Reinforcement Learning

Machine Learning – CSE446

Carlos Guestrin
University of Washington

June 5, 2013

The Reinforcement Learning task

World: You are in state 34.
Your immediate reward is 3. You have possible 3 actions.

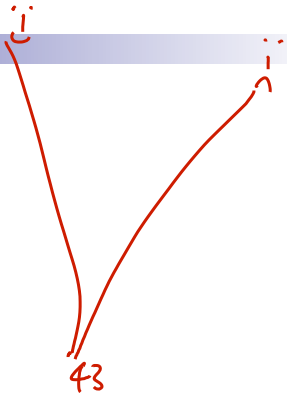
Robot: I'll take action 2.
World: You are in state 77.
Your immediate reward is -7. You have possible 2 actions.

Robot: I'll take action 1.
World: You're in state 34 (again).
Your immediate reward is 3. You have possible 3 actions.

Formalizing the (online) reinforcement learning problem

- Given a set of states **X** and actions **A**
  - in some versions of the problem size of **X** and **A** unknown
- Interact with world at each time step *t*:
  - world gives state $\mathbf{x}_t$ and reward r_t
  - you give next action $\mathbf{a}_t$
- **Goal**: (quickly) learn policy that (approximately) maximizes long-term expected discounted reward

The "Credit Assignment" Problem



I'm in state 43,	reward = 0, action = 2
" " " 39,	" = 0, " = 4
" " " 22,	" = 0, " = 1
" " " 21,	" = 0, " = 1
" " " 21,	" = 0, " = 1
" " " 13,	" = 0, " = 2
" " " 54,	" = 0, " = 2
" " " 26,	" = 100,

Yippee! I got to a state with a big reward! But which of my actions along the way actually helped me get there??

This is the Credit Assignment problem.

Exploration-Exploitation tradeoff

- You have visited part of the state space and found a reward of 100
  - is this the best I can hope for???
- **Exploitation**: should I stick with what I know and find a good policy w.r.t. this knowledge?
  - at the risk of missing out on some large reward somewhere
- **Exploration**: should I look for a region with more reward?
  - at the risk of wasting my time or collecting a lot of negative reward

Two main reinforcement learning approaches

- Model-based approaches:
  - explore environment, then learn model ($P(\mathbf{x}'|\mathbf{x},\mathbf{a})$ and $R(\mathbf{x},\mathbf{a})$) (almost) everywhere
  - use model to plan policy, MDP-style
  - approach leads to strongest theoretical results
  - works quite well in practice when state space is manageable
- Model-free approach:
  - don't learn a model, learn value function or policy directly
  - leads to weaker theoretical results
  - often works well when state space is large

Rmax – A model-based approach

Given a dataset – learn model

Given data, learn (MDP) Representation:

- Dataset: $x_t, a_t \rightarrow r_t, x_{t+1}$
- Learn reward function:
  - R(**x**,**a**) : $R(x_t, a_t) = r_t$
- Learn transition model:
  - P(**x'**|**x**,**a**) ← counts like BNs

next state, current state, MLE

$$\frac{Count(x', x, a) \text{ action} = 1}{Count(x, a) \text{ action} = 1}$$

next state = 43, cur state = 32, current state = 32

43 → 32 → 43 → 31 . . .

a 1 2 2 1 . . .

Planning with insufficient information

- Model-based approach:
  - estimate R(**x**,**a**) & P(**x**'|**x**,**a**)
  - obtain policy by value or policy iteration, or linear programming
  - No credit assignment problem!
    - learning model, planning algorithm takes care of "assigning" credit
- What do you plug in when you don't have enough information about a state?
  - don't reward at a particular state
    - plug in 0?
    - plug in smallest reward (R_{min})?
    - plug in largest reward (R_{max})?
  - don't know a particular transition probability?

Some challenges in model-based RL 2: Exploration-Exploitation tradeoff

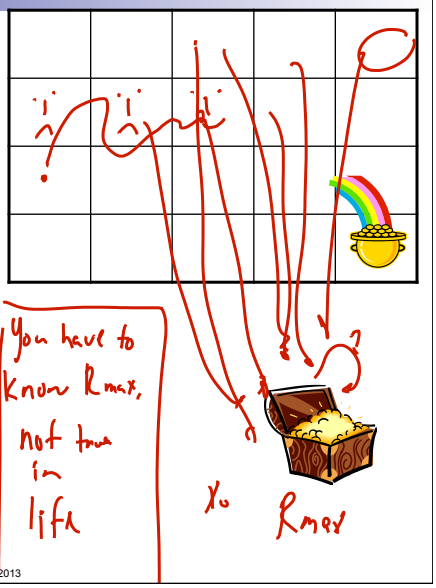
- A state may be very hard to reach
  - waste a lot of time trying to learn rewards and transitions for this state
  - after a much effort, state may be useless
- A strong advantage of a model-based approach:
  - you know which states estimate for rewards and transitions are bad
  - can (try) to plan to reach these states
  - have a good estimate of how long it takes to get there

A surprisingly simple approach for model based RL – The Rmax algorithm [Brafman & Tennenholtz]

- **Optimism in the face of uncertainty!!!!**
  - heuristic shown to be useful long before theory was done (e.g., Kaelbling '90)
- If you don't know reward for a particular state-action pair, set it to R_{max}!!!
- If you don't know the transition probabilities P(**x'**|**x**,**a**) from some some state action pair **x**,**a** assume you go to **a magic, fairytale** new state $\mathbf{x}_0$!!!
  - $R(\mathbf{x}_0,\mathbf{a}) = R_{max}$
  - $P(\mathbf{x}_0|\mathbf{x}_0,\mathbf{a}) = 1$

Understanding R_{max}

- With R_{max} you either:
  - **explore** – visit a state-action pair you don't know much about
    - because it seems to have lots of potential
  - **exploit** – spend all your time on known states
    - even if unknown states were amazingly good, it's not worth it
- Note: you never know if you are exploring or exploiting!!!



Implicit Exploration-Exploitation Lemma

- **Lemma**: every T time steps, either:
  - **Exploits**: achieves near-optimal reward for these T-steps, or
  - **Explores**: with high probability, the agent visits an unknown state-action pair
    - learns a little about an unknown state
  - T is related to *mixing time* of Markov chain defined by MDP
    - time it takes to (approximately) forget where you started

The Rmax algorithm

- **Initialization**:
  - Add state $\mathbf{x}_0$ to MDP
  - $R(\mathbf{x},\mathbf{a}) = R_{max}$, $\forall \mathbf{x},\mathbf{a}$
  - $P(\mathbf{x}_0|\mathbf{x},\mathbf{a}) = 1$, $\forall \mathbf{x},\mathbf{a}$
  - all states (except for $\mathbf{x}_0$) are **unknown**
- Repeat
  - obtain policy for current MDP and Execute policy
  - for any visited state-action pair, set reward function to appropriate value
  - if visited some state-action pair **x**,**a** enough times to estimate $P(\mathbf{x}'|\mathbf{x},\mathbf{a})$
    - update transition probs. $P(\mathbf{x}'|\mathbf{x},\mathbf{a})$ for **x**,**a** using MLE
    - recompute policy

Visit enough times to estimate P(**x'**|**x**,**a**)?

- How many times are enough?
  - use Chernoff Bound!
- **Chernoff Bound**:
  - $X_1,\ldots,X_n$ are i.i.d. Bernoulli trials with prob. θ
  - $P(|1/n \sum_i X_i - \theta| > \varepsilon) \leq \exp\{-2n\varepsilon^2\}$

Putting it all together

- **Theorem**: With prob. at least 1-δ, Rmax will reach a ε-optimal policy in time polynomial in: num. states, num. actions, T, $1/\varepsilon$, $1/\delta$
  - Every T steps:
    - achieve near optimal reward (great!), or
    - visit an unknown state-action pair ! num. states and actions is finite, so can't take too long before all states are known

What you need to know about RL…

- Neither supervised, nor unsupervised learning
- Try to learn to act in the world, as we travel states and get rewards
- Model-based & Model-free approaches
- Rmax, a model based approach: ← learn through optimism
  - Learn model of rewards and transitions
  - Address exploration-exploitation tradeoff
  - Simple algorithm, great in practice

Closing….

What you have learned this quarter

- Learning is function approximation
- Point estimation
- Regression
- LASSO
- Subgradient
- Stochastic gradient descent
- Coordinate descent
- Discriminative v. Generative learning
- Naïve Bayes
- Logistic regression
- Bias-Variance tradeoff
- Decision trees
- Cross validation
- Boosting
- Instance-based learning
- Perceptron
- SVMs
- Kernel trick
- PAC learning
- Bayes nets
  - representation, parameter and structure learning
- K-means
- EM
- Mixtures of Gaussians
- Dimensionality reduction, PCA
- MDPs
- Reinforcement learning

BIG PICTURE

- Improving the performance at some task though experience!!! ☺
  - before you start any learning task, remember the fundamental questions:

What is the learning problem?

From what experience?

What model?

What loss function are you optimizing?

With what optimization algorithm?

Which learning algorithm?

With what guarantees?

How will you evaluate it?

You have done a lot!!!

- And (hopefully) learned a lot!!!
  - Implemented
    - LASSO
    - LR
    - Perceptron
    - Clustering
    - …
  - Answered hard questions and proved many interesting results
  - Completed (I am sure) an amazing ML project
  - And did excellently on the final!

Thank You for the Hard Work!!!