Let $G = (G_1, \dots, G_n)$ be a random vector where G_i is the number of orcs slain on swing i:

1. *(6 points)* Give the log-likelihood function of G given λ.
2. *(8 points)* Compute the MLE for λ in the general case.
3. *(6 point)* Compute the MLE for λ using the observed G.

3 Regularization Constants [16 points]

We have discussed the importance of regularization as a technique to avoid overfitting our models. For linear regression, we could use LASSO (which uses the L_1 norm as a penalty), or ridge regression (which uses the squared L_2 norm as a penalty). In practice, the scaling factor of these penalties has a significant impact on the behavior of these methods, and must often be chosen empirically for a particular dataset. In this problem, we look at what happens when we choose our regularization factor poorly.

For the following, recall that the learning objective under ridge regression is

$$E_R = \sum_{i=1}^{n} (y_i - (\hat{w}_0 + x^{(j)}\hat{w}))^2 + \lambda \|\hat{w}\|_2^2$$

where

$$\lambda \|\hat{w}\|_2^2 = \lambda \sum_{i=1}^{d} (\hat{w}_i)^2 \tag{1}$$

and λ is our regularization constant.

The loss function to be optimized under LASSO regression is

$$E_L = \sum_{i=1}^{n}(y_i - (\hat{w}_0 + x^{(j)}\hat{w}))^2 + \lambda\|\hat{w}\|_1$$

where

$$\lambda\|\hat{w}\|_1 = \lambda \sum_{i=1}^{d} |\hat{w}_i|. \tag{2}$$

1. *(16 points)* Discuss briefly how choosing too small a λ affects the magnitude of the following quantities. Please describe the effects for both ridge and LASSO, or state why the effects will be the same.
   (a) The error on the training set.
   (b) The error on the testing set.
   (c) The elements of $\hat{w}$.
   (d) The number of nonzero elements of $\hat{w}$.
2. *(8 points)* Now discuss briefly how choosing too large a λ affects the magnitude of the same quantities in the previous question. Again describe the effects for both ridge and LASSO, or state why the effects will be the same.

4 Regression, Regularization, and Cross-Validation [40 points]

Ridge regression is the problem of solving

$$\arg\min_{\mathbf{w},w_0} \sum_i (\mathbf{X_i}\mathbf{w} + w_0 - \mathbf{y}_i)^2 + \lambda \sum_j \mathbf{w}_j^2 \tag{3}$$

Here $\mathbf{X}$ is an $N \times d$ matrix of data, and $\mathbf{X_i}$ is the i-th row of the matrix. $\mathbf{y}$ is an $N \times 1$ vector of response variables, $\mathbf{w}$ is a d dimensional weight vector, w_0 is a scalar offset term, and λ is a regularization tuning parameter.
This question contains two learning objectives,

1. Understand the effects of L2 regularization on training and test set errors
2. Use cross-validation to pinpoint the optimal regularization coefficient

You may use any language for your implementation, but we recommend Python. Python is a very useful language, and you should find that Python achieves reasonable enough performance for this problem. You may use common computing packages (such as NumPy or SciPy).

With the exception of computing objective values or initial conditions, the only matrix operations required are adding vectors, multiplying a vector by a scalar, and computing the dot product between two vectors. Try to use as much vector/matrix computation as possible.
Finally here are some pointers toward useful parts of Python:

- `numpy`, `scipy.sparse`, and `matplotlib` are useful computation packages.
- For storing sparse matrices, the `scipy.sparse.csc_matrix` (compressed sparse column) format is fast for column operations.
- Important note for numpy users, `scipy.sparse.csc_matrix` uses matrix semantics instead of `numpy.ndarray` operations. Specifically, the * operation is matrix multiplication instead of the elementwise product.

- If you're new to Python but experienced with Matlab, consider reading NumPy for Matlab Users at `http://wiki.scipy.org/NumPy_for_Matlab_Users`.

This is just our personal advice. We will be able to provide better support for Python users. If you would prefer to use another language, however, that is fine as well.
Recently Yelp held a recruiting competition on the analytics website Kaggle. Check it out at `http://www.kaggle.com/c/yelp-recruiting`. As a side note, browsing other competitions on the website can give you ideas of some cool applications for machine learning!

For this competition, the task is to predict the number of useful upvotes a particular review will receive.

For many Kaggle competitions (and machine learning methods in general), one of the most important requirements for doing well is the ability to discover great features. From an initial list of 1000 features, we have extracted the 100 most relevant for the purpose of this assignment.
Yelp provides a variety of data, such as the review's text, date, and restaurant, as well as data pertaining to each business, user, and check-ins. This information has already been preprocessed for you into the following files:

`upvote_data_100.csv`	Data matrix for predicting number of useful votes
`upvote_labels.txt`	List of useful vote counts for each review
`upvote_features_100.txt`	Names of each feature for interpreting results

For each task, data files contain data matrices, while labels are stored in separate text files. To get you started, the Python following code should load the data:

```
import numpy as np
import scipy.sparse as sp

# Load a text file of integers:
y = np.loadtxt("upvote_labels.txt", dtype=np.int)

# Load a text file of strings:
featureNames = open("upvote_features_100.txt").read().splitlines()

# Load a csv of floats:
A = sp.csc_matrix(np.genfromtxt("upvote_data_100.csv", delimiter=","))
```

Note: You don't have to do use a SciPy sparse matrix for this, but it might make your matrix operations far quicker. Also when using the closed-form solution of linear regression, you must include the bias in your training matrix. Add a column of 1's to the front of the matrix
For this part of the problem, you have the following tasks:

1. *(10 points)* Use the closed-form solution of linear regression to predict the number of useful votes a Yelp review will receive. Use the first 5000 samples for training, and the remaining 1000 samples for testing. Record the RMSE for your training set and test set.

2. *(10 points)* Now let's look at how regularization changes our training and test set errors. Starting at $\lambda = 1$, run ridge regression (using the closed-form solution) on the training set, decreasing λ by 25% 20 times. Use 5-fold cross-validation to test for your optimal λ. For each λ, record the root-mean-squared-error (RMSE) of the entire training set (all 5000 samples) and cross-validation error.

   Plot the RMSE values together on a plot against λ. Which is the optimal λ? Why? Apply your model using the optimal λ on the test set. What RMSE value do you obtain?

3. *(10 points)* Now repeat part 2, using 10-fold cross-validation.

4. *(10 points)* Now split your 5000 training examples into two sets. The first 4000 are your new training set. The final 1000 are a validation set. Repeat part 2 using a validation set as opposed to cross-validation. Describe and explain any differences in performance you see when using a validation set vs. k-fold cross-validation.