This exam is an open book exam. You have 1 hour 50 minutes; budget time carefully. Intermediate steps are rarely required but often useful for partial credit. Good luck!
1 SQL

1. (15 points)
Consider the following Flickr-type database:

Users(uid, name)
Picture(pid, owner, size)
Comment(cid, auth, pict, text)

Where:

- User.uid, Picture.pid, Comment.cid are keys.
- Picture.owner, Comment.auth are foreign keys into Users.
- Comment.pict is a foreign key to Picture.

(a) (5 points) Write a SQL query that counts, for every user, the number of other users who have commented on their pictures. That is, for each user you need to compute the total number of distinct users (including herself) who have commented on any of her pictures. Your answer should return only those answers where the count is ≥ 1 (i.e. you don’t need to return users that have no comments on any of their pictures).

Solution:

```sql
select x.owner, count(distinct y.auth)
from Picutre x, Comment y
where x.pid = y.pict and x.owner != y.auth
```
Users(uid, name)
Picture(pid, owner, size)
Comment(cid, auth, pict, text)

(b) (5 points) A spammer is a user who comments on all pictures that are not owned by him (her). Write a SQL query that returns all spammers.

Solution:
select *
from Users x
where not exists
    select *
    from Picture y
    where y.owner != x.uid and
    not exists select *
        from Comment z
    where x.uid = z.auth and z.pic = y.pid
(c) (5 points) Two users are *friends* if they comment on each others’ pictures: that is, \( x, y \) are friends if \( x \) made a commented on some picture of \( y \), and \( y \) made a comment on some picture of \( x \). Write a SQL query that computes, for each user, the number of his/her friends. Your query should return only those answers where the count is \( \geq 1 \).

**Solution:**

```sql
select x.uid, x.name, count(*)
from Users x, Users y
where exists (select *
    from Picture u, Comment v
    where u.owner = x.uid and v.auth=y.uid
    and v.pict = u.pid)
    and exists ... symmetrically
group by x.uid, x.name
```
2 Transactions

2. (20 points)
Consider a concurrency control manager by timestamps. Below are several sequences of events, including start events, where sti means that transaction Ti starts and coi means Ti commits. These sequences represent real time, and the timestamp-based scheduler will allocate timestamps to transactions in the order of their starts. In each case below, say what happens with the last request.

You have to choose between one of the following four possible answers:

1. the request is accepted,
2. the request is ignored,
3. the transaction is delayed,
4. the transaction is rolled back.

(a) (4 points) st1; st2; st3; r1(A); w1(A); r2(A);
   The system will perform the following action for r2(A): ____________________________

   Solution: The system will perform the following action for r2(A): delayed.

(b) (4 points) st1; st2; r2(A); co2; r1(A); w1(A)
   The system will perform the following action for w1(A): ____________________________

   Solution: The system will perform the following action for w1(A): rolled back.

(c) (4 points) st1; st2; st3; r1(A); w2(A); w3(A); r2(A);
   The system will perform the following action for r2(A): ____________________________

   Solution: The system will perform the following action for r2(A): rolled back.

(d) (4 points) st1; st2; r1(A); r2(A); w1(B); w2(B);
The system will perform the following action for w2(B): ________________

**Solution:** The system will perform the following action for w2(B): accepted.

(e) (4 points) st1; st2; st3; r1(A); w3(A); co3; r2(B); w2(A)

The system will perform the following action for w2(A): ________________

**Solution:** The system will perform the following action for w2(A): ignored.
3 Conceptual Design

3. (10 points)

(a) (5 points) Decompose in BCNF the relation \( R(A, B, C, D, E) \) that satisfies the following functional dependencies. Show your steps, and show the keys in the decomposed relations.

\[ A \rightarrow B \]
\[ CD \rightarrow E \]

Solution:

\[ A^+ = AB \]

Decompose into \( R_1 = AB \), \( R_2 = ACDE \) Continue with

\[ CD^+ = CDE \]

Decompose \( R_2 \) into \( R_3 = CDE \) and \( R_4 = CDA \)
(b) (5 points) For each of the statements below, indicate whether they are true or false. You do not need to justify your answers.

- Every relation with only two attributes is in BCNF.
- If $X$ and $Y$ are super-keys then $X \cup Y$ is also a super-key.
- If $X$ and $Y$ are super-keys then $X \cap Y$ is also a super-key.
- If $X \rightarrow A$ and $Y \rightarrow A$, then $(X \cap Y) \rightarrow A$.
- If $X^+ = X$ and $Y^+ = Y$ then $(X \cap Y)^+ = (X \cap Y)$. 
4 Indexes

4. (10 points)

Consider the following database about word occurrences in Webpages:

Webpage(url, author)
Occurs(url, wid)
Word(wid, text, language)

where:

- Webpage.url and Word.wid are keys.
- Occurs.url and Occurs.wid are foreign keys to Webpage and Word respectively.

Assume the following statistics

\[
T(\text{Webpage}) = V(\text{Occurs}, \text{url}) = 10^9
\]
\[
T(\text{Occurs}) = 10^{12}
\]
\[
T(\text{Word}) = V(\text{Occurs}, \text{wid}) = 10^6
\]
\[
V(\text{Webpage}, \text{author}) = 10^7
\]
\[
V(\text{Word}, \text{language}) = 100
\]

Assume ten records can be fit in one block, hence \( B(\text{Webage}) = \frac{T(\text{Webpage})}{10} \) and similarly for all other tables.

(a) (5 points) Consider the following plan:

\[
(\sigma_{\text{author}='\text{John}'}(\text{Webpage}) \bowtie_{\text{url}=\text{url}} \text{Occurs}) \\
\bowtie_{\text{wid}=\text{wid}} \text{main-memory-hash-join} \sigma_{\text{language}='\text{French}'}(\text{Word})
\]

Compute the cost of the plan in each of the following cases:
1. We have the following indexes:

\[
\begin{align*}
\text{Webpage.url} &= \text{primary index} \\
\text{Webpage.author} &= \text{secondary index} \\
\text{Occurs.url} &= \text{secondary index} \\
\text{Occurs.wid} &= \text{primary index} \\
\text{Word.wid} &= \text{primary index} \\
\text{Word.language} &= \text{secondary index}
\end{align*}
\]

Solution:

\[
\begin{align*}
\text{Unclustered Index Lookup on Author} &= 10^2 \\
\text{Unclustered Index Join} &= 10^2 \times 10^3 \\
\text{Unclustered Index Lookup on Language} &= 10^4 \\
\text{Main Memory Hash Join} &= 0
\end{align*}
\]

Total = $10^2 + 10^5 + 10^4$

2. We have the following indexes:

\[
\begin{align*}
\text{Webpage.url} &= \text{secondary index} \\
\text{Webpage.author} &= \text{primary index} \\
\text{Occurs.url} &= \text{primary index} \\
\text{Occurs.wid} &= \text{secondary index} \\
\text{Word.wid} &= \text{secondary index} \\
\text{Word.language} &= \text{primary index}
\end{align*}
\]

Solution:

\[
\begin{align*}
\text{Clustered Index Lookup on Author} &= 10 \\
\text{Clustered Index Join} &= 10^2 \times 10^2 \\
\text{Clustered Index Lookup on Language} &= 10^3 \\
\text{Main Memory Hash Join} &= 0
\end{align*}
\]

Total = $10 + 10^4 + 10^3$
(b) (5 points) Consider the following plan:

\[(\text{Webpage} \Join_{\text{url}=\text{url}} \text{Occurs}) \Join_{\text{wid}=\text{wid}} \text{Word}\]

Choose a set of indexes that minimizes the total number of disk I/O’s for the plan. Each index should be on a single attribute, and you can choose as many (or as few) indexes as you want. Indicate for each index if it is primary or secondary. For each merge-join we assume to have sufficient main memory to complete it in two pass. (There are no index-join operators in this plan: you need to figure out how indexes can help at all in this query plan.)

**Solution:** Primary Index on Webpage(url)
Primary Index on Occurs(url)
Primary Index on Word(wid)

These clustered indexes allow the two-pas merge join to skip the initial sorting step for each of the tables.
5 Query Optimization

5. (10 points)
For the following questions, we consider the schema \( R(A, B), S(C, D), T(E, F), U(G, H) \).

(a) (5 points) Write a logical plan for the following query:

\[
\text{select } R.A, \text{ sum}(T.F) \\
\text{from } R, S, T \\
\text{where } R.B = S.C \text{ and } S.D = T.E \\
group \text{ by } R.A \\
\text{having } \text{count}(*) > 20
\]

```
\( \Pi \ a, c \) \\
\( \sigma \ s > 20 \) \\
\( \forall \ \ R.A \text{ as } a, \text{ Count}(*) \text{ as } c, \text{ Sum}(T.F) \text{ as } s \) \\
\( \bowtie \ S.D = T.E \) \\
\( \bowtie \ R.B = S.C \) \\
R \\
S \\
T
```

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(b) (5 points) Consider the following query:

```
select *
from R, S, T, U
where R.B = S.C and S.D = T.E and S.D = U.G
```

- Write all left-deep, cartesian-free join trees for this query. Assume that the join operator is not commutative: that is, you should report both $R \bowtie S$ and $S \bowtie R$ as distinct plans. Note: you need to turn in several plans.

Solution:

\[
\begin{align*}
R & \bowtie S \bowtie T \bowtie U \\
R & \bowtie S \bowtie U \bowtie T \\
S & \bowtie R \bowtie T \bowtie U \\
S & \bowtie R \bowtie U \bowtie T \\
S & \bowtie T \bowtie R \bowtie U \\
S & \bowtie T \bowtie U \bowtie R \\
S & \bowtie U \bowtie R \bowtie T \\
S & \bowtie U \bowtie T \bowtie R \\
T & \bowtie S \bowtie R \bowtie U \\
T & \bowtie S \bowtie U \bowtie R \\
T & \bowtie U \bowtie S \bowtie R \\
U & \bowtie S \bowtie R \bowtie T \\
U & \bowtie S \bowtie T \bowtie R \\
U & \bowtie T \bowtie S \bowtie R 
\end{align*}
\]
• Write a full semijoin reducer for this query.

Solution:

\[
\begin{align*}
S' &= S \bowtie T \\
S'' &= S' \bowtie U \\
S''' &= S'' \bowtie R \\
R' &= R \bowtie S''' \\
T' &= T \bowtie S''' \\
U' &= U \bowtie S''' \\
Q &= S''' \bowtie T' \bowtie U' \bowtie R'
\end{align*}
\]
6. (10 points)

Consider the relations \( R(A,B) \), \( S(C,D) \), \( T(E,F) \) and the following histograms on \( R.A \) and \( T.F \):

<table>
<thead>
<tr>
<th>( R.A )</th>
<th>0...999</th>
<th>1000...1999</th>
<th>2000...2999</th>
<th>3000...3999</th>
<th>4000...4999</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( 10^4 )</td>
<td>( 2 \cdot 10^4 )</td>
<td>( 3 \cdot 10^4 )</td>
<td>( 3 \cdot 10^4 )</td>
<td>( 2 \cdot 10^4 )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( T.F )</th>
<th>0...2499</th>
<th>2500...2699</th>
<th>2700...3999</th>
<th>4000...7999</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( 10^4 )</td>
<td>( 10^4 )</td>
<td>( 10^4 )</td>
<td>( 10^4 )</td>
</tr>
</tbody>
</table>

(a) (1 point) What kind of histogram is \( R.A \) and what kind of histogram is \( T.F \) ?

(b) (4 points) Estimate the number of tuples returned by \( \sigma_{500 \leq A \leq 3499}(R) \). Show your work for partial credit.

(c) (5 points) Estimate the number of tuples returned by the following query:

```sql
```

assuming the two histograms above, plus the following statistics:

\[
\begin{align*}
T(R) & = 10^5 & T(S) & = 6 \cdot 10^6 & T(T) & = 4 \cdot 10^5 \\
V(R,B) & = V(S,C) = 3 \cdot 10^3 \\
V(S,D) & = V(T,E) = 2 \cdot 10^4
\end{align*}
\]

Show your work for partial credit.
7 Parallel Databases

7. (15 points)
Consider the following relations $R(A, B), S(C, D)$ and the following query:

```sql
select R.A, sum(S.D)
from R, S
where R.B = S.C
group by R.A
```

(a) (5 points) Write the query in Pig Latin.

(b) (5 points) How many map-reduce tasks does Pig Latin require to evaluate the query?

(c) (5 points) Give the total I/O cost of your Pig Latin program, as a function of $B(R), B(S), B(R \bowtie S)$, and $P$ (the number of processors). Assume that each processor’s main memory is $\geq \max(B(R), B(S), B(R \bowtie S))/P$. Note: your analysis should be based on the actual implementation of map-reduce, and should not assume any other optimizations.
8 Bloom Filters

8. (10 points)

(a) (1 point) Is the following statement true or false? Bloom filters are used to improve cardinality estimation.

(a) ___________

Solution: FALSE. Bloom filter is used for reducing communication costs between two nodes in a distributed system.

(b) (2 points) A Bloom filter using a hash map of 1k Bytes has a false positive error rate of 19%. In order to improve the error rate, the systems administrator decides to double the size of the hash map to 2k Bytes, and, at the same time, to double the number of hash functions used by the Bloom filter. What is the new false positives error rate?

(b) ___________

The new false positive error rate is:

Solution: We are given that \([1 - \exp(-kn/m)]^k = .19\). If \(m\) and \(k\) are now doubled, we have:

\[
[1 - \exp(-2kn/2m)]^{2k} = [1 - \exp(-kn/m)]^{2k} = [(1 - \exp(-kn/m))^2] = .19^2 = .0361
\]

Thus the new false positive rate is 3.61%.

(c) (2 points) A regular hash map of 1k Bytes has a false positive error rate of 19%. In order to improve this rate, the systems administrator decides to double the size of the hash map to 2k Bytes. What is the new false positive error rate?

(c) ___________

The new false positive error rate is:

Solution: We are given that \(1 - \exp(-n/m) = .19\). So \(\exp(-n/m) = .81 \Rightarrow \exp(-n/2m) = \sqrt{.81} = .9\). Thus the new false positive rate is \(1 - \exp(-n/2m) = 1 - .9 = .1\).
(d) (5 points) Data supplier S1 has \( n = 1M (= 10^6) \) documents. Data supplier S2 has also \( n = 1M \) documents. Each document has 1k bytes. They have 50 documents in common and they want to compute these. They will proceed as follows:

- S1 computes a hash map \( M \) with \( cn \) bits, where \( c=8 \) and sends it to S2
- S2 checks its items in \( M \) and sends all matches to S1
- S1 computes the result and sends the matching 50 documents to S2

Indicate the total number of bytes transferred over the network in each step assuming (a) the hash map is a standard hash table, (b) the hash map is a bloom filter.

**Solution:** There are three parts to the cost:

1. S1 computes a hashtable / bloom filter and sends it to S2. This takes \( cn \) bits = 8M bytes = 1MB.

2. S2 sends back documents that is positive when matched against the hashtable / bloom filter. The number of documents is 50 + (number of false positives).

   - For hash table, the false positive rate is \( 1 - \exp(-n/m) = 1 - \exp(-n/8n) = 1 - \exp(-1/8) \), or 11%. So there are 110,000 false positive documents, making the total number of documents 110,050 and size 110,050KB.

   - For bloom filter, the false positive rate is \( (1 - \exp(-k/8))^k \).

   Assuming optimal number of hash functions, the false positive rate is \( 2^{(-8ln2)} \), or 2%. So there are 20,000 false positive documents, making the total 20,050 and size 20,050KB.

3. S1 computes and sends the 50 common documents back to S2. This takes 50KB.

So for hash table, the total cost is \( 1MB + 110.05MB + 0.05MB = 111.10MB \). For bloom filter, the total cost is \( 1MB + 20.05MB + 0.05MB = 21.20MB \).

(Full credit was given if second step was left in terms of exponentials or in terms of \( k \) if optimal bloom filter was not assumed.)